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CONTENTS

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ECONOPHYSICS Section

NEW ECONOMY Section

ECONOPHYSICS Section

CONSISTENT AND INCONSISTENT WAYS TO DYNAMIZE THE NEO-CLASSICAL THEORY

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Abstract. *With equal profit functions, the static neo-classical theory of a firm and its dynamization by dynamic optimization give equal results. Thus without additional assumptions, dynamic optimization does not dynamize the theory. In the existing models, the solution paths obtained by dynamic optimization converge to the static optimum only under very restricting initial conditions. On the other hand, the static neoclassical theory of a consumer and its dynamization by dynamic optimization explain different quantities: the real consumption of various goods and the money allocated for consumption over time. Thus the dynamic neo-classical micro theory obtained by dynamic optimization is either inconsistent with the static one, or models a different quantity. The neo-classical framework is then lacking consistent static and dynamic theories of a firm and a consumer, and here we introduce such. We define the "economic forces" acting upon the production of a firm and the consumption of a consumer, and show that these dynamic theories analogous to Newtonian mechanics are consistent with the static neo-classical ones; the latter correspond to zero-force situations in the former. The proposed theories are expressed with measurable quantities and they explain economic growth too. (JEL D11, D21, D91).*

Keywords: *Economic dynamics, Neo-classical theory, Economic force, Newtonian economics.*

1. Introduction

According to Mirowski (1989a), the progenitors of neo-classical economics imitated classical mechanics. For example, the concept of equilibrium was introduced in economics from physics by Canard at 1801 (Mirowski 1989b). Even though equilibrium is a "balance of forces" situation, in economics the balancing forces have not been defined. In spite of this, the term "force" is common in economics, see Lucas (1988). The "invisible hand" by Adam Smith is an example of how the concept of "force field" has been applied in economics.

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Fisher (1983 pp. 9-12) writes: "... I now briefly consider the feature that a proper theory of disequilibrium adjustment should have ... under what conditions the rational behaviour of individual agents drives an economy to equilibrium. ... Such a theory must involve dynamics with adjustment to disequilibrium over time modelled. ...the most satisfactory situation would be one in which the equations of motion of the system permitted an explicit solution ... as known functions of time. ...the theory of the household and the firm must be reformulated ... to allow agents to perceive that the economy is not in equilibrium and to act on that perception. ... a satisfactory underpinning for equilibrium analysis must be a theory in which the adjustments to disequilibrium made by agents are made optimally."

We follow Fisher in proposing a new framework for modelling in economics. We believe that the *"willingness of economic agents to better their situation"* is the fundamental cause of economic dynamics, and we demonstrate the applicability of this framework in modelling the behaviour of a firm and a consumer. Our approach offers several advantages as compared with the existing way to dynamize the static neo-classical theory by dynamic optimization: 1) The dynamic theories obtained by dynamic optimization are inconsistent with the corresponding static ones. This means that both of them cannot be accepted simultaneously. On the other hand, static neo-classical theory is a special case in our modelling – the zero-force situation – and our framework covers both static and dynamic situations. 2) Our framework does not require dynamic optimization, and 3) it covers also cases where an optimum does not exist. Thus, for example, permanent growth can be modelled in our framework.

2. The Static and the Dynamic Neo-classical Theory of a Firm

We study the dynamization of the neo-classical theory of a firm by dynamic optimization according to Evans (1924), because other such models share the same problems, see e.g. Jorgenson (1963). Evans assumes the cost and demand functions for a monopoly firm as

$$C(\mathrm{q}_\mathrm{s}) = Aq_s^2 + Bq_s + C_0, \;\; q_d = ap + b + hp'(t), \qquad (1)$$

where q_s is the amount of production of the firm in a time unit, q_d the demand of the product of the firm at the time unit, p the price of the product, A, B, C_0, b are positive constants while a is negative. The sign of

constant h is left open. Assuming $q_s = q_d = q,$ that is the whole production gets sold, the profit function becomes the following:

$$\pi(p, p'(t)) = pq - C(q) =$$

$$= p(ap + b + hp'(t)) - A(ap + b + hp'(t))^2 - B(ap + b + hp'(t)) - C_0. \quad (2)$$

Assuming $h = 0,$ we get the corresponding static optimization problem with the solution:

$$\frac{\partial \pi}{\partial \pi} = 0 \Leftrightarrow p^* = \frac{b - 2aAb - aB}{2a(aA - 1)} \Leftrightarrow q^* = ap^* + b = \frac{b + aB}{2(1 - aA)}. \quad (3)$$

The corresponding dynamic optimization problem is following:

$$\max_{p(t), p'(t)} \int_0^{t_1} \pi(p(t), p'(t))dt, \;\; p(0) = p_1, \;\; p(t_1) = p_2. \quad (4)$$

The Euler equation of this problem gives the following differential equation

$$\frac{\partial \pi}{\partial p} - \frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial p'(t)}\right) = 0 \Leftrightarrow 2Ah^2 p''(t) + 2a(1 - aA)p(t) = aB + 2aAb - b, \quad (5)$$

that has solution:

$$p(t) = p_0 + C_1 e^{kt} + C_2 e^{-kt}, \;\; p_0 = p^* = \frac{b - 2aAb - aB}{2a(aA - 1)}, \;\; k = \frac{\sqrt{a(aA - 1)}}{h\sqrt{A}},$$

$$C_1 = \frac{(p_2 - p_0) - (p_1 - p_0)e^{-kt_1}}{e^{kt_1} - e^{-kt_1}}, \;\; C_2 = \frac{(p_1 - p_0)\, e^{kt_1} - (p_2 - p_0)}{e^{kt_1} - e^{-kt_1}}. \quad (6)$$

Now, in thinking whether the static and the dynamic optimization problem are consistent with each other, we see firstly that if $h = 0$ in (5), the Euler equation reduces to that in (3). Thus if we apply the same profit function in the dynamic problem as in the static one, we get the same result: $p(t) = p^*, 0 < t < t_1.$ Dynamic optimization then does not give a differential equation to be solved unless a different profit function is assumed as in the static case, namely, $h \neq 0.$

On the other hand, the solution $p^* = p_0$ of the static problem in (3) is obtained as a special case of Eq. (6) by assuming $C_1 = C_2 = 0,$ i.e. $p_1 = p_2 = p_0.$ Another correspondence between the two solutions is obtained by assuming $C_1 = 0$ in (6) if $h > 0$ and $C_2 = 0$ if $h < 0.$ In this

case $\lim_{t\to\infty} p(t) = p_0$, i.e. the solution path of the dynamic problem converges to the static optimum, even though during time $(0, t_1)$ this convergence does not occur.

Thus the solution of the static problem is obtained as a special case from the dynamic problem only under restricting initial conditions that either degenerate the solution path to a fixed point $p(t) = p_0$, or restrict the solution path to be stable. In cases $C_1, k \neq 0, \lim_{t\to\infty} p(t) = \pm\infty$ depending on the sign of C_1 if $h > 0$, and on the sign of C_2 if $h < 0$. Thus in most cases the solution path of the dynamic problem is inconsistent with that of the static one.

2.1. Modelling the Present Value of Profit

The dynamic optimization problem of the current value of profit of a firm from time unit $(0, t_1)$ is

$$\max_{\mathrm{p(t)}} \int_0^{t_1} F(p(t), t)dt = \max_{\mathrm{p(t)}} \int_0^{t_1} e^{-r\,t}\pi(p(t))dt, \qquad (7)$$

where $e^{-r\,t}$ the discount factor with constant interest rate r. The necessary condition for (7) – together with possible boundary conditions – is the following Euler equation

$$\frac{\partial F}{\partial p(t)} - \frac{d}{dt}\left(\frac{\partial F}{\partial p'(t)}\right) = 0 \Leftrightarrow e^{-r\,t}\frac{\partial \pi}{\partial p(t)} = 0 \Leftrightarrow \frac{\partial \pi}{\partial p(t)} = 0.$$

The necessary condition of problem (7) equals with that in the static theory. Thus assuming present value of profit as the target function in the dynamic problem does not affect the result.

3. The Static and the Dynamic Neo-classical Theories of a Consumer

3.1. The Static Neo-classical Theory of a Consumer

We assume a consumer's decision-making as simple as possible like in all textbooks of elementary economics. The length of the time horizon is assumed to be one week, and the consumer is assumed to choose his/her weekly consumption of only two goods the consumer consumes every

week. For clarity, let good 1 be "food" and good 2 "playing video games" according to the traditional choice between food and fun. The consumer is assumed to have budgeted a fixed amount of money $I(\$/week)$ for his/her weekly consumption, and the unit prices of food and playing video games are $p_1(\$/kg)$ and $p_2(\$/h)$, respectively[1]. The weekly budget of the consumer is $I = p_1q_1 + p_2q_2$ where $q_1(kg/week)$ and $q_2(h/week)$ are the consumption flows of the two goods.

The consumer has a continuous real valued weekly utility function $u = f(q_1, q_2)$. To be able to write well-defined expressions with this function, measurement unit *ut* is given for utility (or satisfaction). However, because the consumer spends all the money he/she has budgeted for the week during the week, the satisfaction he/she gains from consumption takes place at the week. Thus function u with unit $ut/week$ measures the consumer's weekly flow of satisfaction.

The consumer's marginal utilities of the two goods,

$$\frac{\partial f(q_1, q_2)}{\partial q_1} > 0, \quad \frac{\partial f(q_1, q_2)}{\partial q_2} > 0,$$

have units $(ut/week)/(kg/week) = ut/kg$ and $(ut/week)/(h/week) = ut/h$, respectively. Substituting the budget in the utility function gives the following unrestricted problem:

$$\max_{q_1 q_2} u, u = f(q_1, q_2) = f(q_{1,}(I - p_1q_1)/p_2) \equiv F(q_1, p_1, p_2, I). \tag{8}$$

The optimal weekly consumption of food q_1^* can be solved from the following equation:

$$\frac{du}{dq_1} = 0 \Leftrightarrow \frac{\partial f}{\partial q_1} - \frac{p_1}{p_2}\frac{\partial f}{\partial q_2} = 0 \Leftrightarrow \frac{1}{p_1}\frac{\partial f}{\partial q_1} = \frac{1}{p_2}\frac{\partial f}{\partial q_2} \Rightarrow q_1^* = g(p_1, p_2, I),$$

that can also be presented according to Eq. (8) as:

$$\frac{\partial F}{\partial q_1} = \frac{\partial f}{\partial q_1} - \frac{p_1}{p_2}\frac{\partial f}{\partial q_2} = 0. \tag{9}$$

The sufficient condition for maximum is:

$$\frac{d^2u}{dq_1^2} = \frac{\partial^2 f}{\partial q_1^2} - \frac{p_1}{p_2}\frac{\partial^2 f}{\partial q_2 \partial q_1} - \frac{p_1}{p_2}\frac{\partial^2 f}{\partial q_1 \partial q_1} + \frac{\partial^2 f}{\partial q_2^2}\left(\frac{p_1}{p_2}\right)^2 < 0. \tag{10}$$

[1] Measurement units are in parenthesis after the quantities, see De Jong (1967).

Non-increasing marginal utility makes $\partial^2 f / \partial q_1^2$ and $\partial^2 f / \partial q_2^2$ non-positive, and if the partial functions are continuous, then $\partial^2 f / \partial q_1 \partial q_2 = \partial^2 f / \partial q_2 \partial q_1$ (Apostol 1979 p. 360). Assuming the partial functions continuous, the sufficient condition for maximum is: $\partial^2 f / \partial q_1 \partial q_2 > 0$. Thus, the greater q_1 is, the higher the marginal utility of good 2 and vice versa.

3.2. Dynamic Consumer Behaviour by Dynamic Optimization

Here we dynamize the theory given in section 3.1 by dynamic optimization. The consumption flows of the two goods are assumed to depend on time t with unit *week*, while the other quantities are assumed fixed. Substituting the budget equation in the utility function, gives:

$$u(t) = f\left(q_1(t), \frac{I - p_1 q_1(t)}{p_2} \right) \equiv F(q_1(t), p_1, p_2, I). \qquad (11)$$

Assuming that the consumer lives an infinite time with ρ as his/her time preference, the unrestricted dynamic optimization problem for the consumer becomes the following:

$$\max_{q_1(t)} \int_0^\infty e^{-\rho t} u(t) dt = \max_{q_1(t)} \int_0^\infty e^{-\rho t} F(q_1(t), I, p_1, p_2) dt$$

$$= \max_{q_1(t)} \int_0^\infty G(q_1(t), I, p_1, p_2, \rho, t) dt.$$

The Euler equation of this problem is:

$$\frac{\partial G}{\partial q_1} - \frac{d}{dt}\left(\frac{\partial G}{\partial q_1'(t)} \right) = 0 \Leftrightarrow e^{-\rho t} \frac{\partial F}{\partial q_1} = 0 \Leftrightarrow \frac{\partial F}{\partial q_1} = 0 \text{ since } e^{-\rho t} > 0.$$

The necessary condition for this dynamic problem then equals with that in the static case in (9). Thus for dynamic optimization to give an equation of motion for the consumption of a consumer, either the target function or the budget equation must be changed from that of static analysis. However, in that case the two frameworks would not be consistent with each other as we observed in the case of a firm.

3.3. The Ramsey-Cass-Koopmans Model of Consumption

The inability of dynamic optimization to yield an equation of motion for the consumption of a consumer described in Section 3.2 has led to the situation that a dynamic model for the real consumption of a consumer does not exist. On the other hand, the allocation of money for consumption over time has been modelled by Ramsey (1928), Cass (1965), and Koopmans (1965). The RCK-model is presented here in the form of Romer (1996, pp. 39-44). In the model there are H identical households with the size of each household growing at rate n. The household's lifetime utility function of infinite horizon with discount rate ρ is the following

$$U = \int_0^{\infty} e^{-\rho t} u(C(t)) \frac{L(t)}{H} dt, \tag{12}$$

where $C(t)$ is the consumption of each member of the household at time t. $u(C(t))$ is the utility function that gives each member's utility at a given date, $L(t)$ the total population in the economy, and $L(t)/H$ average number of members in a household. Thus $u(C(t))L(t)/H$ is the household's utility at time t. The budget constraint of this optimization problem is:

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} A(t)\, w(t) \frac{L(t)}{H} dt,$$

where $R(t) = \int_{\tau=0}^{t} r(\tau) d\tau, r$ is the interest rate, $K(0)/H$ the initial capital holdings of the household, $A(t)$ the efficiency factor of a worker in work, and $A(t)w(t)$ a worker's labour income. The budget constraint takes care that the present value of lifetime consumption does not increase the present value of lifetime income of the household. To get clear results, the following utility function is assumed for the household,

$$u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta}, \ \theta > 0, \ \rho - n - (1-\theta)g > 0, \tag{13}$$

where θ is the coefficient of risk aversion, and $g = A'(t)/A(t)$ the growth rate of efficiency of labour. The latter condition guarantees that the lifetime utility does not diverge. This constrained dynamic optimization problem yields the following optimum condition (Romer 1996 p. 44):

$$\frac{C'(t)}{C(t)} = \frac{r(t) - \rho}{\theta}.$$

The optimal growth rate of consumption of a worker $C'(t)/C(t)$ is thus increasing, if interest rate exceeds the time preference of the household and vice versa.

Now, the RCK-model explains the time path of money used in consumption of a household, but it does not dynamize the static neo-classical theory of real consumption of a consumer given in section 3.1. The two theories explain different quantities and we have found no article where the link between these theories has been studied. The difference in the theories is seen in the arguments of the utility functions. The static theory assumes that utility originates from the real consumption of various goods, and the dynamic theory assumes that utility originates from the total money used in consumption without noticing the structure of consumption.

4. A Dynamic Theory of Production Consistent with the Static Neo-classical One

4.1. Kinematics of Production and Consumption

Let $q(t)(unit/time)$ be the flow of production or consumption of a good at time moment t. The accumulated production (consumption) of the good till time moment t, $Q(t)(unit)$, is then:

$$Q(t) = Q(t_0) + \int_{t_0}^{t} q(s)ds,\ Q'(t) = q(t),\ Q''(t) = q'(t),$$

where $Q(t_0)(unit)$ is the accumulated amount of production (consumption) of the good till moment t_0, $Q'(t) = q(t)(unit/time)$ the momentous flow, and $Q''(t) = q'(t)(unit/time^2)$ the momentous acceleration of accumulated production (consumption) at time moment t. This kinematics serves as a prelude for Newtonian theories of production and consumption.

4.2. Newtonian Theory of Production

The decision-making for the dynamics of production of a firm can be studied by assuming the decision-makers to plan whether to increase the accumulated production of the firm by a certain amount or not, or by assuming that they are planning to change the flow of production of the firm from the last week, month, or year by a certain quantity. We study

here only the latter case by the profit function applied in Evans (1924) with $h = 0$. Assuming the units for the quantities as $q(unit/week), p(\$/unit)$, the Newtonian theory of a firm in Estola (2001) gives the following equation of motion for production of the profit-seeking firm:

$$mq'(t) = \frac{\partial \pi}{\partial q} \Leftrightarrow mq'(t) = \left(\frac{2}{a} - 2A\right) q(t) - \frac{b}{a} - B. \qquad (14)$$

The units of the constants are:

$A : \$ \times week/unit^2, B : \$/unit, a : unit^2/(\$ \times week), b : unit/week,$ and positive constant m with unit $\$ \times week^2/unit^2$ is the inertial factor of production. It takes care that changing the flow of production does not occur instantaneously but takes some time. These units make the equation dimensionally correct, see De Jong (1967). Marginal profitability $\partial \pi / \partial q$ with unit $\$/unit$ is the "force" acting upon the production of the firm. According to Eq. (14), there is positive acceleration in production if $\partial \pi / \partial q > 0$ and vice versa. The solution of the differential equation in (14) is:

$$q(t) = \frac{b + aB}{2(1 - aA)} + C_1 e^{\frac{2(1-aA)}{am}t}, \quad q(\infty) = \frac{b + aB}{2(1 - aA)}, \quad \frac{2(1 - aA)}{am} < 0,$$

$$p(\infty) = \frac{b - 2aAb - aB}{2a(aA - 1)}, \quad C_1 \text{ constant.}$$

This dynamic model gives the static neo-classical theory as asymptotic case with $t \to \infty$. The static neo-classical equilibrium is obtained in (14) also as the "zero-force" situation. Notice that instead of production dynamics we could have modelled price dynamics and get the same solution because the demand function defines a one-to-one relation between p and q_1. Non-stable solution paths, that correspond to permanent growth, are obtained from (14) by assuming time dependencies in the demand or the cost function, or assuming increasing returns to scale, see Estola (2001).

5. A Dynamic Theory of Real Consumption Consistent with the Static One

Here we model dynamic consumer behaviour so that the static neo-classical framework corresponds to a "zero-force" situation in this. We

continue analyzing the two-good situation in section 3.1 and assume a Cobb-Douglas – form for utility, $u = A(bq_1)^{\alpha}(cq_2)^{1-\alpha}$, where the units of the positive constants are: $A : ut/week, b : week/kg, c : week/h,$ and α is a pure number. Utility is thus measured in units $ut/week$ and the two power functions are dimensionless. Substituting the budget in the utility function, we get:

$$u(t) = A(bq_1(t))^{\alpha}\left(\frac{c(I - p_1 q_1(t))}{p_2}\right)^{1-\alpha}. \qquad (15)$$

Omitting the time dependency in q_1 we get the corresponding static neo-classical solution for optimal q_1 from Eq. (15) as follows:

$$\frac{\partial u}{\partial q_1} = \frac{Ac(bq_1)^{\alpha}\left(\frac{c(I - p_1 q_1)}{p_2}\right)^{-\alpha}(\alpha I - p_1 q_1)}{p_2 q_1} = 0 \Leftrightarrow q_1^* = \frac{\alpha I}{p_1}.$$

According to Estola & Hokkanen (2008), we can write the following Newtonian equation of motion for q_1 for a utility-seeking consumer with time:

$$m_1 q_1^{'}(t) = \frac{\partial u}{\partial q_1}$$

$$\Leftrightarrow m_1 q_1^{'}(t) = \frac{Ac(bq_1(t))^{\alpha}\left(\frac{c(I - p_1 q_1(t))}{p_2}\right)^{-\alpha}(\alpha I - p_1 q_1)(t))}{p_2 q_1(t)} \qquad (16)$$

where positive constant m_1 with unit $ut \times week^2/kg^2$ has the same role as inertial mass has in physics; it takes care that the adjustment in consumption does not occur immediately but takes some time. Marginal utility $\partial u/\partial q_1$ is the "force" acting upon the consumption of good 1. Notice that the budget equation is included in the utility function before calculating the marginal utility. Because of the non-linearity of Eq. (16), we solve it numerically by assuming the values for the constants as: $\alpha = 0.5, I = 100,\ p_1 = 5,\ p_2 = 10, m_1 = 2, A = b = c = 1.$ With $t \to \infty,$ the solution path of Eq. (16) in Figure 1 converges to the static neo-classical optimum: $q_1^* = \alpha I / p_1 = 10.$

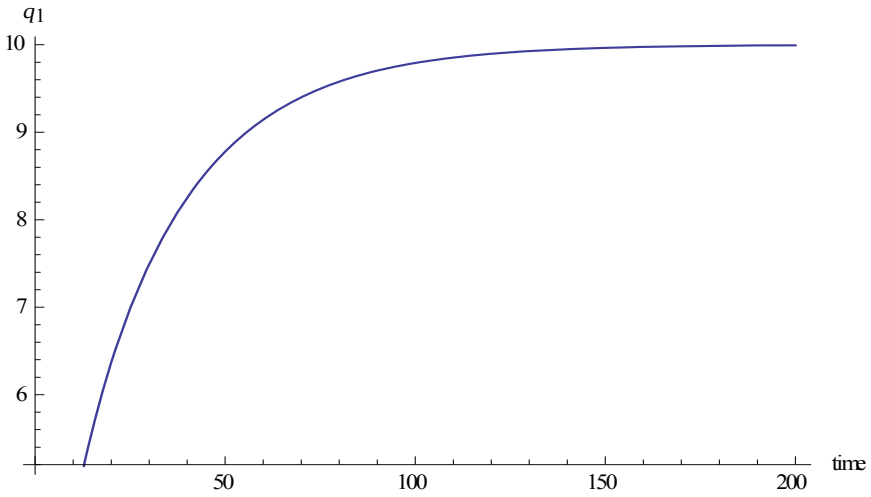

Figure 1. The solution path for consumption of good 1.

In order to test the theory in (16), the marginal utility of the consumer should be measurable. However, the consumer's marginal utility of food consumption with unit ut/kg is not measurable because we have no way to measure a consumer's satisfaction. Thus, for testing the theory we need to transform the force in a measurable form. We can factor $\partial u/\partial q_1$ as:

$$\frac{\partial u}{\partial q_1} = \frac{(1-\alpha)Ac(bq_1)^{\alpha}\left(\frac{c(I-p_1q_1)}{p_2}\right)^{-\alpha}}{p_2}\left[\left(\frac{\alpha}{(1-\alpha)}\right)\left(\frac{I-p_1q_1}{q_1}\right)-p_1\right], \quad (17)$$

where positive term Z,

$$Z \equiv \frac{(1-\alpha)Ac(bq_1)^{\alpha}\left(\frac{c(I-p_1q_1)}{p_2}\right)^{-\alpha}}{p_2},$$

does not affect the sign of the force. The active part of the force – the term in brackets in (17) – can be understood so that the positive term

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{I-p_1q_1}{q_1}\right)=\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{I}{q_1}-p_1\right)$$

is the "marginal willingness-to-pay" of the consumer for food, and p_1 is its price, see Estola & Hokkanen (2008). Thus the term in brackets in (17) can

be understood so that the force is positive if the marginal willingness-to-pay of the consumer exceeds the price of food, and vice versa. This term has the same zero point as $\partial u / \partial q_1$ has, namely $q_1^* = \alpha I / p_1$. Thus the acceleration of food consumption of this consumer is positive if the consumer's marginal willingness-to-pay exceeds the price of food, and vice versa.

To simplify the equation, we leave the positive factor Z out from the force and denote this new measurable force with unit $\$/kg$ as F_1,

$$F_1 = \frac{\alpha}{1-\alpha}\left(\frac{I}{q_1} - p_1\right) - p_1.$$

The corresponding equation of motion for food consumption is

$$\hat{m}_1 q_1^{'}(t) = \frac{\alpha}{(1-\alpha)}\left(\frac{I}{q_1(t)} - p_1\right) - p_1, \tag{18}$$

where positive inertial constant $\hat{m}_1$ ("mass") with unit $\$ \times week^2 / kg^2$ makes the equation well defined, see De Jong (1967). Due to the non-linearity of the equation in (18), we solve it numerically. Assuming $\alpha = 0.5,\ p_1 = 5,\ I = 100$ as before, the solution path is shown in Figure 2. The consumer's food consumption converges to the static neo-classical optimum: $q_1^* = \alpha I / p_1 = 10$. Thus analogous results are obtained by the two forces. The advantage of the former is that it is derived from the assumption that the consumer's aim is to increase his/her utility with time, and that of the latter is that it is measurable in units $\$/kg$.

Our framework implies that the explicit measuring of utility is not needed in modelling consumer behaviour. A consumer's decision-making can always be expressed in the form "*marginal willingness-to-pay minus price*". Utility is only an auxiliary quantity required in defining the marginal willingness-to-pay of a consumer for various things. All utility functions with the same preference order give equal marginal willingness-to-pay values for goods near the consumer's optimum, see Estola & Hokkanen (2008).

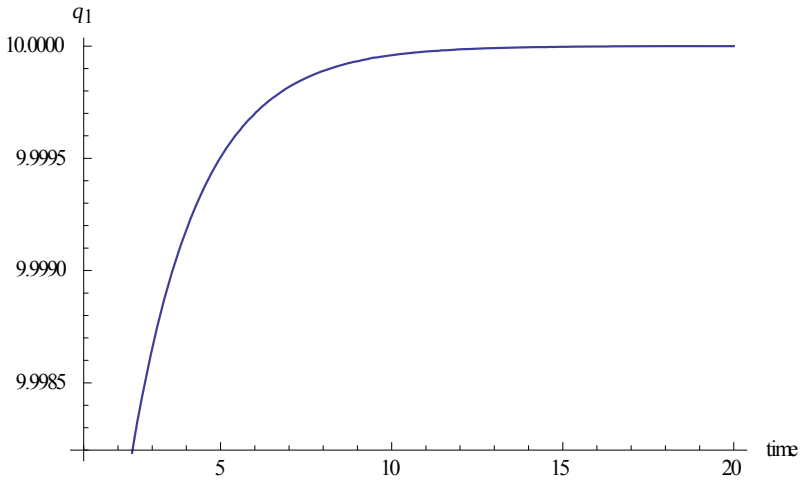

Figure 2. The path of food consumption of the consumer.

The dynamic consumer behaviour introduced in this section has still one advantage as compared with the static neo-classical framework. Because time is omitted in the static neo-classical analysis, in that framework we cannot model how changes in consumers' wealth or in prices with time affect their consumption. In our framework this can be done. Suppose a consumer gains wealth so that he/she can steadily increase funds for his/her consumption. The budgeted funds for his/her weekly consumption become then a function of time, and in the real world the prices of goods are functions of time. Assuming the Newtonian equation for food consumption as in (18), we can set there functions $I(t)$, $p_1(t)$ and solve the equation in this case. Thus our framework can be extended for modelling changes in consumption due to changes in income and prices that cannot be done in the static neo-classical one. Time dependent consumer preferences can be included in the model analogously.

6. Conclusions

We showed that the static neo-classical theory of a firm and a consumer are either inconsistent with the corresponding theories obtained by dynamic optimization, or they model a different quantity. As a solution to this, we extended the static neo-classical theory of a firm and a consumer to a dynamic form consistent with the static analysis. In this we defined the "economic forces" acting upon the production of a profit-seeking firm and a utility-seeking consumer. An isomorphism between

economic dynamics and classical mechanics was thus proposed that gives equilibrium and non-equilibrium analysis by using a single framework.

REFERENCES

[1] Apostol, T., 1979, *Mathematical Analysis*, Second Edition. Addison-Wesley Publishing Company, Inc.

[2] Cass, D., 1965, Optimum Growth in an Aggregative Model of Capital Accumulation. *Review of Economic Studies* 32 (July): 233-240.

[3] De Jong, F. J., 1967, *Dimensional Analysis for Economists*. North-Holland Publishing Company, Amsterdam.

[4] Estola, M., 2001, A Dynamic Theory of a Firm: An Application of 'Economic Forces'. *Advances in Complex Systems,* Vol. **4,** No.1.

[5] Estola, M. & Hokkanen, V-M., 2008, Consumer, Firm, and Price Dynamics: An Econophysics Approach. Modeling by Economic Forces. VDM Verlag Dr. Muller, Saarbrücken, Germany.

[6] Evans, G. C. 1924, The Dynamics of Monopoly. *American Mathematical Monthly*, February, pp. 77-83.

[7] Fisher, F. M., 1983, *Disequilibrium Foundations of Equilibrium Economics*. Cambridge University Press.

[8] Jorgenson, D. T., 1963, Capital Theory and Investment Behavior. *American Economic Review*, May, pp. 247-259.

[9] Koopmans, T. C., 1965, On the Concept of Optimal Economic Growth. In *The Economic Approach of Development Planning.* Amsterdam: North Holland.

[10] Lucas, R. E. Jr., 1988, On the Mechanics of Economic Development. *Journal of Monetary Economics*, 22, pp. 3-42.

[11] Mirowski, P., 1989a., More Heat than Light, Economics as Social Physics, Physics as Nature's Economics. Cambridge University Press.

[12] Mirowski, P., 1989b, The Rise and Fall of the Concept of Equilibrium in Economic Analysis. *Recherches Economiques du Louvain*, vol. 55:4.

[13] Ramsey, F. P., 1928, A Mathematical Theory of Saving. *Economic Journal 38* (December): 543-559. Reprinted in Stiglitz and Uzawa (1969).

[14] Romer, D., 1996, *Advanced Macroeconomics*. McGraw-Hill, USA.

INVESTIGATION OF THE COMPLETE GROUP OF PERMUTATIONS S_4 BY THE CONSTRUCTION OF THE MULTI-VALUED FUNCTION THAT DESCRIBES THE COMPLEX VECTOR FIELD

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Abstract. *We investigate analytically the complete group of permutations S_4 that is responsible for the construction of the multi-valued function which describes the appropriate complex vector field. The latter is determined as over a torus in the commutative case of the group S_2, as over the ordinary complex plane in the case of the non commutative subset of the group S_4.*

The last situation is generated by the consideration of S_2 on a torus.

The aforesaid statements directly connect with the solution of Landau-Lifshitz equation when its spectral parameter varies on a torus, and complete anisotropy is taken into account.

Thus, the main goal of the present paper consists of the explicit mathematical study of the non commutative part of S_4 on the complex plane, that is raised by S_2 on a torus, and which concerns an analytical solution of the general Landau-Lifshitz equation in the framework of the more or less unified, but not universal method. The solution technique bases on the respective vector boundary Riemann problem with so called permutation matrix coefficient.

Keywords and phrases: *multi-valued function, complex vector field, non commutative case, four-dimensional complete group of permutations, vector boundary Riemann problem, diagonalization procedure.*

1. Introduction

Before we come to the reasonable explanation of the given problem explicit solution, some necessary notions must be proposed.

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At first, we associate to the permutation $\omega_m = \begin{pmatrix} 1 & \dots & m \\ j_1 & \dots & j_m \end{pmatrix}$ the $m \times m$ matrix of zeroes and ones whose k th row contains a one in the j_k th column and zeroes in other columns $(k, j_k = \overline{1,m})$. Such matrices are called permutation matrices and are in a natural one-to-one correspondence with the permutations ω_m. When instead of units the permutation matrix contains Holder functions, we come to the so called permutation matrix-function, e.g.

$$\omega_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} == (123) \leftrightarrow \Omega_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \leftrightarrow \Omega_3^* = \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{pmatrix},$$

and a,b,c are the Holder functions over the relevant compact / algebraic Riemann surface.

Now, let an arbitrary algebraic/compact Riemann surface R be given, and its finite genus is $r \geq 0$. We are looking for the function $f(z,u), (z,u) \in R$ (when $r = 0$, then instead of (z,u) only z is considered). This function is analytic everywhere on R, except the finite set of ramification points

$$(\alpha_i, u(\alpha_i)) \in R \ \ (i = \overline{1,n}), \tag{1.1}$$

and its functional values undergo the following m-dimensional permutations when come around the given points (1.1):

$$T_i = \begin{pmatrix} 1 & \dots & m \\ j_1^{(i)} & \dots & j_m^{(i)} \end{pmatrix} = (j_1^{(i)} \quad \dots \quad j_m^{(i)}) \quad (j_k, k = \overline{1,m}; i = \overline{1,n}). \tag{1.2}$$

Additionally, $f(z,u)$ is assumed to have a finite order at infinity.

The search of such multi-valued function $f(z,u)$, or $f(z)$ in the case of the complex plane, by its monodromy group (1.1), (1.2) [1], leads to the solution of the corresponding homogeneous vector boundary Riemann problem [2]-[4] whose boundary condition looks like

$$F^+(t,v) = M(t,v)F^-(t,v), \quad (t,v) \in L, \ D^{-1} \mid (F). \tag{1.3}$$

Here: $F(z,u) = \{F_j(z,u)\}_{j=1}^m$ are the unknown m-dimensional vector-valued functions that are analytic everywhere on the surface R outside the

finite system of open Jordan contours L without self-intersections, and whose endpoints are the ramification points of $F(z,u)$:

$$L=\bigcup_{i=1}^{n}L_i,\ L_i=(\alpha_i,v(\alpha_i);\beta_i,v(\beta_i)),\quad L_l\bigcap L_k=\varnothing,\ (l\neq k;\ l,k,i=\overline{1,n}).\ (1.4)$$

Functions $F(z,u)$ are bounded at the endpoints of L from (1.4), extend to L $H-$ continuously from the left and right, have a finite order at infinity, and $F^{\pm}(t,v)$ are the limited values of $F(z,u)$ at L from the left and right respectively.

Such functional class is designated as $h_0(L;R)$.

The matrix coefficient from (1.3) looks like

$$M(t,v)=\sum_{i=1}^{n}M_i\delta(t,v;L_i),\quad \delta(t,v;L_i)=\begin{cases}1,\ (t,v)\in L_i\\ 0,\ (t,v)\notin L_i\end{cases},\qquad (1.5)$$

and M_i $(i=\overline{1,n})$ is the $m\times m$ permutation matrix that is raised by the appropriate permutation (1.2).

The m-dimensional vector-divisor of infinities is D in (1.3), and $F(z,u)$ is divisible by it. When the genus of the surface R equals zero ($r=0$), then the divisibility condition $D^{-1}\,|\,(F)$ in (1.3) is missing.

Each the sought for scalar component $F_j(z,u)$ $(j=\overline{1,m})$ is the j th branch of the initially wanted multi-valued function $f(z,u)$ when $r>0$, and $f(z)$ – in the case of $r=0$.

It is well known [2] that the explicit solution of the problem (1.1)-(1.5) generates, in its turn, the construction of the relevant algebraic equation of the covering with respect to the surface R. The last fact and the research of the original vector problem (1.1)-(1.5) have a lot of industrial applications in optics, acoustics, various wave propagation and modern technical electrodynamics, as well [5]-[13].

Moreover, the most interesting case is non commutative. It means that there is no any linear transformation that is constant everywhere on the contour (1.4) and which takes all the matrices (1.5) to the diagonal form [14]. Otherwise, the vector boundary Riemann problem (1.1)-(1.5) is reduced to the corresponding system of m scalar boundary homogeneous Riemann problems on the given algebraic/compact surface R. The solutions of such scalar problems are well known [6]-[8], [12], [13] and are not of great interest. That is the main reason why in this paper, we deal only with the non commutative matrix coefficients.

Additionally, it should be noted that the proposed explicit solution of the aforesaid class of the boundary vector problems is done here in terms of the so called canonical and normal solution matrices (c.s.m. and n.s.m. correspondingly). This fact is very important, since we avoid here the habitual rather difficult investigation approach. This procedure deals with the analytical study of the respective integral equations' system that is raised by the original vector boundary Riemann problem [8].

Justifying the proposed here method of c.s.m. and n.s.m. construction for (1.1)-(1.5) problem solution, we should like to remind of the following fact. The c.s.m. and n.s.m. form the algebraic equations of the covering surfaces over the original algebraic/compact Riemann surface R. This object is principal in the inverse scattering problem and soliton theory [5], [9], [11]-[13], as well.

So, $m \times m$ matrix $\mathrm{X}(z,u),(z,u) \in R$, is the c.s.m. (canonical solution matrix) of the vector boundary problem (1.1)-(1.5) if: a) $\mathrm{X}(z,u)$ satisfies the given boundary condition; b) $\det \mathrm{X}(z,u)$ has no poles anywhere in the finite part of R and can be equal to zero only at the endpoints of the open contour L; c) the orders r_j $(j = \overline{1,m})$ of columns at infinity in the matrix $\mathrm{X}(z,u)$ can not be lessened, i.e. $\underset{\infty}{\mathrm{ord}} \det \mathrm{X}(z,u) = \sum_{j=1}^{m} r_j$. Matrix $\mathrm{X}(z,u),(z,u) \in R,$ is the n.s.m. (normal solution matrix) of the same type of vector boundary Riemann problem (1.1)-(1.5) if $\mathrm{X}(z,u)$ satisfies conditions a), b) and does not satisfy the last condition c), i.e. $\underset{\infty}{\mathrm{ord}} \det \mathrm{X}(z,u) < \sum_{j=1}^{m} r_j$.

Now, when the last necessary notions are introduced, we can move to the actual problem statement.

2. The problem statement and preliminary results

Let the complete group of permutations S_4 be given. It is well known [15] that S_4 is non commutative. In our situation, it means that not all its representatives commute in pairs.

Therefore, if S_4 is the base of the monodromy group (1.1), (1.2) [1] of the sought for multi-valued function, then we encounter one of the most difficult case of such function construction. Even the complex plane, as an

initial surface of the zero genus cannot propose the explicit solution of the aforesaid problem.

Really, in terms of the so called permutation matrices [3], the last fact implies the nonexistence of the constant linear transformation that reduces all these matrices to the diagonal form [14]. In its turn, the latter shows the impossibility of the classical diagonalization procedure [4] of the respective homogeneous vector boundary Riemann problem [2] that is responsible for the construction of the appropriate multi-valued function [1]. The coefficient of the present homogeneous vector boundary Riemann problem is the permutation matrix that is in one-to-one correspondence with the group S_4.

Diagonalization means the reduction of the original vector problem to the equivalent system of scalar ones whose explicit solution is either well known, or can be done effectively by the respective technique [4]-[8].

Further, the requirements of the current industrial scientific development in majority, deal nowadays with the complete anisotropy of the studied medium that generates the urgent investigation of so popular and important nanostructures.

Such phenomena and the relevant vector fields can be described analytically only by means of the multi-valued functions over the algebraic / compact Riemann surfaces. The most interesting and non explored situation is non commutative monodromy group (1.1), (1.2) [1], as it was mentioned above.

So, **the main purpose of the given paper** *is the analytic investigation of the complete four-dimensional non commutative group of permutations* S_4, *as the base of the multi-valued function construction over the complex plane, at least. The application of these results concerns the explicit study of the Landau-Lifshitz equation whose spectral parameter varies on a torus and the complete anisotropy is taken into account [5], [11]-[13].*

This equation is principal in soliton theory, describes the dynamics of magnetization of the ferromagnetic patterns in the magnetic field and the process of the low-frequent wave propagation in ferromagnetic. These specified materials are used in the creation of the nonreciprocal wave devices [10]. Nowadays, an explicit solution of the generalized Landau-Lifshitz equation (L.-L.e.) becomes rather urgent owing to its application to the spin moment transference in the magnetic nanostructures, construction of the Magnetic Resistive Random Access Memory (MRAM) elements and magnetic logical elements as well.

Investigation of the same equation in the case of complete anisotropy [5] was done in terms of the two-sheeted covering of a torus using geometrical and physical characters of the original applied problem statement. As it was marked in [5], in other more complicated covering versions the proposed solving procedure was not effective at all. In its turn, the aforesaid mostly important current case of the L.-L.e. was studied in [11] only approximately. In papers [12], [13], the suggested general analytic approach [3], [4] deals, in particular with an explicit algebraic equation construction of the two-sheeted covering of a torus including the solution of L.-L.e.

However, the problem of an explicit study of other surfaces that can be generated by the mentioned torus' two-sheeted covering in the case of L.-L.e. was not either raised or affected at all. The last fact explains the reason why the main purpose of the suggested here research is formulated just as it is done above.

At last, from our viewpoint, the virtue of the proposed, probably deliberately simplified approach of the given article is its future leading to the effective study of the relevant problems on the surfaces of the nonzero genera.

Returning again to the studied group S_4 and all its permutations whose number equals $4! = 24$, we select and form 5 subsets of S_4 with respect to their cyclic structure. Namely,

1) 6 elements with 1 cycle of the length 4 in all 3 "similar" pairs:

$$\omega_1 = (1234),\ \omega_2 = (1243);$$

$$\omega_3 = (1324),\ \omega_4 = (1342);$$

$$\omega_5 = (1423),\ \omega_6 = (1432);$$

2) 1 identical element with 4 cycles of the length 1:

$$\omega_7 = (1)(2)(3)(4) = I;$$

3) 3 elements with 2 cycles of the length 2:

$$\omega_8 = (12)(34),\ \omega_9 = (13)(24),\ \ \omega_{10} = (14)(23);$$

4) 6 elements with 2 cycles of the length 1 and 1 cycle of the length 2 in all 3 "similar" pairs:

$$\omega_{11} = (1)(2)(34),\ \omega_{12} = (3)(4)(12);$$

$$\omega_{13} = (1)(4)(23),\ \omega_{14} = (2)(3)(14);$$

$$\omega_{15} = (2)(4)(13),\ \omega_{16} = (1)(3)(24);$$

5) 8 elements with 2 cycles of the lengths 1 and 3 correspondingly in all 4 “similar” pairs:

$$\omega_{17} = (1)(234),\ \omega_{18} = (1)(243);$$

$$\omega_{19} = (2)(134),\ \omega_{20} = (2)(143);$$

$$\omega_{21} = (3)(124),\ \omega_{22} = (3)(142);$$

$$\omega_{23} = (4)(123),\ \omega_{24} = (4)(132). \qquad (2.1)$$

Checking (2.1), it is easy to find that not all representatives inside even of one and the same subset commute in pairs.

Thus, all non trivial subsets 1), 3)-5) from (2.1) are non commutative. Only some pairs of permutations from 1), 3)-5) commute. They are the following:

1) ω_2, ω_4; ω_1, ω_6; ω_3, ω_5. The result of their multiplication is I from the subset 2). All other remained non commutative pairs, after multiplication, transform to the element from the subset 5). Thus, only 3 pairs from $C_6^2 = 15$ commute.

3) It is completely commutative subset, and each multiplying result represents the third permutation that was not included into the product as the factor. Therefore, all $C_3^2 = 3$ elements commute in pairs.

4) Only elements of 3 “similar” pairs commute, i.e. ω_{11}, ω_{12}; ω_{13}, ω_{14}; ω_{15}, ω_{16}, and the result of multiplication is the complete subset 3). All other non commutative permutations from $C_6^2 = 15$ pairs, form the two-factor products that belong to the subset 5).

5) Permutations of 4 “similar” pairs commute in our aforesaid meaning: ω_{17}, ω_{18}; ω_{19}, ω_{20}; ω_{21}, ω_{22}; ω_{23}, ω_{24}. The remained non commutative pairs from the $C_8^2 = 28$ representatives of the subset 5) form the products whose result is the permutation that belongs either to the subset 3), or 5).

Turning to the interrelations between subsets (2.1), it is easy to notice that the number of commutative in pairs permutations is even less than those which are from one and the same subset.

3. Results

As it was said, the purpose of the given paper is an explicit construction of the covering algebraic equations that are raised by the

L.-L.e. [5], [11]-[13] and whose respective vector boundary Riemann problem (1.1)-(1.5) has non commutative matrix coefficient. So, first of all we briefly introduce some results that concern the construction of the two-sheeted covering of a torus T, and include simultaneously an analytic solution of the L.-L.e. [5], [11]-[13]. As it will be shown later, the study of such simple problem will allow constructing the appropriate coverings with the non commutative monodromy groups (1.1), (1.2).

Hence, the sought for vector function is the following:

$$F(z,u)=\{F_1(z,u),\ F_2(z,u)\}\in h_0(L;T)-? \tag{3.0}$$

The well-known algebraic equation of a torus T and its genus ρ are given below:

$$T: u^2=\prod_{j=1}^{2}(z-a_j)(z-b_j);\ \ \rho=1. \tag{3.1}$$

The corresponding homogeneous vector boundary Riemann-Hilbert problem that raises the two-sheeted covering of T and its algebraic equation [1], [2], including an investigation of the L.-L.e. [5], [11], has the following boundary condition [12], [13]:

$$F^{+}(t,v)=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}F^{-}(t,v),\quad (t,v)\in L,\quad D^{-1}\,|\,(F), \tag{3.2}$$

where:

$$v^2=\prod_{j=1}^{2}(t-a_j)(t-b_j);\quad L=(a,v(\alpha);\beta,v(\beta)); \tag{3.3}$$

D is the two-dimensional vector-divisor of infinities on T, and all other symbols are described in the Section 1.

Geometrically, it means that two copies of T are pasting cross-wise together along the contour-cut L.

Since the matrix coefficient in (3.2) is commutative, there exists the linear transformation S:

$$\exists S=S^{-1}=\frac{1}{\sqrt{2}}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix},\quad \Phi(z,u)=S\times F(z,u) \tag{3.4}$$

that is constant everywhere on L and which takes (3.2) to the equivalent system of homogeneous boundary scalar Riemann-Hilbert problems on T [12]-[14]:

$$\Phi^{+}(t,v)=\operatorname{diag}(-1,1)\,\Phi^{-}(t,v),\quad (t,v)\in L,\ \ D^{-1}\,|\,(\Phi). \tag{3.5}$$

Because of [3], [4], [6], [7] we get the required solutions of (3.5):

$$\Phi_1(z,u) = \Phi_{11}(z,u) \bigcup \Phi_{12}(z,u);$$

$$\Phi_{11}(z,u) = g_1(z)\chi(z,u),\ \Phi_{12}(z,u) = (h_2(z)+u)\ \chi(z,u),$$

$$\Phi_2(z,u) = 1; (z,u) \in T, \tag{3.6}$$

whose functions $g_1(z), h_2(z)$ are explicitly obtained polynomials with the known coefficients and respective powers 1 and 2.

$$\chi(z,u) = \exp\left\{\frac{1}{2}\int_L \mathrm{d}\,w - \int_{(z*,\xi^*)}^{(m_1,\mu_1)} \mathrm{d}\,w + \int_{(z*,\xi^*)}^{(a,v(a))} \mathrm{d}\,w\right\}. \tag{3.7}$$

In (3.7): $\mathrm{d}\,w$ is the gap-like analogy of Cauchy kernel on T; the point (m_1,μ_1) is found by means of the Jacobi conversion problem [1] and is written explicitly in terms of elliptic Jacobi functions [4], [6], [16]; (z^*,ξ^*) is an arbitrary point of T.

Then the written below matrices represent the c.s.m. (while $j=1$) and n.s.m. (while $j=2$) of (3.5), respectively:

$$Y_j(z,u) = \mathrm{diag}\,(\Phi_{1j}(z,u), 1) \quad (j = 1, 2). \tag{3.8}$$

Taking into account formula (3.4), matrices (3.8) can be transformed into the c.s.m. $(j=1)$ and n.s.m. $(j=2)$ of the original problem (3.0)-(3.3):

$$\mathrm{X}_j(z,u) = \frac{1}{\sqrt{2}}\begin{bmatrix} -\Phi_{1j}(z,u) & 1 \\ \Phi_{1j}(z,u) & 1 \end{bmatrix} \quad (j = 1, 2). \tag{3.9}$$

In its turn, the sought for vector function from (3.0) looks like:

$$F(z,u) = \begin{bmatrix} F_{1j}(z,u) \\ F_{2j}(z,u) \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} -\Phi_{1j}(z,u)+1 \\ \Phi_{1j}(z,u)+1 \end{bmatrix}\ (j = 1,\ 2). \tag{3.10}$$

Further, we get two corresponding algebraic equations of the two-sheeted coverings of the torus T, that are given by (3.1):

$$\lambda^2 + c_{1j}(z,u)\lambda + c_{2j}(z,u) = 0; \quad c_{1j}(z,u) = -(F_{1j}(z,u) + F_{2j}(z,u)) = \sqrt{2},$$

$$c_{2j}(z,u) = F_{1j}(z,u) \times F_{2j}(z,u) = (1 - \Phi_{1j}^2(z,u))/2. \tag{3.11}$$

Using the substitution:

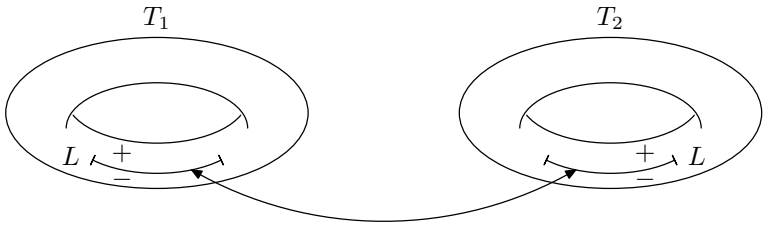
$$W = \sqrt{2}\lambda + 1$$

for λ, we reduce (3.11) to the unified algebraic equation of the two-sheeted covering T^* over the torus T:

$$T^*: \quad W^2 - \Phi_{1j}^2(z,u) = 0 \quad (j = 1, 2). \tag{3.12}$$

The latter includes both equations (3.11) and is in conformity with other known results [5], [12], [13].

Geometrically, the covering T^* over T is shown here as the crosswise pasting of two torus' copies along the crosswise cut of the contour L from (3.3).



$$\begin{aligned} &\rho^* = 2. \\ &L = (\alpha, v(\alpha); \beta, v(\beta)): \quad (12). \end{aligned} \tag{3.13}$$

In (3.13), the commutative monodromy group is given on the torus T, and its relevant permutation from (1.2) is (12) with the appropriate ramification points $(\alpha, v(\alpha))$; $(\beta, v(\beta))$ that are the endpoints of the open contour L.

As it is well known [17], the genus ρ^* of the covering R^* over the compact / algebraic Riemann surface R can be computed by the following formula:

$$\rho^* - 1 = m(\rho - 1) + \frac{1}{2}V, \tag{3.14}$$

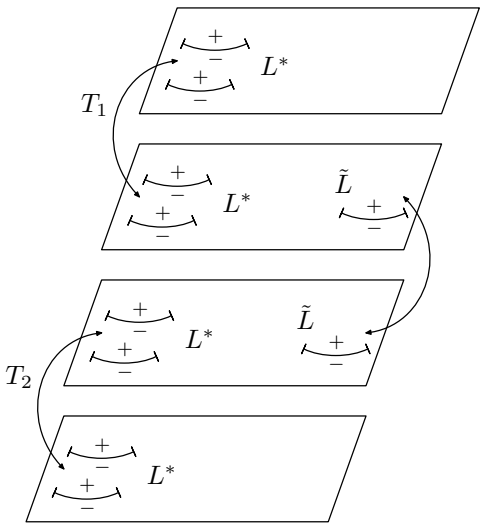
where: ρ is the genus of the initial surface R; m is the dimension of the permutations from the monodromy group that is fixed on R; $V = \sum_{k=1}^{p}(q_k - 1)$ is the ramification index of the covering R^*, whose item q_k in the sum symbol is the order of the respective k th ramification point,

and all of these p points are summed as many times, as their multiplicities are considered.

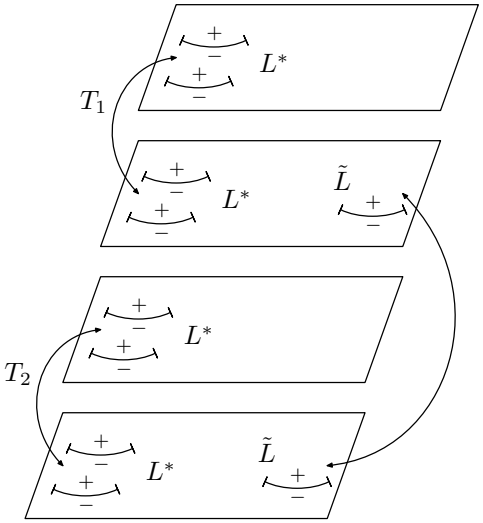
In (3.13), ρ^* is computed by the formula (3.14), and the monodromy group of T^* is written according to (1.1), (1.2).

Using the results of [3], [4] we can assert that the original vector problem (3.0)-(3.3) on T with the second-order commutative permutation matrix coefficient generates several other homogeneous vector boundary Riemann-Hilbert problems on the complex plane **C** with the fourth-order non commutative permutation matrix coefficients. An explicit solution of such problems and construction of the algebraic equations of the respective coverings are done by means of the general method and technique of [3], [4].

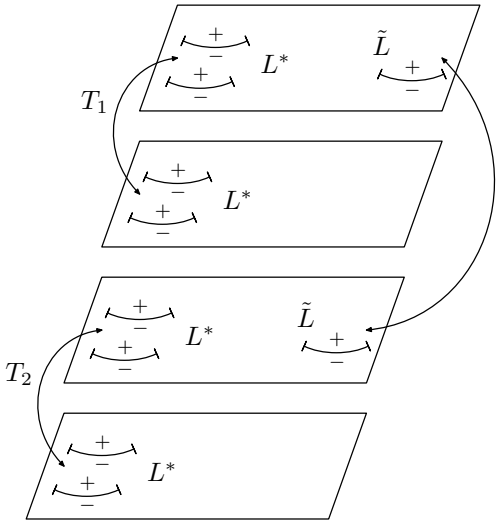
Since a torus is the two-sheeted covering of the complex plane **C,** then the surface from (3.13) is the four-sheeted covering over **C**. Turning again to the results of [3], [4] which describe the non commutative monodromy groups generated by the commutative ones that are given on the surfaces with the bigger genera, we can introduce geometrically the new coverings R_μ^* $(\mu = \overline{1, 4})$ over **C**. These surfaces are evoked by (3.13), are equivalent to T^* topologically, but not conformally, and their monodromy groups are non commutative. So, the aforesaid coverings R_μ^* $(\mu = \overline{1, 4})$ geometrically look like:



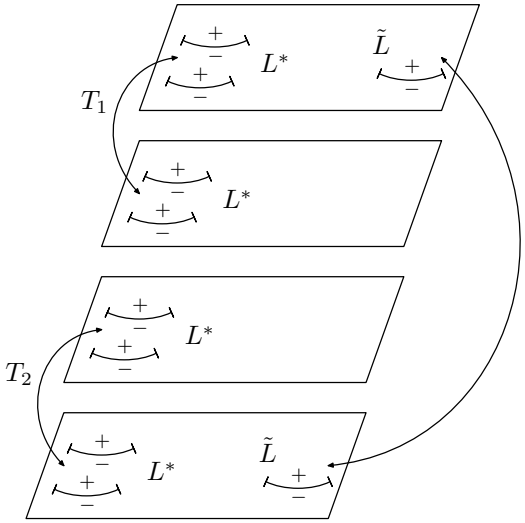
$$L^* = \bigcup_{j=1}^{2} (a_j, b_j): \quad (12)(34)$$
$$\tilde{L} = (\alpha, \beta): \quad (1)(23)(4); \ \rho_1^* = 2. \qquad (3.15)$$



$$L^* = \bigcup_{j=1}^{2} (a_j, b_j): \quad (12)(34)$$
$$\tilde{L} = (\alpha, \beta): \quad (1)(24)(3);\ \rho_2^* = 2. \tag{3.16}$$



$$L^* = \bigcup_{j=1}^{2} (a_j, b_j): \quad (12)(34)$$
$$\tilde{L} = (\alpha, \beta): \quad (13)(2)(4);\ \rho_3^* = 2. \tag{3.17}$$



$$L^* = \bigcup_{j=1}^{2} (a_j,\, b_j): \quad (12)(34)$$
$$\tilde{L} = (\alpha,\, \beta): \quad (14)(2)(3);\ \rho_4^* = 2. \tag{3.18}$$

Hence, according to (1.1)-(1.5) and (3.15)-(3.18) the respective vector boundary Riemann-Hilbert problems on **C** have the non commutative matrix coefficients, the unknown vector-valued functions are the following:

$$f(z) = \{f_j(z)\}_{j=1}^{4} \in h_0(L^* \bigcup \tilde{L}; \mathbf{C}) \ \textbf{- ?} \tag{3.19}$$

and satisfy the boundary condition:

$$f^+(t) = \left(\sum_{i=1}^{2} M_i \delta(t; L_i) \right) f^-(t), \quad t \in L_1 \bigcup L_2, \quad L_1 = L^*, \quad L_2 = \tilde{L}\,. \tag{3.20}$$

In (3.20), M_i $(i = 1,\, 2)$ are the permutation matrices that are raised by the corresponding permutations from (3.15)-(3.18) non commutative monodromy groups and look as:

$$M_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ or } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \text{ or } \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\text{or} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{3.21}$$

It should be noted again that the problem (3.19)-(3.21) is equivalent geometrically to the cross-wise pasting of two copies of the torus T along the contour-cut L from the problem (3.0)-(3.3). This fact is true in the topological, but not in the conformal meaning. In the considered case of (3.19)-(3.21), the required pasting together construction is done not as the two-sheeted covering of the torus T but in terms of the four-sheeted covering of **C** and represents the above mentioned surfaces R_μ^* $(\mu = \overline{1, 4})$.

The normal basis of R_μ^* $(\mu = \overline{1, 4})$, in accordance with [2]-[4], [6] is written below:

$$\{\Phi_{11}(z,u), \Phi_{12}(z,u), u, 1\}, \tag{3.22}$$

where the functions $\Phi_{1j}(z,u)$ $(j = 1, 2)$ are from (3.6), (3.7), and u is described by the equation (3.1).

If r_j $(j = \overline{1, 3})$ are the respective orders of the functions $\{\Phi_{11}(z,u), \Phi_{12}(z,u), u\}$ at infinity, then $\sum_{j=1}^{3}(r_j - 1) = 2$ and equals the genus ρ^* of the four-sheeted covering R_μ^* $(\mu = \overline{1, 4})$. This fact is in conformity with the well-known Riemann-Roch theorem [1], [17] as one of its corollaries.

The sought for c.s.m. of the problem (3.19)-(3.21) is got due to the technique of [3], [4] and c.s.m., n.s.m. (3.9) that were obtained solving the problem (3.0)-(3.3):

$$\mathrm{X}(z) = \begin{bmatrix} \Phi_{11}(z,u) & \Phi_{12}(z,u) & u & 1 \\ \Phi_{11}(z,-u) & \Phi_{12}(z,-u) & -u & 1 \\ -\Phi_{11}(z,-u) & -\Phi_{12}(z,-u) & -u & 1 \\ -\Phi_{11}(z,u) & -\Phi_{12}(z,u) & u & 1 \end{bmatrix}, \tag{3.23}$$

$$\mathrm{X}(z)=\begin{bmatrix} \Phi_{11}(z,u) & \Phi_{12}(z,u) & u & 1 \\ \Phi_{11}(z,-u) & \Phi_{12}(z,-u) & -u & 1 \\ -\Phi_{11}(z,u) & -\Phi_{12}(z,u) & u & 1 \\ -\Phi_{11}(z,-u) & -\Phi_{12}(z,-u) & -u & 1 \end{bmatrix}. \qquad (3.24)$$

Matrices (3.23), (3.24) are the c.s.m. for the first and the fourth, the second and third values of M_2 from (3.21) respectively.

Results (3.22)-(3.24) are obtained as the particular simplest case of [4]. Taking into consideration formulae (3.23), (3.24) we construct the sought for algebraic equations [15] of the coverings R_μ^* $(\mu=\overline{1,4})$ whose geometrical pictures are (3.15)-(3.18):

$$R_\mu^*\ (\mu=\overline{1,4}):\ V^4+d_1(z)V^3+d_2(z)V^2+d_3(z)V+d_4(z)=0,$$

$$\Phi_{1j\pm}=\Phi_{1j}(z,\pm u)\ \ (j=1,2);\ \ d_1(z)=-4;$$

$$d_2(z)=4+2(1-u^2)-(\Phi_{11+}+\Phi_{12+})^2-(\Phi_{11-}+\Phi_{12-})^2;$$

$$d_3(z)=2(2(u^2-1)+(1+u)(\Phi_{11-}+\Phi_{12\ -})^2+(1-u)(\Phi_{11+}+\Phi_{12+})^2); \qquad (3.25)$$

$$d_4(z)=((u+1)^2-(\Phi_{11+}+\Phi_{12+})^2)((u-1)^2-(\Phi_{11-}+\Phi_{12-})^2).$$

Thus, we have obtained the required algebraic equation (3.25) that appears as unified for all coverings R_μ^* $(\mu=\overline{1,4})$.

In comparison with the joint formula (3.12) that unites the pair of different algebraic equations for the surface T^*, the unified equation (3.25) for the topologically equivalent coverings R_μ^* $(\mu=\overline{1,4})$ is unique for all four cases. It is quite natural, though (3.25) is generated by (3.12). Really, R_μ^* $(\mu=\overline{1,4})$ cover the complex plane, and the corresponding vector boundary Riemann-Hilbert problem (3.19)-(3.21) has no n.s.m., but only c.s.m. [2], [4]. This completely explains uniqueness of the existing algebraic equation (3.25) [2], [4].

Therefore, we have constructed explicitly the unknown algebraic equations of the surfaces that are raised by the two-sheeted covering of a torus (3.12) from the solution of L.-L.e. [5], [12], [13]. Moreover, the monodromy groups of these surfaces are non commutative. It means that that the originally given problem is solved completely and the goal of the present article is attained.

4. Remarks and conclusions

Though the suggested here problem is solved explicitly, it is clear that the given group of permutations S_4 is not studied completely even in the meaning of commutativity in pairs and over the complex plane. It implies that not all possible existing multi-valued functions with non commutative in pairs monodromy groups from S_4 are investigated yet. In its turn, the relevant vector fields are not constructed on **C**, at least.

Further, if commutativity in bigger sets than pairs is required, the problem of the monodromy group investigation becomes almost unobservable. Hence, the future analytical research of S_4 remains urgent as from the pure mathematical viewpoint, as from the applied industrial and engineering.

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REFERENCES

[1] B. Riemann, *Gesammelte Mathematische Werke,* **2**, Aufl., Teubner, Leipzig (1892).

[2] J. Plemelj, *Riemannsche Funktionenscharen mit gegebener Monodromien Gruppe*, Monatsh. Math. Phys., **19**, 211-245 (1908).

[3] I. Dmitrieva, *On some generalizations of the classical Riemann problem,* PAMM **7**. Issue 1. Special Issue: 6th Congress on Industrial and Applied Mathematics (ICIAM07) and GAMM Annual Meeting, Zurich 2007. Published On-line 21 May 2009. DOI: 10.1002/pamm.200701128, 216003-216004.

[4] I. Dmitrieva, *Factorization of non commutative permutation matrices on an algebraic surface of non zero genus,* transl. in St. Petersburg Math. J. **4**, No. 2, 309-318 (1993).

[5] A. I. Bobenko, *Real algebro-geometric solutions of the Landau-Lifshitz equation in Prim theta-functions*, transl. in Funct. Anal. Appl., **19**, 6-19 (1985).

[6] L. I. Chibrikova, *Boundary problems of the analytic function theory on Riemann surfaces,* Scientific and Engineering Results. Current Mathematical Problems. Fundamental Trends. Mathematical Analysis, Nauka, Moscow, **18**, 3-66 (1980). (Russian).

[7] F. D. Gakhov, *Boundary value problems,* Pergamon, Oxford (1966).

[8] N. P. Vekua, *Systems of singular integral equations and some boundary value problems*, Noordhoff, Groningen (1967).
[9] S. P. Novikov, V. E. Zakharov & S. V. Manakov, *Soliton theory. Method of the inverse problem*, Nauka, Moscow (1980) (Russian).
[10] Yu. I. Pimenov, V. I. Volman & A. D. Muravtsov, *Technical electrodynamics*, Radio and Communications, Moscow (2002) (Russian).
[11] A. K. Zvezdin, K. A. Zvezdin, & A. V. Hvalkovskiy, *Generalized Landau-Lifshitz equation and transference processes of the spin moment in magnetic nano-structures*, UFN **178**, No. 4, 236-442 (2008) (Russian).
[12] I. Dmitrieva, *Vector boundary Riemann problems in soliton theory*, IEEE (2010), Print ISBN: 978-1- 4244- 8859-9. DOI : 10.1109/MMET 2010.5611433.
[13] I. Yu. Dmitrieva, *Solution of the Landau-Lifshitz equation by means of the matrix Riemann problem*, Scient. Works of Odessa National Acad. of Telecommunications (ONAT), No. 1, 102-108 (2011) (Russian).
[14] V. V. Morozov, *On commutative matrices*, Kazan. Gos. Univ. Uchen. Zap. **112**, 17- 20 (1952). (Russian).
[15] A.G. Kurosh, *General algebra course*, Nauka, Moscow (1975). (Russian).
[16] M. Abramowitz & I.A. Stegun, *Handbook of mathematical functions*, National Bureau of Standards. Applied Mathematics Series, **55**, New York (1964).
[17] A. Hurwitz & R. Courant, *Funktiontheorie*, Springer, Berlin – Heidelberg – New York (1964).

NON-STATIONARY VOLATILITY WITH HIGHLY ANTI-PERSISTENT INCREMENTS: EVIDENCE FROM RANGE-BASED VOLATILITY

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Abstract. *We analyze range-based volatility of three highly capitalized companies to show that the volatility process is non-stationary and its logarithmic transformation together with logarithmic increments is approximately normally distributed. Further, the increments are shown to be highly anti-persistent. Together with the assertion that logarithmic returns are normally distributed, and uncorrelated with time-varying volatility, we propose a new returns-generating process. The whole procedure is based on empirical observations without any limiting assumptions. We reconstruct returns series which remarkably mimic the real-world series and posses the standard stylized facts – uncorrelated returns with heavy tails, strongly auto correlated absolute returns and volatility clustering.*

Keywords*: volatility modelling, anti-persistence, non-stationarity.*

1. Introduction

Accurate modelling and forecasting of volatility is one of the biggest challenges in financial economics and financial econometrics. Historically, there are four major groups of volatility forecasting approaches – historical volatility, conditional heteroskedasticity models, stochastic volatility models, and implied volatility models. Historical volatility models contain many different approaches such as random walks, historical averages, HAR models, autoregressive moving averages and their fractional generalizations, exponential smoothing approaches and others. Conditional heteroskedasticity group contains a wide portfolio of (G)ARCH models and their specifications such as EGARCH, IGARCH, FIGARCH and many others. Stochastic volatility models add a stochastic term into the conditional variance equation contrary to the previous group. Implied volatility models are based on the Black-Scholes option pricing formula

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[2, 27] and its various specifications and generalizations. These approaches are nicely reviewed and compared in two studies of Poon & Granger [30, 31]. In their second study [31], the authors argue that the implied volatility models outperform the others, followed by the historical volatility models, in volatility forecasting. This is a rather interesting, and disturbing, finding since the Black-Scholes formula is known to be based on highly unrealistic assumptions of the returns process. This might imply that the other approaches are actually not optimal and there is some other approach closer to reality.

As nicely summarized by Morana & Beltratti [28], there are two basic approaches to volatility modelling – structural breaks (and regime switching) and long-range dependence. In this paper, we propose an alternative approach to the volatility modelling. Based on a simple statistical analysis, we show that volatility can be effectively modelled as a non-stationary process with highly anti-persistent logarithmic increments, which are, moreover, normally distributed. By doing so, we are able to mimic the basic stylized facts of the financial returns – no autocorrelation, highly persistent absolute returns, non-normality, fat tails, and volatility clustering [7]. Note that mainly the ability to produce fat tails is a great achievement as the leptokurtosis is usually imposed to the simulated processes by various fat-tailed distributions of the innovations – Student's *t*, generalized error distributions, and others. We approach the problem from the opposite direction, starting from the real-world data. By analyzing the series of three American highly capitalized stocks, we are able to state eight basic Claims. Based on these Claims, we argue that the logarithmic returns are uncorrelated and normally distributed with approximately zero mean and time-varying standard deviation (volatility). The logarithm of the standard deviation is non-stationary and approximately normally distributed with approximately normally distributed increments, which are stationary and highly anti-persistent. Based on these findings, we are able to reconstruct the series of returns, which resemble the actual financial returns and the stylized facts very closely. Note that the construction of returns is based solely on the normal distributions. These results might have a crucial effect on volatility modelling and connected topics – mainly portfolio analysis and (conditional) Value-at- Risk.

The paper is structured as follows. In Section 2, we present the basic methodology needed for the statistical analysis conducted in the paper. In Section 3, we describe the dataset and present the crucial findings about the process of volatility. In Section 4, we show whether the findings can be used for simulation of series, which would resemble the real-world

financial series. Section 5 concludes and states directions for further research in this area.

2. Methodology

In this section, we describe the basic methodology used in the rest of the paper. Hence, we describe the notions of long-range dependence, tests for stationarity (and unit roots) and tests for long-range dependence (specifically anti-persistence in our case).

2.1. Long-range dependence and stationarity

Long-range dependence is highly connected to the Hurst effect, i.e. a situation characteristic with long periods when the series is above the mean which are followed by long periods when the series is below the mean of the series while the series still remains stationary, and so also Hurst exponent H. A critical value of Hurst exponent is 0.5 and suggests two possible processes. H being equal to 0.5 implies either an uncorrelated or a short-term dependent process [22, 1]. Uncorrelated process has zero auto-covariances at all non-zero lags and short-term dependent process shows non-zero auto-covariances at low lags vanishing exponentially to zero for higher lags. For $H > 0.5$, auto-covariances of the process are positive at all lags so that the process is called long-range dependent with positive correlations or persistent. Auto-covariances of such process are hyperbolically decaying and non-summable [1]. On the other hand, for $H < 0.5$, the process is said to be long-range dependent with negative correlations or anti-persistent. Similarly to the previous case, auto-covariances of such process are slowly decaying but summable. The persistent process is visibly locally trending while the anti-persistent process switches signs more frequently than a random process would [34, 25, 13, 1].

Hurst exponent H is connected to parameter d of fractional integration so that $H = d + 0.5$. Long-range dependent processes are frequently defined in two domains – time and frequency:

- *(time domain)* A stationary process with an autocorrelation function $\rho(k)$ decaying as $\rho(k) \alpha k^{2H-2}$ for $k \to +\infty$ is called long-range dependent with Hurst exponent H.
- *(frequency domain)* A stationary process with a spectral density $f(\lambda)$ following $f(\lambda) \alpha |\lambda|^{1-2H}$ for $\lambda \to 0$ is called long-range dependent with Hurst exponent H.

Note that a notion of long-range dependence is tightly connected to stationarity of the process, which is obvious from both presented definitions. Without stationarity, there can be no standard long-range dependence. Going back to the two given definitions, we can construct neither autocorrelation function nor spectrum for a non-stationarity process due to infinite variance; hence we cannot talk about standard long-range dependence. To check for stationarity, we apply two standard tests – ADF [10] and KPSS [19]. ADF has a null hypothesis of a unit root and can take lags of the differenced series into consideration and thus controlling for the memory effects. KPSS, on contrary, has a stationarity null hypothesis and it also can control for memory effects with a use of auto-covariance adjusted variance with Barlett weights [14].

2.2. Anti-persistence tests

As we are interested primarily in long-range dependence and not necessarily in a specific value of H or d, we use (modified) rescaled range and rescaled variance analyses for testing the presence of long-range dependence in the studied process.

2.2.1. Classical rescaled range analysis

Rescaled range analysis (R/S) is the traditional Hurst exponent estimation method proposed by Hurst [16] and further adjusted by Mandelbrot & Wallis [26]. In the procedure, the series of returns of length T is divided into N adjacent sub-periods of length υ so that $N\upsilon = T$. For each sub-period, a rescaled range of a profile X_t (cumulative deviations from an arithmetic mean) is calculated as R/S, where $R = max(X_t) - min(X_t)$ is a range of the corresponding profile and S is a standard deviation of the corresponding returns. Rescaled ranges are calculated for each sub-period of length υ and an average rescaled range is calculated [29]. The rescaled ranges scale as:

$$(R/S)_\upsilon \, \alpha \, \upsilon^H.$$

V statistic, which is also used for a cycles detection, a stability testing of the Hurst exponent or a change in scaling behaviour (crossover) detection, is defined as:

$$V_\upsilon = (R/S)_\upsilon / \sqrt{\upsilon}$$

and converges to the distribution with a rather complicated distribution function F_V for independent processes (see Refs. [24, 16, 29] for details

and critical values). For our purposes, we set $\upsilon = T$ to test for the presence of long-range dependence in the underlying process by the means of V statistic.

2.2.2. Modified rescaled range analysis

Due to the bias of *R/S* analysis for short-term memory processes, Lo [24] proposed the modified rescaled range analysis (*M*−*R/S*) which utilizes a modified standard deviation S_M. S_M is defined with a use of auto-covariances of the original increment series γ_j up to lag ξ as:

$$S^M = \sqrt{S^2 + 2\sum_{j=1}^{\xi} \gamma_j \left(1 - \frac{j}{\xi + 1}\right)} \; .$$

R/S is thus a special case of $M - R/S$ with $\xi = 0$. The distribution of the modified V statistic converges to F_V not only for independent processes but also for short-range dependent ones. Choice of the correct lag ξ is critical for the estimation of the modified rescaled ranges [35, 33]. Lo [24] suggested optimal lag based on the sample first-order autocorrelation coefficient of the original series $\rho(1)$. However, we set some fixed values of ξ (defined later) since the automatic lag assumes that the underlying process is $AR(1)$, which is rather strict.

2.2.3. Rescaled variance analysis

Rescaled variance analysis (V/S) was proposed by Giraitis and co-authors [14] as a modified version of KPSS statistic [19], which is usually used for testing of stationarity but was also shown to have good power for series with long-term memory [20, 21]. The procedure is very similar to the modified rescaled range analysis and differs in a use of a sample variance of the profile of the series instead of the range. As an alternative to the V statistic, the M statistic is defined as:

$$M_\upsilon = \frac{\mathrm{var}(X_t)}{\upsilon (S^M)^2} .$$

Note that the modified standard deviation is used so that the method is robust to short-range dependence as well. Variance of the M statistic is

much lower than the one of *R/S* and *M* − *R/S* so that the confidence intervals are much narrower and the method is thus more efficient. Similarly to *R/S* and *M* − *R/S,* one needs to set $\upsilon = T$ and construct *M* statistic if testing for the presence of long-range dependence in the process.

2.2.4. Moving block bootstrap

The bootstrap method [11] has been proposed to deal with the statistical properties of small samples. The basic notion behind the procedure is resampling with replacement from the original series and repeated estimation of a specific parameter. By shuffling, a distribution of the original series remains unchanged while possible dependencies are distorted [8]. Hypothesis can be then tested with a use of *p*-values based on the bootstrapped estimates. For our purposes and for the time series analysis in general, the simple bootstrap is not enough as the shuffling rids us not only from the long-range dependence but the short-range dependencies as well. Srinivas & Srinivasan [32] proposed a modified method which retains the short-term dependence characteristics but lacks the long ones – the moving block bootstrap with pre-whitening and post-blackening.

In the procedure, the original time series $\{x_t\}^T_{t=1}$ =1 is firstly pre-whitened (filtered) by a specific process – usually *AR*(*p*) or *MA*(*q*) – and residuals ε_t are obtained. Residuals are centred so that the centred residuals $\{\varepsilon_{t,c}\}^T_{t=1}$ are just the demeaned residuals. The series $\{\varepsilon_{t,c}{}^T_{t=1}$ is divided into *m* blocks of length ζ while $m\zeta = T$. The blocks are reshuffled and post-blackened with the use of the model from the pre-whitening part and residuals $\varepsilon_{t,c}$ to form the new bootstrapped time series $\{x_{t,b}\}^T_{t=1}$. Such time series retains the short-range dependence, potential heteroskedasticity and short-term trends as well as the distribution of the original time series. Importantly for the long-range dependence testing, for small enough ζ, the long-term correlations are torn. The parameter of interest is then estimated on the new time series. Bootstrapping is repeated *B* times so that distributions of M_υ and V_υ are obtained and used to get *p*-values for the null hypothesis of no long-range dependence in the series.

For our purposes, we use *AR*(1) process for pre-whitening and post-blackening, which is standard in finance literature [12, 17], and $\zeta = 20$.

Such choice of ζ should be sufficient for ridding of the potential long-range dependence while the other properties remain similar to the original process. The procedure is repeated for lags $\xi = 0,1,2,5,10,15,30,50,75,100$ for both modified rescaled range analysis and rescaled variance analysis with $B = 1000$ bootstrap repetitions. As will be visible in the following sections, the analyzed series are on the edge between stationarity and non-stationarity, and we will eventually analyze the first differences of the series. However, for such boundary cases, there is a high risk of over-differencing which would inflict MA(1) process in the series. To control for this, we also apply the moving block bootstrap with the same parameters as noted before but with MA(1) for pre-whitening and post-blackening. This way, we can be more confident about our findings while controlling for the most problematic cases.

2.3. Garman-Klass daily variance estimator

There are various estimators of daily volatility (or variance) ranging from very simple absolute and squared returns through model-based estimators (e.g. GARCH or implied volatility based) to range-based estimators and realized variance [6]. From many possibilities, we choose Garman-Klass estimator:

$$\widehat{\sigma^2_{GK,t}} = \frac{(\log(H_t / L_t))^2}{2} - (2\log 2 - 1)(\log(C_t / O_t))^2,$$

where H_t and L_t are daily highs and lows, respectively, and C_t and O_t are daily closing and opening prices, respectively. This estimator does not take overnight volatility into consideration but is very simple and efficient (much more efficient than absolute and various power-returns, comparable with other range-based measures and less efficient than the realized variance) [6]. Even though the realized variance would be a more efficient choice, it is not easily obtainable for all assets while for the Garman-Klass estimator, all necessary variables are available freely for practically all financial assets. Moreover, the Garman-Klass estimator does not impose any specific model on the daily variance process, compared to (*G*)ARCH and stochastic volatility models.

3. Data and statistical analysis

We analyze series of three stocks with some of the highest capitalizations in the US markets – AAPL (Apple Inc.), IBM (International Business Machines Corporation) and MSFT (Microsoft Corporation) – between 3.1.2000 and 29.2.2012 (3059 observations). Normalized prices are shown in figure 1. We can see that even though all three companies are technological, they underwent very different dynamics during the analyzed period. Apple, as a favourite of the most recent days, has grown remarkably while IBM and Microsoft have been rather stagnant. Estimates of daily volatility are shown in figure 2. IBM and MSFT experienced very similar dynamics of volatility as well as its levels. AAPL, on contrary, shows markedly higher average values of volatility with more extreme values than the other two. Nevertheless, we do not observe any drastic jumps in the volatility levels and we rather find smooth transitions from lower to higher levels or vice versa. Moreover, all three series seem non-stationary at least visually as we notice phases with varying means. This observation is checked on statistical basis later in the text.

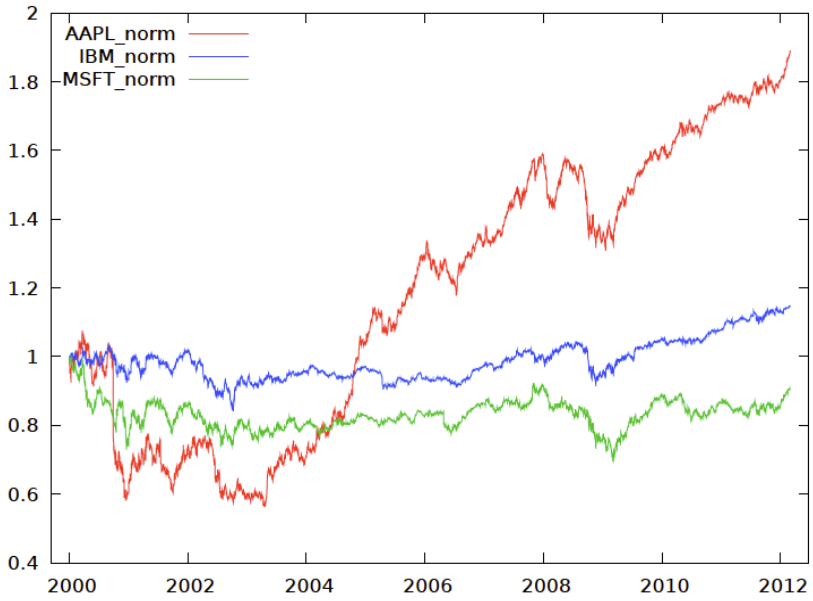

Figure 1. Normalized stock indices prices. Logarithmic prices divided by the first observation of the respective series for a better comparison.

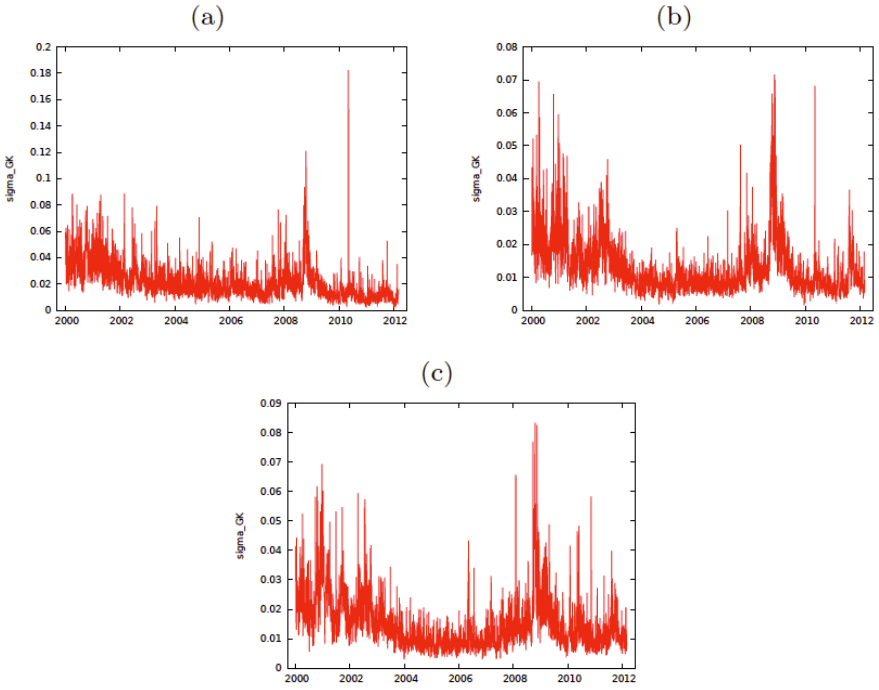

Figure 2. Garman-Klass estimates $\hat{\sigma}_{GK}$ of volatility for (a) AAPL, (b) IBM and (c) MSFT.

As the Garman-Klass estimator does not take overnight dynamics into consideration, we analyze daily logarithmic returns defined as $r_t = \log(C_t) - \log(O_t)$. Descriptive statistics of r_t are summarized in table 1. The table also includes the descriptive statistics of the standardized returns (returns standardized by the square root of the estimated daily variance). We can see that the raw returns are fat-tailed and positively skewed while the standardized returns are very close to having normal tails. This result is supported by the Jarque-Bera test [18] – the raw returns are not normally distributed but the standardized returns are very close to being normally distributed. This is also supported by the quantile-quantile (*QQ*) plots in figure 3. Based on Ljung-Box test [23], we find no significant autocorrelations in the first thirty lags for the standardized returns. Based on these findings, we propose the first Claim[1]:

Claim 1. *Logarithmic open-close returns are uncorrelated and normally distributed with time varying volatility* $\approx N(0, \sigma_t)$.

[1] All "Claims" presented in this paper should be taken as approximate results. Nevertheless, we show later in the text that these "Claims" can be used to construct the series which strongly resemble the basic stylized facts of the financial returns.

Table 1. *Descriptive statistic of returns and returns standardized by* $\widehat{\sigma_{GK}}$.

	mean	SD	skew.	ex. kurt.	JB	p-val.	Q(30)	p-val.
r_{AAPL}	0.0001	0.0247	0.0432	2.4504	766	0.0000	65	0.0000
r_{IBM}	0.0008	0.0150	0.0836	4.7032	2823	0.0000	69	0.0000
r_{MSFT}	-0.0001	0.0173	0.2105	3.4390	1530	0.0000	46	0.0300
$r_{st,AAPL}$	0.0336	1.0374	0.0510	-0.3711	19	0.0000	40	0.1120
$r_{st,IBM}$	0.1059	1.0111	0.0551	-0.1963	6	0.0396	47	0.0250
$r_{st,MSFT}$	0.0027	0.9880	0.0552	-0.2216	8	0.0201	36	0.2180

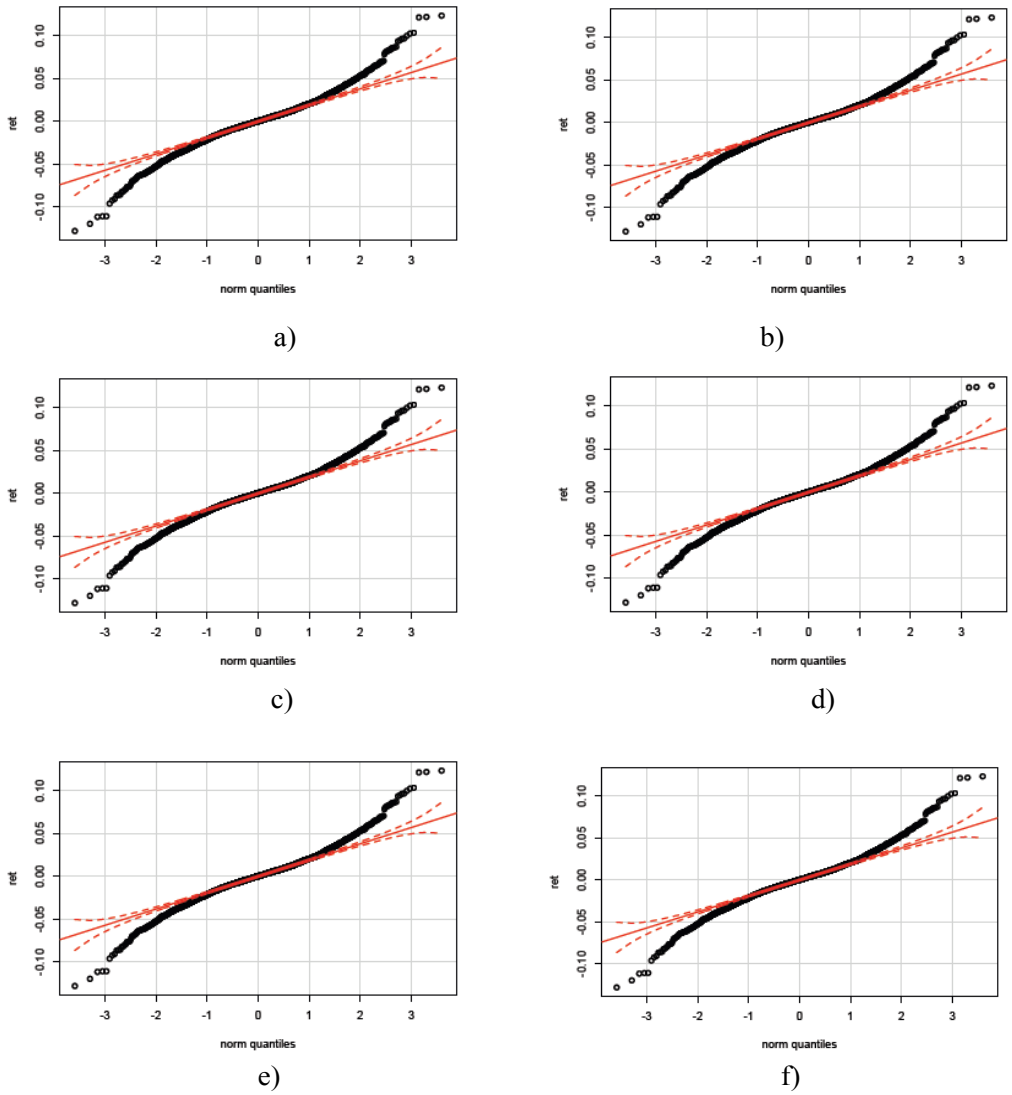

Figure 3. *QQ plots for raw (first row) and standardized (second row) returns*. AAPL (first column), IBM (second column) and MSFT (third column). 95% confidence intervals for normal distribution are marked by red dashed lines.

Let us now focus on the approximate distribution of the volatility process. Figure 4 presents the *QQ* plots against the normal distribution and uncovers that the process of volatility is far from being normally distributed whereas its logarithmic transformation is approximately normally distributed and evidently, so are the increments of logarithmic volatility. Note that these findings also indicate that there are no obvious structural breaks which would either cause the logarithmic increments to be fat-tailed (if there were some sudden jumps in the volatility levels) and/or the logarithmic volatility to be bi- or multi-modal (neither the *QQ* plots nor the histograms, not shown here, indicate such a characteristic). Therefore, we propose two other Claims:

Claim 2. *Logarithmic volatility is close to being normally distributed with a mean* $\mu_{\log\sigma}$.

Claim 3. *Increments of logarithmic volatility are close to being normally distributed* $\approx N(0,\sigma_{\Delta})$.

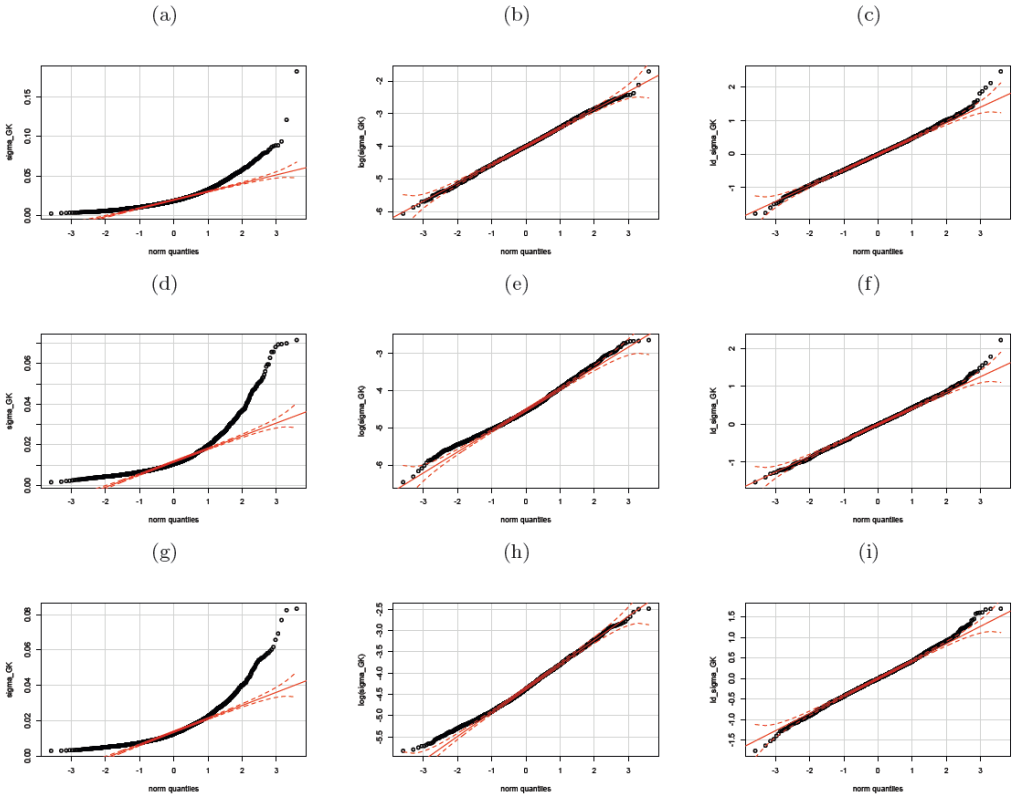

Figure 4. *QQ plots for* $\widehat{\sigma_{GK}}$ *(first column),* $\log\widehat{\sigma_{GK}}$ *(second column) and* $\Delta\log\widehat{\sigma_{GK}}$ *(third column).* AAPL (first row), IBM (second row) and MSFT (third row). 95% confidence intervals for normal distribution are marked by red dashed lines.

Table 2. Stationarity tests for $\widehat{\sigma_{GK}}$. KPSS with the null hypothesis of stationarity and ADF with the null hypothesis of a unit root. *, ** and *** for significance at 10%, 5% and 1% significance level, respectively.

	KPSS(1)	KPSS(10)	KPSS(100)	ADF(1)	ADF(10)	ADF(100)
$AAPL$	43.8283***	11.2969***	1.7515***	−18.8717***	−6.3622***	−2.2830
IBM	20.5034***	4.7213***	0.7410***	−15.0817***	−5.7168***	−3.1221**
$MSFT$	21.2672***	5.0439***	0.7780***	−16.1605***	−5.9202***	−2.6700*
$\log AAPL$	57.5655***	13.8110***	1.9881***	−16.5976***	−5.6643***	−1.7975
$\log IBM$	26.2098***	5.9606***	0.8626***	−14.8127***	−5.4914***	−2.6609*
$\log MSFT$	24.8671***	5.7543***	0.8176***	−15.9073***	−5.2080***	−2.2931
$\Delta \log AAPL$	0.0012	0.0055	0.0391	−62.8961***	−24.8072***	−7.7096***
$\Delta \log IBM$	0.0009	0.0038	0.0241	−63.5708***	−23.2364***	−6.7480***
$\Delta \log MSFTL$	0.0015	0.0072	0.0431	−64.9717***	−24.9563***	−7.1949***

Now, we focus on an essential question of stationarity of the series. The results for ADF and KPSS tests for various lags are summarized in Table 3. We use lags 1, 10 and 100 to control for practically no memory, short memory and long-term memory, respectively. The results are quite straightforward. Firstly, volatility process is neither stationary not an evident unit root process. For long lags, we are not able to reject the unit root null for AAPL and MSFT. Secondly, the same is true for the logarithmic transformation of volatility but for all three stocks. Moreover, the results are much stronger here as the series are very close to being normally distributed which is assumed for the tests. Here again, after controlling for long-range dependence (100 lags), we cannot reject the unit root of the series. This indicates that after controlling for long-range dependence in the increments, we cannot reject unit root for the logarithmic volatility. Thirdly, increments of the logarithmic volatility are asymptotically stationary even after controlling for long-term memory. These indicate that the first differences of the logarithmic volatility are likely to be long-range dependent. Based on these findings, we propose four other Claims:

Claim 4. *Volatility and logarithmic volatility are non-stationary.*

Claim 5. *Logarithmic volatility is close to a unit root process after controlling for the long-term memory.*

Claim 6. *Increments of logarithmic volatility are asymptotically stationary.*

Claim 7. *Increments of logarithmic volatility are likely to be long-range dependent.*

Table 3. Anti-persistence tests – bootstrapped *p*-values for the null of "no anti-persistence" controlling for *AR*(1) process. ξ stands for the number of lags taken into consideration for standard deviation S^M for *V* and *M* statistic.

ξ	V_{AAPL}	M_{AAPL}	V_{IBM}	M_{IBM}	V_{MSFT}	M_{MSFT}
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0090	0.0000	0.0040	0.0000	0.0000	0.0000
50	0.0710	0.0000	0.0150	0.0000	0.0060	0.0030
75	0.3930	0.0150	0.0040	0.0000	0.0280	0.0180
100	0.7700	0.0680	0.0090	0.0000	0.1610	0.0830

Therefore, it is needed to analyze the increments of logarithmic volatility and its correlation structure. Figure 5 presents the autocorrelation and partial autocorrelation functions. We see a common pattern for all three analyzed series – strongly negative autocorrelation at the first lag for ACF which vanishes for further lags, and negative partial autocorrelations which decay relatively slowly to zero for PACF. This is indicative for two possible processes – a strong MA(1) process [4] or a strongly anti-persistent ARFIMA(0,*d*,0) process [15, 3, 9]. Debowski [9] actually shows that ARFIMA processes can be generalized so that we obtain stationary and invertible processes even for anti-persistent processes with $d \in (-1, 0)$, i.e. $H \in (-0.5, 0.5)$. To test for anti-persistent processes while still controlling for short-term memory as well as potential over-differencing[2], we use the modified rescaled range analysis and rescaled variance analysis with moving block bootstrap *p*-values for the null hypothesis of no anti-persistence with AR(1) and MA(1) processes in pre-whitening and post-blackening procedures. The results for ξ = 0, 1, 2, 5, 10, 15, 30, 50, 75, 100 are summarized in tables 3 and 4. We observe that the results for both AR(1) and MA(1) controls are practically the same – for short to medium lags, we reject "no anti-persistence" null hypothesis, while for long lags, we do not. However, it is hard to distinguish between short and long-term memory for such long lags so that we treat the series as anti-persistent. For further discussion of the issue, see Beran [1]. Based on these results, we propose the last Claim:

[2] Since the unit root tests have low power when too many lags are taken into consideration, it is possible that taking the first differences of potentially spuriously detected unit root process imposes a strong moving average process, MA(1).

Claim 8. *Increments of logarithmic volatility are strongly anti-persistent with* $d \in (-1, 0)$, *i.e.* $H \in (-0.5, 0.5)$.

Table 4. Anti-persistence tests – bootstrapped *p*-values for the null of "no anti-persistence" controlling for $MA(1)$ process. ξ stands for the number of lags taken into consideration for standard deviation S^M for V and M statistic.

ξ	V_{AAPL}	M_{AAPL}	V_{IBM}	M_{IBM}	V_{MSFT}	M_{MSFT}
0	0.0080	0.0000	0.0050	0.0000	0.0000	0.0000
1	0.0070	0.0000	0.0040	0.0010	0.0000	0.0030
2	0.0040	0.0000	0.0040	0.0000	0.0000	0.0010
5	0.0040	0.0000	0.0020	0.0000	0.0000	0.0030
10	0.0100	0.0000	0.0030	0.0000	0.0010	0.0020
15	0.0030	0.0000	0.0030	0.0000	0.0000	0.0020
30	0.0210	0.0000	0.0070	0.0000	0.0000	0.0050
50	0.1030	0.0010	0.0220	0.0000	0.0090	0.0220
75	0.4050	0.0500	0.0030	0.0000	0.0430	0.0480
100	0.7140	0.1930	0.0040	0.0000	0.1550	0.1390

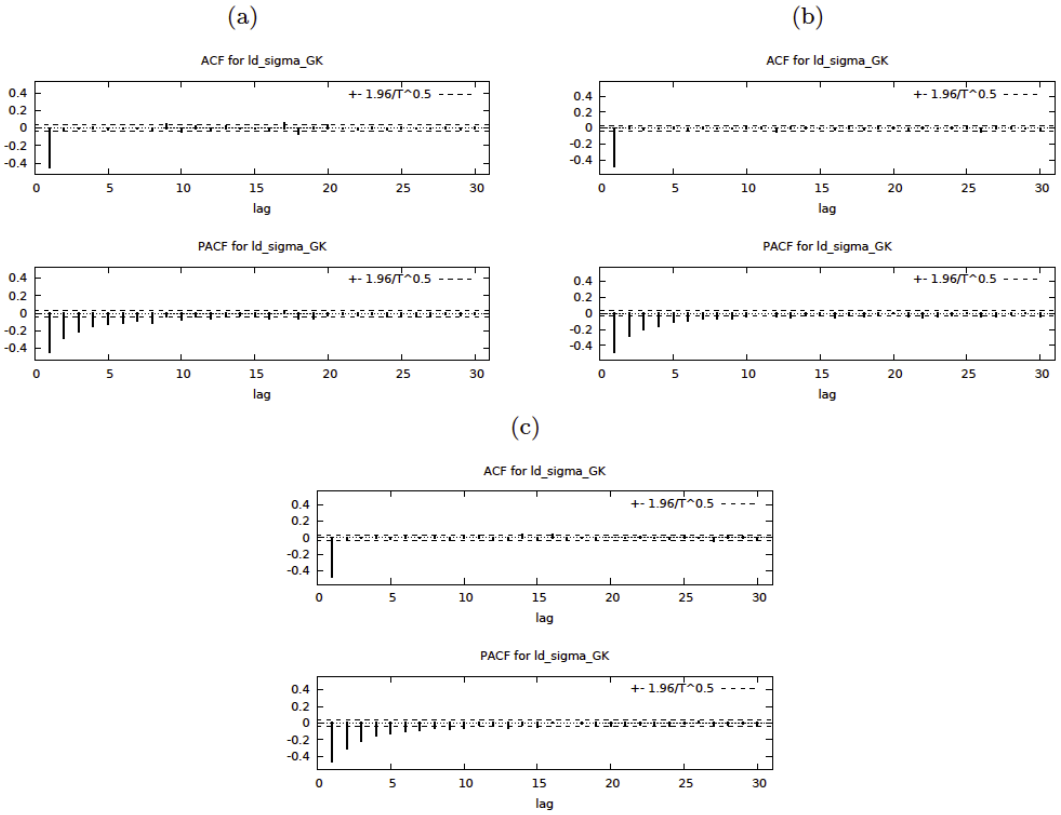

Figure 5. Autocorrelation and partial autocorrelation functions of logarithmic differences Of $\widehat{\sigma_{GK}}$ (a) AAPL, (b) IBM and (c) MSFT.

4. Simulations

Based on all the Claims we made, we now try to reconstruct the series with the observed statistical properties and compare whether these are in hand with the real financial series. To do so, we need to estimate three parameters – $\mu_{log\sigma}$, σ_Δ and d. We then simulate the series of the logarithmic returns in the following way. First, we simulate ARFIMA (0,d,0) process[3] for the increments of the logarithmic volatility with a standard deviation of σ_Δ. Second, we integrate the series and adjust it so that the average value of the integrated series is $\mu_{log\sigma}$. Third, we take the exponential of the integrated series to get the series of the time-dependent standard deviation. In the last step, we use the standard deviation for uncorrelated normally distributed series with zero mean.

The average logarithmic volatilities $\mu_{log\sigma}$ are –3.9962, –4.4959 and –4.3439 for AAPL, IBM and MSFT, respectively. The standard deviations of the increments of the logarithmic volatility σ_Δ are 0.4817, 0.4388 and 0.4463 for AAPL, IBM and MSFT, respectively. The biggest issue is the estimation d because majority of the d and H estimators are constructed primarily for the persistent processes with $d > 0$, i.e. $H > 0.5$, and their finite sample performance for anti-persistent processes has not been seriously discussed in the literature yet. To overcome this issue, we present the simulations for $-0.9 \leq d \leq -0.1$ with a step of 0.1. The other two parameters are set to $\mu_{log\sigma} = -4.5$ and $\sigma_\Delta = 0.45$.

In figure 6, we present the simulated series of the standardized logarithmic returns for various values of d. We can see that for $d = -0.1$, the series is degenerate and shows some extreme values and does not resemble the empirical series of logarithmic returns. The lower the d parameter gets the less extreme the simulated values are and also the less extreme the volatility clustering is. The series which resemble the real world data the most are the ones with $d = -0.4, -0.5, -0.6$. For the lower values of d, the volatility clustering becomes very negligible. The best fit to the real-world series is found for $d = -0.5$ where we can observe very different phases of market behaviour – calm periods are followed by highly volatile and vice versa. It is needed to note that the returns for $-0.9 \leq d \leq -0.4$ are uncorrelated (the ACF and PACF are not shown here).

[3] We choose ARFIMA(0,d,0) because we need an anti-persistent Gaussian process which allows for strong anti- persistence. ARFIMA is an obvious and logical choice.

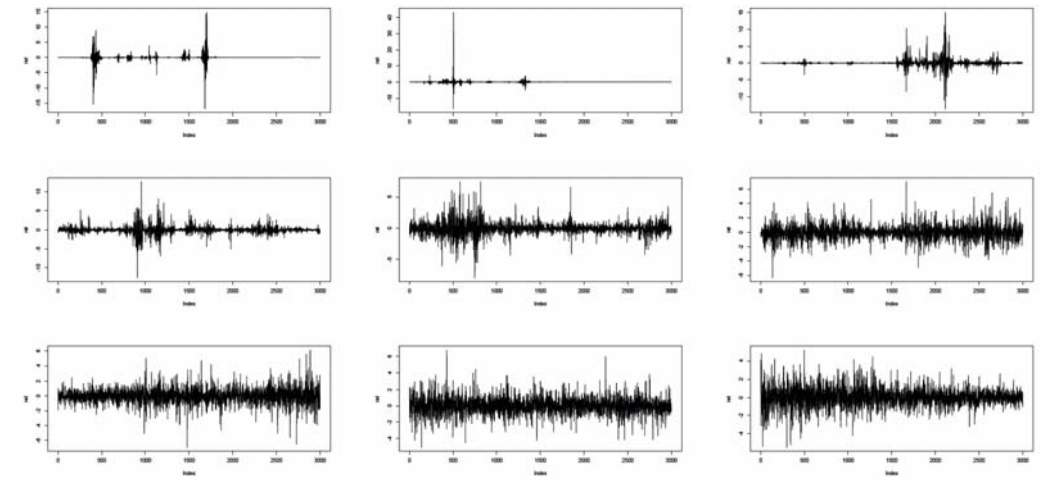

Figure 6. Simulated standardized returns. Starting from $d = -0.1$ (top left) and $d = -0.2$ (top middle) to $d = -0.9$ (down right).

Simulated time-dependent standard deviations are summarized in figure 7. Here again, we observe that for d parameter close to zero, the standard deviation process is degenerate with several extreme spikes. The time-dependent volatility is the most realistic for $d = -0.4$ and $d = -0.5$ and it is actually very hard to distinguish between these two and the standard deviations in figure 2 for the three analyzed stocks. For lower d, the standard deviation series become too volatile and do not produce phases with high volatility switching with phases of low volatility. Volatility persistence is further illustrated by autocorrelation functions of absolute returns in figure 8. Persistence of absolute returns is considered to be a stylized fact of financial markets. Even though the question of stationarity of absolute returns themselves remains, we do not deal with it and just check whether the stylized fact is observed for the simulated series as well. We observe that for the processes with d between –0.1 and –0.5, the autocorrelations decay slowly and remain positive for high lags. Again for $d = -0.5$, the autocorrelations remain slightly above 0.2 for all analyzed lags, which is most closely in hand with what is reported for the real-world time series [7].

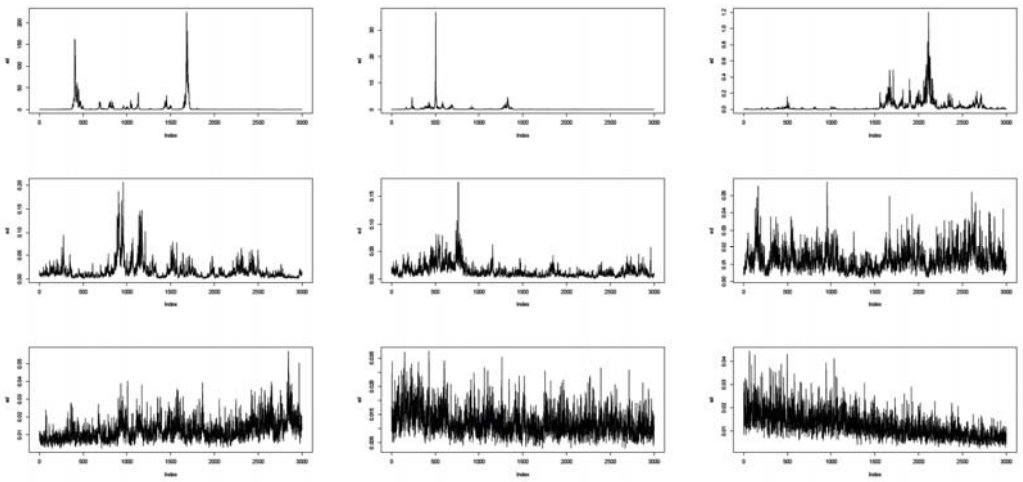

Figure 7. Simulated standard deviation processes. Starting from $d = -0.1$ (top left) and $d = -0.2$ (top middle) to $d = -0.9$ (down right).

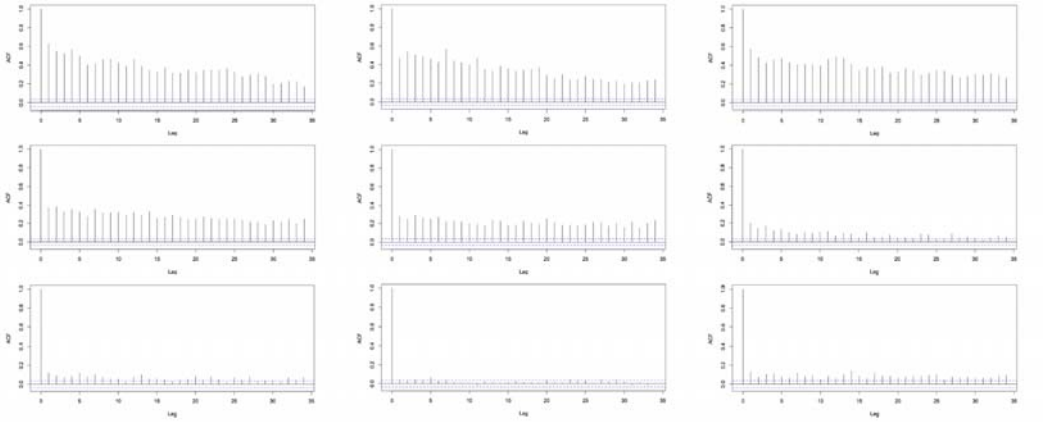

Figure 8. Autocorrelation function for absolute returns of the simulated processes. Starting from $d = -0.1$ (top left) and $d = -0.2$ (top middle) to $d = -0.9$ (down right).

5. Conclusions and discussion

We introduced the alternative paradigm to volatility modelling of financial securities. On the example of three stocks of highly capitalized companies, we showed that volatility process is non-stationary and its logarithmic transformation together with logarithmic increments is approximately normally distributed. Further, the increments have been shown to be highly anti-persistent. Together with the assertion that logarithmic returns are normally distributed, and uncorrelated with time-varying volatility, we proposed the new returns-generating process. Note

that the whole procedure is based on empirical observations without any limiting assumptions. We are able to construct the returns series which remarkably mimic the real-world series and posses the standard stylized facts – uncorrelated returns with heavy tails, strongly auto correlated absolute returns and volatility clustering. Therefore, the proposed methodology opens a rather unexplored field in research of financial volatility. As this paper rather introduces the framework, there are many possibilities for further research in the field.

First of all, the crucial distinguishing between stationary and non-stationary series may raise questions. However, all standard stationarity tests have relatively low power and size against alternatives presented in this paper. Therefore, the correctness of this approach should be decided based on its ability to forecast volatility. This leads us to other future directions. Second, the proposed methodology should be compared to standardly used methods on basis of their forecasting accuracy. Third, the Value-at-Risk usefulness of the methodology is to be checked. Fourth, the correct specification of the logarithmic differences of the volatility process should be more thoroughly analyzed. ARFIMA(0,d,0) is only the initial proposition and there are more possibilities such as general ARFIMA(p,d,q) specifications or multifractal generalizations. Fifth, consistent and efficient estimators of d and H for highly anti-persistent processes should be discussed. And finally, the proposed methodology needs to be checked on other estimators of volatility (mainly various estimators of realized volatility).

Acknowledgments

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REFERENCES

[1] Beran, J. (1994), *Statistics for Long-Memory Processes,* Volume **61** of Monographs on Statistics and Applied Probability. New York: Chapman and Hall.

[2] Black, F. and M. Scholes (1973), *The pricing of options and corporate liabilities*. Journal of Political Economy **81**(3), 637-654.

[3] Bondon, P. and W. Palma (2006), *A class of antipersistent processes*. Journal of Time Series Analysis **28**(2), 261-273.

[4] Box, G. and G. Jenkins (1970), *Time Series Analysis: Forecasting and Control. Holden Day*, San Francisco.
[5] Cajueiro, D. and B. Tabak (2005), *The rescaled variance statistic and the determination of the hurst exponent. Mathematics and Computers in Simualtion* **70**(3), 172-179.
[6] Chou, R.-Y., H.-C. Chou, and N. Liu (2010), *Handbook of Quantitative Finance and Risk Management, Chapter Range Volatility Models and Their Appliations in Finance,* pp. 1273-1281. Springer.
[7] Cont, R. (2001), *Empirical properties of asset returns: stylized facts and statistical issues*. Quantitative Finance **1**(2), 223-236.
[8] Davison, A. and D. Hinkley (1997), *Bootstrap Methds and Their Application*. Cambridge University Press.
[9] Debowski, L. (2011), *On processes with hyperbolically decaying autocorrelations*. Journal of Time Series Analysis **32,** 580-584.
[10] Dickey, D. and W. Fuller (1979), *Estimators of autoregressive time series with a unit root.* Journal of the American Statistical Association **74,** 427-431.
[11] Efron, B. (1979), *Bootstrap methods: Another look at the jacknife*. Annals of Statistics **7,** 1-26.
[12] Efron, B., R. Tibshirani, and R. Tibshirani (1993), *An introduction to the bootstrap*. Chapman & Hall.
[13] Embrechts, P. and M. Maejima (2002), *Selfsimilar Processes*. Princeton University Press. [14] Giraitis, L., P. Kokoszka, R. Leipus, and G. Teyssiere (2003). Rescaled variance and related tests for long memory in volatility and levels. Journal of Econometrics **112,** 265-294.
[15] Hosking, J. (1981), *Fractional differencing*. Biometrika **68**(1), 165-176.
[16] Hurst, H. (1951), *Long term storage capacity of reservoirs*. Transactions of the American Society of Engineers **116,** 770-808.
[17] Izzeldin, M. and A. Murphy (2000), *Bootstrapping the small sample critical values of the rescaled range statistic*. The Economic and Social Review **31**(4), 351-359.
[18] Jarque, C. and A. Bera (1981), *Efficient tests for normality, homoscedasticity and serial independence of regression residuals: Monte carlo evidence*. Economics Letters **7**(4), 313-318.
[19] Kwiatkowski, D., P. Phillips, P. Schmidt, and Y. Shin (1992), *Testing the null of stationarity against alternative of a unit root: How sure are we that the economic time series have a unit root?* Journal of Econometrics **54,** 159-178.
[20] Lee, D. and P. Schmidt (1996). *On the power of the kpss test of stationarity against fractionally-integrated alternatives*. Journal of Econometrics **73,** 285-302.
[21] Lee, H. and C. Amsler (1997), *Consistency of the kpss unit root against fractionally integrated alternative.* Economics Letters **55,** 151-160.
[22] Lillo, F. and J. Farmer (2004). *The long memory of the efficient market. Studies in Nonlinear Dynamics and Econometrics* **8**(3).
[23] Ljung, G. and G. Box (1978). *On a measure of a lack of fit in time series models*. Biometrika **65**(2), 297-303.
[24] Lo, A. (1991), *Long-term memory in stock market prices*. Econometrica **59**(5), 1279-1313.
[25] Mandelbrot, B. and J. van Ness (1968), *Fractional brownian motions, fractional*

noises and applications. SIAM Review **10**(422), 422-437.
[26] Mandelbrot, B. and J. Wallis (1968), *Joah, joseph and operational hydrology*. Water Resources Research **4,** 909-918.
[27] Merton, R. (1973), *Theory of rational option pricing*. Bell Journal of Economics and Management Science **4**(1), 141-183.
[28] Morana, C. and A. Beltratti (2004), *Structural change and long-range dependence in volatility of exchange rates: either, neither or both?* Journal of Empirical Finance **11,** 629-658.
[29] Peters, E. (1994), *Fractal Market Analysis – Applying Chaos Theory to Investment and Analysis.* New York: John Wiley & Sons, Inc.
[30] Poon, S.-H. and C. Granger (2003), *Forecasting volatility in financial markets: A review*. Journal of Economic Literature **41,** 478-539.
[31] Poon, S.-H. and C. Granger (2005), *Practical issues in forecasting volatility.* Financial Analysts Journal **61**(1), 45-56.
[32] Srinivas, V. and K. Srinivasan (2000), *Post-blackening approach for modeling dependent annual streamflows*. Journal of Hydrology **230,** 86-126.
[33] Teverosky, V., M. Taqqu, and W. Willinger (1999), *A critical look at lo's modified r/s statistic.* Journal of Statistical Planning and Inference **80**(1-2), 211-227.
[34] Vandewalle, N., M. Ausloos, and P. Boveroux (1997), *Detrended fluctuation analysis of the foreign exchange market.* In Econophysic Workshop, Budapest, Hungary.
[35] Wang, W., P. van Gelder, J. Vrijling, and X. Chen (2006), *Detecting long-memory: Monte Carlo simulations and application to daily streamflow processes*. Hydrology and Earth System Sciences Discussions **3,** 851-862.

THE CHAOTIC PRODUCTION GROWTH MODEL OF THE MONOPOLY FIRM AND INCENTIVES TO INNOVATE

Vesna D. JABLANOVIC*

Abstract. *Monopoly is a market with only one seller. Characteristics of the model of a monopolistic market structure are: a) a firm is a price-maker; b) a firm do not behave strategically; c) entry into the industry is completely blocked; d) buyers are price taker; e) no close substitutes for the firm's product; f) well informed buyers.*

Monopoly has the ability to influence the market price. Except a monopolist's choice of price and output, there are other decisions a monopolist must make. One of the most important is how much to invest in research and development activities. Process innovation explains an idea the lowers the cost of producing existing products. Monopoly will invest in a new technology whenever doing so lowers its costs.

The basic aim of this paper is to construct a relatively simple chaotic growth model of the monopoly quantity that is capable of generating stable equilibria, cycles, or chaos. Incentives to innovate are included in model.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}$, *plays a crucial role in explaining local stability of the monopoly's production, where, b – the coefficient of the marginal cost function of the monopoly firm, m – the coefficient of the inverse demand function, e – the coefficient of the price elasticity of the monopoly's demand,* α *– the coefficient which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.*

Keywords: *Monopoly, Production, Process innovation, Chaos.*

1. Introduction

Monopoly is the form of market structure in which a single firm sells a commodity for which there are no close substitutes. Under monopoly, there is a single producer and entry into the market by additional sellers is

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blocked. Monopoly is a price maker because it can influence the price through this choice of quantity. A monopolist recognizes that the amount it sells influences the price it receives for its output. Except a monopolist's choice of price and output, there are other decisions a monopolist must make. One of the most important is how much to invest in research and development activities. For example, process innovation is an idea that lowers the cost of producing existing products. The innovation leads to lower production costs. Monopoly will invest in a new technology whenever doing so lowers its costs.

Chaos theory can explain effectively unpredictable economic long time behavior arising in a deterministic dynamical system because of sensitivity to initial conditions. It must be emphasized that a deterministic dynamical system is perfectly predictable given perfect knowledge of the initial condition, and is in practice always predictable in the short term. The key to long-term unpredictability is a property known as sensitivity to (or sensitive dependence on) initial conditions.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983, 1997), Grandmont (1985), Goodwin (1990), Medio (1993, 1996), Lorenz (1993), among many others.

The basic aim of this paper is to provide a relatively simple chaotic growth model of the monopoly's production that is capable of generating stable equilibria, cycles, or chaos. Incentives to innovate are included in the model.

A Simple Chaotic Model of a Profit-Maximizing Monopoly

In the model of a profit-maximizing monopoly, take the inverse demand function:

$$P_t = n - m\,Q_t \tag{1}$$

where P – monopoly price; Q – monopoly output; n, m – coefficients of the inverse demand function.

Further, suppose the quadratic marginal-cost function for a monopoly is:

$$MC_t = a + b\,Q_t + c\,Q_t^{\,2} \tag{2}$$

where *MC* – marginal cost; *Q* – monopoly output; *a, b, c* – coefficients of the quadratic marginal-cost function.

Monopoly will invest in a new technology whenever doing so lowers its costs. With innovation, the firm's marginal cost curve falls to:

$$MC_t = (a + bQ_t + c\,Q_t^2) - d\,(a + bQ_t + c\,Q_t^2) \tag{3}$$

or:

$$MC_t = a(1-d) + b(1-d)\,Q_t + c\,(1-d)\,Q_t^2, \tag{4}$$

where *MC* – marginal cost; *Q* – monopoly output; *a, b, c* – coefficients of the quadratic marginal-cost function, *d* – the coefficient which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.

Marginal revenue is:

$$MR_t = P_t\left[1+\left(\frac{1}{e}\right)\right] \tag{5}$$

where *MR* – marginal revenue; *P* – monopoly price; *e* – the coefficient of the price elasticity of demand.

A monopoly firm maximizes profit by producing the quantity at which marginal revenue equals marginal cost. Thus the profit-maximizing condition is that:

$$MRt = MC \tag{6}$$

Further,

$$Pt+1 = Pt + \Delta P \tag{7}$$

or:

$$Pt+1 = Pt + \alpha\, Pt+1, \tag{8}$$

i.e.

$$(1-\alpha)\,P_{t+1} = P_t. \tag{9}$$

Thus, the chaotic model of the profit-maximizing monopoly is presented by the following equations:

$$(1-\alpha)\,P_{t+1} = P_t \tag{10}$$

$$MR_t = MC_t \tag{11}$$

$$MR_t = P_t\left[1+\left(\frac{1}{e}\right)\right] \tag{12}$$

$$MC_t = a\,(1-d) + b\,(1-d)\,Q_t + c\,(1-d)\,Q_t^{\,2} \tag{13}$$

$$P_t = n - m\,Q_t \tag{14}$$

where: Q – output of the monopoly firm; MC – marginal cost; MR – marginal revenue; P – monopoly price; e – the coefficient of the price elasticity of demand; n, m – coefficients of the inverse demand function; a, b, c – coefficients of the quadratic marginal-cost function, d – the coefficient which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.

Firstly, it is supposed that $a = 0$ and $n = 0$.

By substitution one derives:

$$Q_{t+1} = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}Q_t - \frac{c\,(1-d)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}Q_t^{\,2}. \tag{15}$$

Further, it is assumed that the monopoly quantity is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the monopoly output growth rate depends on the current size of the monopoly output, Q, relative to its maximal size in its time series Q^m. We introduce q as $q = Q/Q^m$. Thus q range is between 0 and 1. Again we index q by t, i.e., write q_t to refer to the size at time steps $t = 0, 1, 2, 3,...$ Now, growth rate of the monopoly output is measured as:

$$Q_{t+1} = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}Q_t - \frac{c\,(1-d)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}Q_t^{\,2}. \tag{16}$$

This model given by equation (11) is called the logistic model. For most choices of b, c, d, m, and e there is no explicit solution for (11). Namely, knowing b, c, d, m, and e and measuring q_0 would not suffice to predict q_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (11) can lead to very interesting dynamic behaviour, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behaviour of q_t. This difference equation (11) will possess a chaotic

region. Two properties of the chaotic solution are important: firstly, given a starting point q_0 the solution is highly sensitive to variations of the parameters *b, c, d, m*, and *e*; secondly, given the parameters *b, c, d, m*, and *e* the solution is highly sensitive to variations of the initial point q_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

2. Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation:

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0, 4], \quad z_t \in [0, 1] \tag{17}$$

is equivalent to the iteration of growth model (9) when we use the following identification:

$$z_t = \frac{c(1-d)}{b(d-1)} q_t \tag{18}$$

and:

$$\pi = \frac{b(d-1)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} . \tag{19}$$

Using (11) and (13) we obtain:

$$z_{t+1} = \left[\frac{c(1-d)}{b(d-1)}\right] q_{t+1} =$$

$$= \left[\frac{c(1-d)}{b(d-1)}\right] \left\{ \frac{b(d-1)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t - \frac{c(1-d)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t^2 \right\} =$$

$$= \frac{c(1-d)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t - \frac{c^2 (1-d)^2}{mb(d-1)(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t^2 .$$

On the other hand, using (12), (13), and (14) we obtain:

$$z_{t+1} = \pi z_t (1 - z_t) = \left\{ \frac{b(d-1)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} \right\} \left[\frac{c(1-d)}{b(d-1)} \right] q_t \left\{ 1 - \left[\frac{c(1-d)}{b(d-1)} \right] \right\} =$$

$$= \frac{c(1-d)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t - \frac{c^2 (1-d)^2}{mb(d-1)(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t^{\,2}.$$

Thus we have that $q_{t+1} = \frac{b(d-1)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t - \frac{c(1-d)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t^{\,2}$

iterating is really the same as iterating $z_{t+1} = \pi z_t(1 - z_t)$ using (13) and (14).

It is important because the dynamic properties of the logistic equation (12) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that:

(i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;

(ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;

(iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$;

(iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;

(v) For $3 < \pi < 4$ all solutions will continuously fluctuate;

(vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Conclusions

This paper suggests conclusion for the use of the simple chaotic model of a profit – maximizing monopoly in predicting the fluctuations of

the monopoly output. The model (11) has to rely on specified parameters b, c, m, d, and e, and initial value of the monopoly output, q_0. But even slight deviations from the values of parameters b, c, m, d, and e and initial value of the monopoly output, show the difficulty of predicting a long-term behaviour of the monopoly output, q_0.

A key hypothesis of this work is based on the idea that the coefficient:

$$\pi = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}$$

plays a crucial role in explaining local stability of the monopoly output where, b – the coefficient of the marginal cost function of the monopoly firm, m – the coefficient of the inverse demand function, e – the coefficient of the price elasticity of monopoly's demand, α – the coefficient of the price growth, d – the coefficient which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.

The quadratic form of the marginal cost function of the monopoly firm is important ingredient of the presented chaotic monopoly output growth model (11).

REFERENCES

[1] J. Benhabib, R. H. Day, *Rational Choice and Erratic Behaviour*, Review of Economic Studies 48: 459-471, 1981.

[2] J. Benhabib, R. H. Day, *Characterization of Erratic Dynamics in the Overlapping Generation Model*, Journal of Economic Dynamics and Control 4: 37-55, 1982.

[3] J. Benhabib, K. Nishimura, *Competitive Equilibrium Cycles*, Journal of Economic Theory 35: 284-306, 1985.

[4] R. H. Day, *Irregular Growth Cycles*, American Economic Review 72: 406-414, 1982.

[5] R. H. Day, *The Emergence of Chaos from Classic Economic Growth*, Quarterly Journal of Economics 98: 200-213, 1983.

[6] R. H. Day, *Complex Economic Dynamics Volume I: An introduction to dynamical systems and market mechanism*, In: Discrete Dynamics in Nature and Society, Vol. **1,** 177-178, MIT Press, 1997.

[7] R. M. Goodwin, *Chaotic Economic Dynamics*, Clarendon Press, Oxford, 1990.

[8] J. M. Grandmont, *On Endogenous Competitive Business Cycles*, Econometrica 53: 994-1045, 1985.

[9] D. V. Jablanović, *Agricultural Monopolistic Competitor and the Pigovian Tax*, Studies in Agricultural Economics, 115 (2013) 57-60 http://dx.doi.org/ 10.7896/ j.1224, 2013.

[10] D. V. Jablanović, *The Lerner Index and the Chaotic Monopoly Output Growth Model,* Asian Journal of Business and Management Sciences (AJBMS) ISSN: 2047-2528, Vol. **1,** Number 12, pg. 102-106, 2012.
[11] D. V. Jablanović, *The Chaotic Long-Run Monopolistic Competitor's Output Growth*, African Journal of Business Management African Journal of Business Management Vol. **6**(48), pp. 11758-11761, 2012
[12] D. V. Jablanović, *The Chaotic Production Growth Model of the Monopoly Firm*, Hyperion International Journal of Econophysics &New Economy, Volume **4**, Issue 2, 2011, pp. 285-292.
[13] T. Li, J. Yorke, *Period Three Implies Chaos*, American Mathematical Monthly 8: 985-992, 1975.
[14] E. N. Lorenz, *Deterministic nonperiodic flow*, Journal of Atmospheric Sciences 20: 130-141, 1963.
[15] H. W. Lorenz, *Nonlinear Dynamical Economics and Chaotic Motion*, 2nd edition, Springer-Verlag, Heidelberg, 1993.
[16] R. M. May, *Mathematical Models with Very Complicated Dynamics*, Nature 261: 459-467, 1976.
[17] A. Medio, *Chaotic Dynamics: Theory and Applications to Economics*, Cambridge University Press, Cambridge, 1993.
[18] A. Medio, *Chaotic dynamics. Theory and applications to economics*, Cambridge University Press, In: De Economist 144 (4), 695-698, 1996.
[19] A. Medio and M. Lines, *Nonlinear Dynamics. A primer*, Cambridge University Press 2001, In: De Economist 152, pp. 143-145, 2004.
[20] Peitgen, Heinz-Otto, Jürgens Hartmut, and D. Saupe, *Chaos and Fractals-New Frontiers of Science*, Springer-Verlag, New York, 1992.
[21] O. E. Rössler, *An equation for continuous chaos*, Phys. Lett. 57A: 397-398, 1976.
[22] P. N. V. Tu, *Dynamical Systems*. Springer-Verlag, 1994.

A NEW ECONOPHYSICS MODEL TO STUDY HUMAN RESOURCES IN EUROPEAN UNION

Mirela F. NICOLOV[1]

Abstract. *In this paper a new econophysics model of Human Resources evolution is proposed and discussed. For this purpose an analogy between the mechanical model especially the dynamic model and the Human Resources in European Union as regression function of second degree consideration is used. The model is tested on the values for the period 1990-2010.*

Keywords: *econophysics, mechanical dynamic model, regression functions, human resources.*

1. Introduction

Econophysics applies various models and concepts imported from condensed matter and statistical physics to analyze economic and financial phenomena. This new field of research has generated a lot of methodological debate (Schinckus, 2010a). It is often presented as a positivist discipline that provides a more empirical basis to economics .Despite the novelty of this new approach, more and more papers about econophysics have been published in journals devoted to Physics and Statistical Mechanics. Several meeting series dedicated to this topic are regularly organized and moreover, new Ph.D. programs in Econophysics recently appeared in some universities. Nowadays, Econophysics appears to be a new step in the history and the evolution of Physics Sciences and then the question about the disciplinary characteristics of Econophysics must then be asked (Schinckus, 2010a; Gallegati et.al, 2006; McCauley, 2006).

For the 1970s, a new theoretical movement has been initiated by some physicists who began publishing articles devoted to the study of social phenomena, such as the formation of social groups (Weidlich, 1971) or social mimicry (Callen&Shapiro, 1970). The next decade confirmed this new theoretical trend (labelled sociophysics), as the number of physicists

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publishing papers devoted to the explanation of social phenomena and the number of themes analysed continued to increase. During the 1990s, physicists turned their attention to economics, and particularly financial economics, giving rise to econophysics. Although the movement's official birth for example announcement came in a 1996 article by Stanley and his team (Stanley et. al., 1996), econophysics was at that time still a young field. Econophysics can be defined as "a quantitative approach using ideas, models, conceptual and computational methods of statistical physics". Today, econophysics is an institutionalized field, with different journals proposing a prolific literature about the way of characterizing the evolution of financial prices. There is an "extreme diversity" of models recently developed by econophysicists and many theoretical frameworks still emerge. (Bucsa et. al., 2011; Stanley et. al., 1996)

Econophysics presents itself as a new way of thinking about the economic and financial systems through the "lenses" of physics (Schinckus, 2010b). As much as neoclassical economics imported models from classical physics as formulated by Lagrange (Mirowski, 1989) and financial economics built on the model of Brownian motion also imported from physics, econophysics tries to model economic phenomena using analogies taken from modern condensed matter physics and its associated mathematical tools and concepts. Using the standard tools of statistical mechanics including microscopic models like Ising model and scaling laws, econophysicists aim at explaining how complex economic systems behave. Broadly speaking, econophysics is founded on general statistical properties that reappear across many and diverse phenomena. This statistical regularity can be characterized by scaling laws that are considered as the heart of econophysics (Bouchard, 2002; Staley et.al., 2000). These scaling laws can take a variety of forms. The objective of the next section is to offer a generic formula characterizing the main distributions usually used by econophysicists (Bucsa et. al., 2011).

The theories of Human Resources Management have the origin in United States of America (Brewster, 2004; Brewster, 1994). The management of Human Resources in particular has been heavily influenced by thinking in USA. The impact of the globalization was influenced on the way that people are managed, particularly in Europe Union (Brewster, 2004; Brewster, 1994). In human resources management (HRM), in particular is the US model of HRM one that will inevitably be followed in Europe? Our understanding of management in general and human resources management in particular has been heavily influenced by thinking in the United States of America. This is perhaps not surprising

from a country that has been for decades the largest economy in the world. HRM like many other aspects of management was originally conceptualized and developed in the United States of America. The study of personal management was partly a file clerk's job, partly a housekeeping job, partly a social worker's job and partly a fire-fighting to head of union trouble (Drucker, 1989). The American's theories about HRM would work anywhere in the world. Relationships between the structural characteristics of work organizations and variables or organization context will be stable across societies (Hickson et.all, 1974). Their main findings from cases in the USA, Canada and then UK were that companies are subject to the same relationships in terms of size, dependence on parent group and technology irrespective of country. Hickson et.al offered a culture free context of organization structure (Hickson et.al, 1974). Kidger appreciated that grew in isolation from the world economy are having their approaches supersede by universally applicable techniques (Kidger, 1991). Other models of Human Resources Management (HRM) are continually being developed. And the rhetoric at least of HRM has been spread to many other countries both in the theoretical discourse and within employing organizations. From the early days, there have been calls for comparative HRM, studying similarities and differences in management systems and the way people are managed in different countries (Brewster, 2004; Brewster, 1994; Redding, 1994). Whether the US derived visions of Human Resource Management (HRM) apply everywhere in the world is an important question for both theory and practice since following US prescription in either area may be detrimental if theories are not transferable, for example, the need for a contingent approach encompass cultural, sectorial and regional differences. Similarly, other theorists have also argued for the need to cover both national differences and organizational contingencies, although they have used different terminologies: macro economy, micro economy (Gronghaug, 1992) exogenous, endogenous (Brewster, 2004; Brewster, 1994), external and internal variable (Morley, 2004). How the theory applies particularly to the European setting remains a conceptual and empirical challenge. One problem is that the complications noted above make research difficult and there is a scarcity of empirical data (Brewster, 2004; Brewster, 1994). Despite these theories and the complexities of understanding different national contexts geographical differences are apparent, and the effort to understand them is important. There are a number of critical differences between the North American context and the European (Brewster, 1994). Such a comparison involves substantial generalization. We must remain

aware of the substantial differences of the substantial differences within North America, even within individual states in the USA and the differences between European countries (Nicolov, 2011).

Present paper will present a new econophysics model for modelling the human resources of the European RDI sector, especially for EU15 respectively EU25. The new econophysics model will be a dynamic one.

2. The Dynamic Model

In figure 1 we can see the point *M*, represented in a Cartesian coordinate system OXYZ by the position vector: $\overrightarrow{r_M}$. Each coordinate axis corresponds to a line unit vector called: $\vec{i}, \vec{j}, \vec{k}$ are the unit vectors of the coordinate axes and axes and property meaning: $|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$ (Nicolov, 2009).

Position vector of point *M*1 has analytical expression: $\vec{r}_1 = r_x.\vec{i} + r_y.\vec{j} + r_z.\vec{k}$.

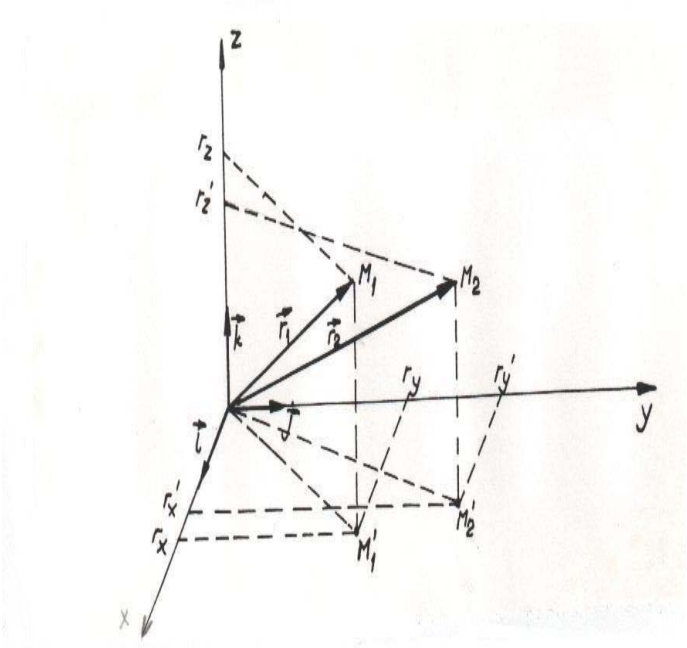

Figure 1. Cartesian coordinate system. (Nicolov, 2009).

Since the coordinates *rx*, *ry* and *rz* can depend on time, you can write the following parametric equations: $r_x = r_x(t)$, $r_y = r_y(t)$, $r_z = r_z(t)$. Elimination of parametric equations leads to the trajectory equation: $z = z(x, y)$.

If a material point moves from point M_1 to point M_2 by using *a* path *c*, its position vector of time-varying r_1 to r_2, the analytical expression: $\vec{r}_2 = r'_x.\vec{i} + r'_y.\vec{j} + r'_z.\vec{k}$

Velocity of the material point in the Cartesian coordinate system by definition is expressed as:

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} = \dot{r}_x \cdot \vec{i} + \dot{r}_y \cdot \vec{j} + \dot{r}_z \cdot \vec{k}.$$

So you can write the following relations: $v_x = \dot{r}_x = \frac{dx}{dt}$; $v_y = \dot{r}_y = \frac{dy}{dt}$; $v_z = \dot{r}_z = \frac{dz}{dt}$.

According to the definition, the acceleration of the material point is given by: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt}$.

Force acting on the material point is a Newtonian type force. Knowing the material point mass (*m*) and applying Principle 2 of Newtonian mechanics, we have the relations: $\vec{F} = m \cdot \vec{a}$ or $\vec{F} = m \cdot a_x \cdot \vec{i} + m \cdot a_y \cdot \vec{j} + m \cdot a_z \cdot \vec{k}$.

Knowing his position vector of a material point, i.e. trajectory equation and parametric equations, one can determine the speed, acceleration and type of force acting on it. Knowing the type of force acting on the material point trajectory equation can be determined that he is travelling based on the forces acting: $\vec{F} = m \cdot \vec{a}; \vec{a} = \frac{\vec{F}}{m}$.

If acceleration is shaped as $\vec{a} = \frac{d\vec{v}}{dt}$, separate variables: and then we make the integration by using initial conditions and then we obtain: Speed Law: $\vec{v} = \vec{v_0} + \int_{t_0}^{t} \vec{a} \cdot dt$.

Given the definition of velocity: $\vec{v} = \frac{d\vec{r}}{dt}$ and, separate variables are $d\vec{v} = \vec{a} \cdot dt$, we make the integration by using initial conditions and obtained the law of motion: $\vec{r} = \vec{r_0} + \int_{t_0}^{t} \vec{v} \cdot dt$.

If we write on the 3 axes are called parametric equations:

on Ox: $x = x_0 + \int_{t_0}^{t} v_x \cdot dt$; on Oy: $y = y_0 + \int_{t_0}^{t} v_y \cdot dt$; on Oz: $z = z_0 + \int_{t_0}^{t} v_z \cdot dt$.

By eliminating the parametric equations of the trajectory equation we obtain:

$$z = z(x, y) \Rightarrow \vec{r} = x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k}.$$

3. Results

In the present paper is studied the determinants indicators for European competitiveness by linking RDI indicators from the next databases: EIS2004, EIS2005, IUS2010, IUS2011.

The IUS2010 contain a list of 25 indicators that captures national performance CDI. 19 indicators were carried over from the previous EIS 009 and 12 indicators were not modified, two indicators were combined and 5 indicators were partially modified by using broader or narrower definitions or different names. Considering the merging of 2 indicators, 18 indicators from IUS2010 are equivalent to those of EIS2009 and in addition were introduced seven new indicators. IUS uses the most recent statistics from EUROSTAT and other internationally recognized sources as available at the time of analysis.

IUS2011 distinguishes between three main types of innovation indicators on 8 dimensions, for a total of about 25 different indicators. Input key factors in the analysis of external innovation performance of firms covering the three dimensions of innovation are: human resources, open, excellent and attractive research systems, finance and support. Business activities relate to the company's innovation efforts, grouped into three dimensions of innovation are as follows: investment firms and spirit connections entrepreneurial and intellectual assets. Output factors related to effects on innovation activities of firms in innovation have 2 sizes: Innovators and economic effects. These are indicators for presenting the performance of RDI. Some indicators of innovation at EU level, such as public spending on RDI, can be more easily influenced by policy interventions than others, such as private innovation SMEs.

It is deal in the next part with Science and engineering graduates aged 20-29 per 1,000 inhabitants, for EU15 respectively EU25.

Science and engineering graduates aged 20-29 (per 1,000 inhabitants) (‰) (Source EIS2004, EIS2005) is defined as all post-secondary education graduates belongs to ISCED 5A with and above in life sciences (ISC42), physics (ISC44), mathematics and statistics (ISC46) engineering

calculation (ISC48) and engineering trades (ISC52), production and processing (ISC54), architecture and building (ISC58) (Canberra Manual, 1995). Population reference was for age groups between 20 and 29 years inclusive. This indicator is a measure of the supply of graduates in science and engineering education. Due to problems of comparability of educational qualifications in various parts of the EU countries, this indicator uses broad categories of education. This means that it covers everything from graduate degree. A broad coverage can also be an advantage, because even a year graduates of programs are valuable for incremental innovation in manufacturing and services.

In 2004 EIS the data are recorded from the period 1993-2004.We have the only valid data for these indicators for the period 1997-2002, for represented category EU15 and EU25. So, for the case of EU15 recorded values was 10.3 ‰ in 1997, a figure which increased to 12.47 ‰ in 2002. These values can vary for EU25 between 9.28 ‰ and 11.49 ‰.

In table 1 and figure 2, we can see the Human Resources modelling.

Table 1. Human Resources modelling

Science and engineering graduates aged 20-29 (per 1,000 inhabitants) (‰) (EU15)	$Y_{EU15(SE)} = 0.10x^2+0.310x+7.449$
Science and engineering graduates aged 20-29 (per 1,000 inhabitants) (‰) (EU25)	$Y_{EU25(SE)} = 0.10x^2+0.311x+8.468$

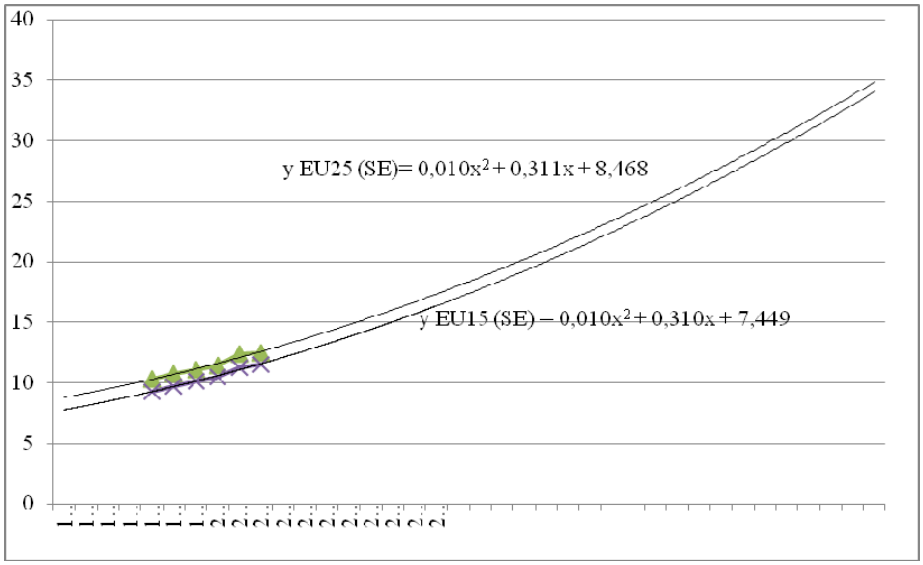

Figure 2. Modelling Human Resources for European Union for the period 1990-2010 and for the next 20 years.

We can see from here that the curves which represent HR modelling are as follows:

- For the case of EU15 of science and engineering graduates aged 20-29 per 1,000 inhabitants represented in ‰ is:

$$Y_{EU15(SE)} = 0.10x^2+0.310x+7.449.$$

- For the case of EU25 of science and engineering graduates aged 20-29 per 1,000 inhabitants represented in ‰ is:

$$Y_{EU25(SE)} = 0.10x^2+0.311x+8.468.$$

4. Conclusions

From the present study of the indicators in the modelling used for the last 20 years and the next 20 years we can see that the majority observe an accelerated trend which respect the econophysics dynamic model presented above, the individual trajectories for each indicator.

In the study of human resources can draw the following conclusions: equations generated to characterize the evolution and future trends for each studied case are all second degree equations similar to parametric equations of econophysics model already presented. It is noted that the equations that characterize the development of human resources are generated by a set of differential equations of the form $y = AX^4 + BX^3 + +CX^2 + Dx + E$, which by differentiating twice even give equations obtained. These equations characterize a dynamic phenomenon in accelerated system.

Most of the previous studies were performed on samples of firms in each country while this study is estimated on a sample consisting of EU15, EU25. Our sample size is different from the samples used by other authors. The difference can also be attributed to the choice of explanatory variables. In addition, input data on innovation and firm size, previous studies have simulated the output of RDI studies on depending on the demand side and technological factors, sources of information for innovation, cooperation models and firm specific characteristics, such as age, existence RDI facilities from the impact of change on performance management (Hashi & Stojcic, 2012). Hashi and Stojcic results presented in their paper in 2012, confirms the shape model introduced in this study (Hashi & Stojcic, 2012).

REFERENCES

[1] Bouchaud J.-P. (2002), An Introduction to statistical finance, Physica A 313 (1), 238-251.

[2] Brewster C. (1994), European HRM: reflection of, or challenge to the American Concept? In P. S. Kirkbride Ed., *Human Resource management in Europe*, London, Routledge.

[3] Brewster C. (2004), European perspectives on human resource management, *Human Resource Management Review*, 14, 365-382.

[4] Bucsa G., Jovanovic F., Schinckus C. (2011), A unified model for price return distributions used in econophysics, Physica A 390 3435-3443,

[5] Callen E., Shapiro D. (1974), A theory of social imitation, Physics Today 12(2) (1974) 23-28.

[6] Canberra Manual (1995), The measurement of scientific and technological activities manual on the measurement of human resources devoted to Science & Technology organization for economic co-operation and development, OECD and ECSC-EC-EAEC, Brussels-Luxembourg.

[7] Drucker P. (1989*),* The new realities: In government and politics, in economics and business, in society and world view. New York: Harper & Row,

[8] Fagerberg J. Srholec M (2008), *National Innovation Systems, Capabilities And Economic Development,* Research Policy 37 (2008) 1417-1435

[9] Fagerberg J. (1996), *Competitiveness, Scale and R&D*, No 1996545, Working Papers Archives from Centre for Technology, Innovation and Culture, University of Oslo, http://EconPapers.repec.org/RePEc:tik:wparch:1996545,

[10] Gallegati M., S. Keen, T. Lux, Ormerod P. (2006), *Worrying trends in econophysics*, Phys. A 370, 1-6.

[11] Hashi, I., Stojcic, N. (2012), The impact of innovation activities on firm performance using a multi-stage model: Evidence from the Community Innovation Survey 4. Res. Policy (2012), http://dx.doi.org/10.1016/j.respol.2012.09.011

[12] Hickson D., Hinnings C. R. McMillan & J. P. Schwitter (1974), The culture – free context of organization structure: A tri national comparison, *Sociology*, 8, 59-80.

[13] http://www.proinno-europe.eu/metrics

[14] Kidger P. J. (1991), The emergence of international human resource management, *International Journal of Human Resource Management,* 2(2) 149-163.

[15] Lam A. (2006), *Organizational Innovation*, pag.115-147, in The Oxford Handbook of Innovation edited by Jan Fagerberg, D. Mowery, Richard Nelson, O.U.P, New York.

[16] Lam A., Lundvall B. (2008), *The Learning Organisation and National Systems of Competence Building and Innovation*, Online at http://mpra.ub.uni-muenchen.de /12320/ MPRA Paper No. 12320.

[17] McCauley J. (2004), *Dynamics of Markets: Econophysics and Finance*, Cambrigde University Press, Cambrigde.

[18] McCauley J. (2006), Response to `worrying trends in econophysics', Physica A 371, 601.

[19] Mirowski P. (1989), More Heat Than Light: Economics as Social Physics, Physics as Nature's Economics, Cambridge University Press, Cambridge, New York.

[20] Morley M. J. (2004), Contemporary debates in European Human Resource Management: Context and content, *Human Resource Management Review*, 14 353-364.
[21] Morten Berg Jensen, Bjorn Johnson, Edward Lorenz, Bengt Ake Lundvall (2007), *Forms of knowledge and modes of innovation*, Research Policy 36, pp. 680-693.
[22] Mowery D, Sampat B (2006), *Universities in National Innovation System*, p.209-239, in The Oxford Handbook of Innovation edited by Jan Fagerberg, D. Mowery, Richard Nelson, Oxford University Press, New York.
[23] Narula R, Zanfei A. (2006), *Globalization of Innovation: The Role of Multinational Enterprises*, pp. 318-347, in The Oxford Handbook of Innovation edited by Jan Fagerberg, D. Mowery, Richard Nelson, Oxford University Press, New York.
[24] Naubahar S. (2006), *Emergence and development of the National Innovation Systems concept*, Research Policy, 35, 745–766.
[25] Nicolov M. F. (2009), *Physics for Engineers (in Romanian), Aurel Vlaicu* University Publishing House, Arad.
[26] Nill Jan, Kemp René (2009), Evolutionary approaches for sustainable innovation policies: From niche to paradigm?, Research Policy, **38,** 668–680.
[27] Redding S. G. (1994), Comparative management theory: Jungle, zoo or fossil bed?, *Organization St.*, 15, 323-359.
[28] Rickles D. (2008), *Econophysics and the complexity of the financial markets*, in: J. Collier, C. Hooker (Eds.), Handbook of the Philosophy of Science, in: Philosophy and Complex Systems, vol. **10,** North Holland, Elsevier Editions, New York.
[29] Schinckus C. (2010a), Is econophysics a new discipline? The neopositivist argument, Phys. **A** 389, 3814-3821
[30] Schinckus C. (2010b), *Econophysics and economics: sister disciplines?* Am. J. Physics 78 **(1),** 325-327.
[31] Stanley Eugene, Nunes Amaral, et al. (2000), *Econophysics: what can Physicists contribute to Economics?* International Journal of Theoretical and Applied Finance 3 (3), 335-346.
[32] Stanley H., Afanasyev V., et al. (1996), Anomalous fluctuations in the dynamics of complex systems: from DNA and physiology to econophysics, Physica A, 224 (1), 302-321.
[33] Weidlich W. (1971), *The statistical description of polarization phenomena in society*, British Journal of Mathematical and Statistical Psychology, 24 **(1),** 251-266.

NEW ECONOMY Section

PRODUCTS AND SERVICES LIFE-CYCLE IN THE ENTERPRISE DEVELOPMENT

Ioana ARMAS*

Abstract. *The entire evolution and development of the enterprise, its goals, policies, and strategies are expressed by its products and services. Thus, all the dimensions of the enterprise, respectively management, organizational structure, financial and economic strength etc., are wrapped in one global dimension – the offer of products and services. From this point of view, the development of the products and/or services must integrate the enterprise's dimensions, such that the corresponding management areas to attain through their results the enterprise's goals. The concept that unifies all these aspects, by integrating the enterprise's dimensions relative to the enterprise's development and evolution, is the life-cycle of the products and services.*

In this context, the present paper defines in a process – oriented manner the life-cycle concept for products and services, highlighting their dimensional complexity, and develops, considering the synergistic aspects, the framework of the life-cycle oriented development of the enterprise. Also, the dynamic integrated quality concept is determined as the context for evaluation and positive evolution of the enterprise.

Keywords: *products and services life-cycle, dimensional complexity, synergistic oriented process maps, life-cycle oriented development of competitiveness, dynamic integrated quality.*

1. Introduction

The main aspects of economy are determined by the enterprise's creation activities, expressed through products and services offered to markets as complex areas of the society.

Today, the concepts attached to products and services present high complexities both at their definition, and implementation levels. Thus, the following definitions, oriented on global quality, will be considered [1]:

A product represents any system offered to the market in order to be purchased and used, that satisfies or creates a necessity at a given requested

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time by fulfilling a set of requirements according to the working environment, user, competition etc. at an exclusively economical price accepted by the user, without inducing negative mutations (physical, psychological, behavioural etc.) of the humans, users or not, of the society, or of the natural environment.

A service represents a process supported by an organization or by a technical system, that satisfies or creates customer's necessities and requests at any desired time, for an accepted time period, and at an exclusively economical price agreed by the user, without inducing negative mutations (physical, psychological, behavioural etc.) of the humans, users / clients or not, of the society, or of the natural environment.

2. Products and services dimensional complexity

According to the global quality oriented definitions for products and services, results that their complexity is 3-dimensional (3-D) enfolded, as in table 1 and figures 1 and 2, where each general dimension, D_i, $i = 1, 2, 3,$ s in fact the envelope of other dimensions integrated in it.

Table 1.

The general and detailed dimensions of the products and services

Dimensions of products	**Dimensions of services**
D_1 **– The functional dimension**	D_1 **– The functional dimension**
D_{11} – necessities	D_{11} – necessities
D_{12} – requests	D_{12} – requests
D_{13} – implementation level	D_{13} – implementation level
D_{14} – technical and technological context	D_{14} – response capabilities
	D_{15} – technical and technological context
D_2 **– The quality / existence dimension**	D_2 **– The quality / existence dimension**
D_{21} – performance	D_{21} – performance
D_{22} – quality by conformity	D_{22} – quality by conformity
D_{23} – safety and security	D_{23} – adaptability
D_{24} – induced mutations	D_{24} – safety and security
	D_{25} – induced mutations

D_3 – The economic dimension	D_3 – The economic dimension
D_{31} – development and manufacturing costs	D_{31} – development and implementation costs
D_{32} – price and other costs for the client / user	D_{32} – support costs and corresponding resources
D_{33} – temporal opportunity for the market	D_{33} – price and other costs for the client / user
	D_{34} – temporal opportunity for the market

Thus, any product, *P*, or service, *S*, will be described by its dimensional coordinates, according to figures 1 and 2 that can be expressed as matrices:

$$[P] = [D_1 \quad D_2 \quad D_3], \tag{1}$$

and respectively,

$$[S] = [D_1 \quad D_2 \quad D_3], \tag{2}$$

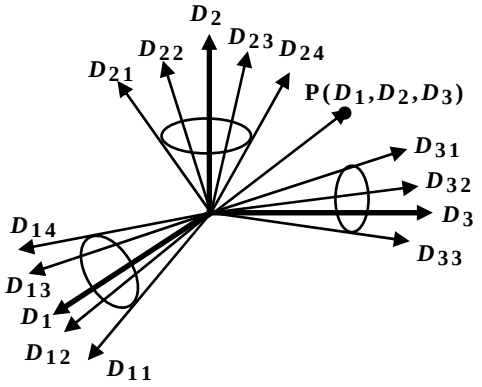

Figure 1. The dimensional complexity of products.

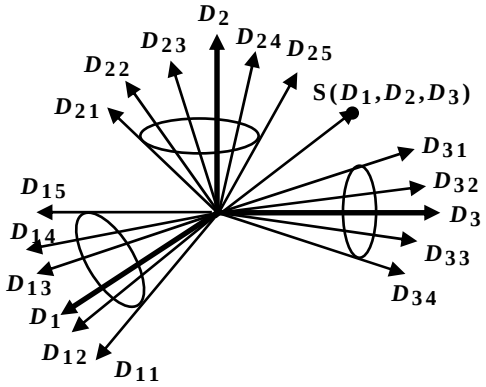

Figure 2. The dimensional complexity of services.

If the general dimensions are detailed, then the product, respectively the service will be described by the matrices:

$${}^{e}[P] = \begin{bmatrix} D_1 & D_2 & D_3 \\ D_{11} & D_{21} & D_{31} \\ D_{12} & D_{22} & D_{32} \\ D_{13} & D_{23} & D_{33} \\ D_{14} & D_{24} & * \end{bmatrix} \quad \text{and respectively,} \quad {}^{e}[S] = \begin{bmatrix} D_1 & D_2 & D_3 \\ D_{11} & D_{21} & D_{31} \\ D_{12} & D_{22} & D_{32} \\ D_{13} & D_{23} & D_{33} \\ D_{14} & D_{24} & D_{34} \\ D_{15} & D_{25} & * \end{bmatrix},$$

that explicitly highlight their dimensional complexity through the partition:

$$ {}^{e}[P]=\begin{bmatrix}[P]\\ [\Pi(D)]\end{bmatrix}, \qquad \text{with} \qquad [\Pi(D)]=\begin{bmatrix} D_{11} & D_{21} & D_{31}\\ D_{12} & D_{22} & D_{32}\\ D_{13} & D_{23} & D_{33}\\ D_{14} & D_{24} & * \end{bmatrix}, \qquad (3) $$

$$ {}^{e}[S]=\begin{bmatrix}[S]\\ [\Sigma(D)]\end{bmatrix}, \qquad \text{with} \qquad [\Pi(D)]=\begin{bmatrix} D_{11} & D_{21} & D_{31}\\ D_{12} & D_{22} & D_{32}\\ D_{13} & D_{23} & D_{33}\\ D_{14} & D_{24} & D_{34}\\ D_{15} & D_{25} & * \end{bmatrix}, \qquad (4) $$

where $\Pi(D)$ and $\Sigma(D)$ are the dimensional complexity matrices, that are not uniquely determined, but contain as kernel the above determined components as under-dimensions or dimensions to be enfolded.

In this context the results for the development of any product or service are strongly related with the enterprise's dimensions of competitiveness. The corresponding concept and framework that unifies products and respectively, services with the enterprise's perspective of development and evolution is the *life-cycle* defined as the interconnected assemble of the processes by which a product or a service are developed, implemented, used, reused or withdrawn from the user area and reprocessed, all these in a related manner with the enterprise's external and internal environments.

3. The products life-cycle

According to the product's life-cycle definition determined in §1, the following constitutive dominant processes will be determined:

P1 – requests, necessities, and market opportunity analysis;

P2 – product development and design;

P3 – product manufacturing and/or implementation;

P4 – distribution and sales;

P5 – product use and operation;

P6 – technical assistance and maintenance;

P7 – after-use and after-operation processes (e.g., recycling, reuse etc.);

P8 – communication processes;

P9 – learning processes.

The above processes build the global life-cycle context as in figure 3, where $\Theta_{k,j}$, $k=\overline{1,9}$, $j=j(k)$ represent the components of the synergistic integration relation that interconnects the **P1 ÷ P9** processes through the information set:

$$I_{PLC}=\{\Theta_{k,j}: k=\overline{1,9},\ j=j(k),\ j(k)=1,...,k-1,k+1,...,9\}, \qquad (5)$$

such that the product's life-cycle is determined by:

$$PLC=\mathbf{P1}\cup_{sg}\mathbf{P2}\cup_{sg}...\cup_{sg}\mathbf{P9}\cup_{sg}I_{PLC}, \qquad (6)$$

where $\cup_{sg}$ represents the synergistic integration through informational set, characterized by the following defining properties:

- any fault in performing any process $(\neg\mathbf{P}\boldsymbol{i}, \boldsymbol{i}=\overline{1,9})$ determines corresponding faults in performing the product's global life-cycle $(\neg PLC)$;
- any fault in implementing and accomplishing the informational set $(\neg I_{PLC})$ determines faults in performing and implementing the product's global life-cycle $(\neg PLC)$.

Thus, results the necessity to consider the design of the life-cycle for any product type, such that the objectives of the product management as ***project management*** will be oriented towards the development, implementation and management of the specific **P1 ÷ P9** processes, and the informational set I_{PLC} in a strong relation with the internal management aspects of the enterprise, its strategies, policies, internal organization, and external environment. In this context results the defining framework for the product's life-cycle oriented project management, as in figure 4.

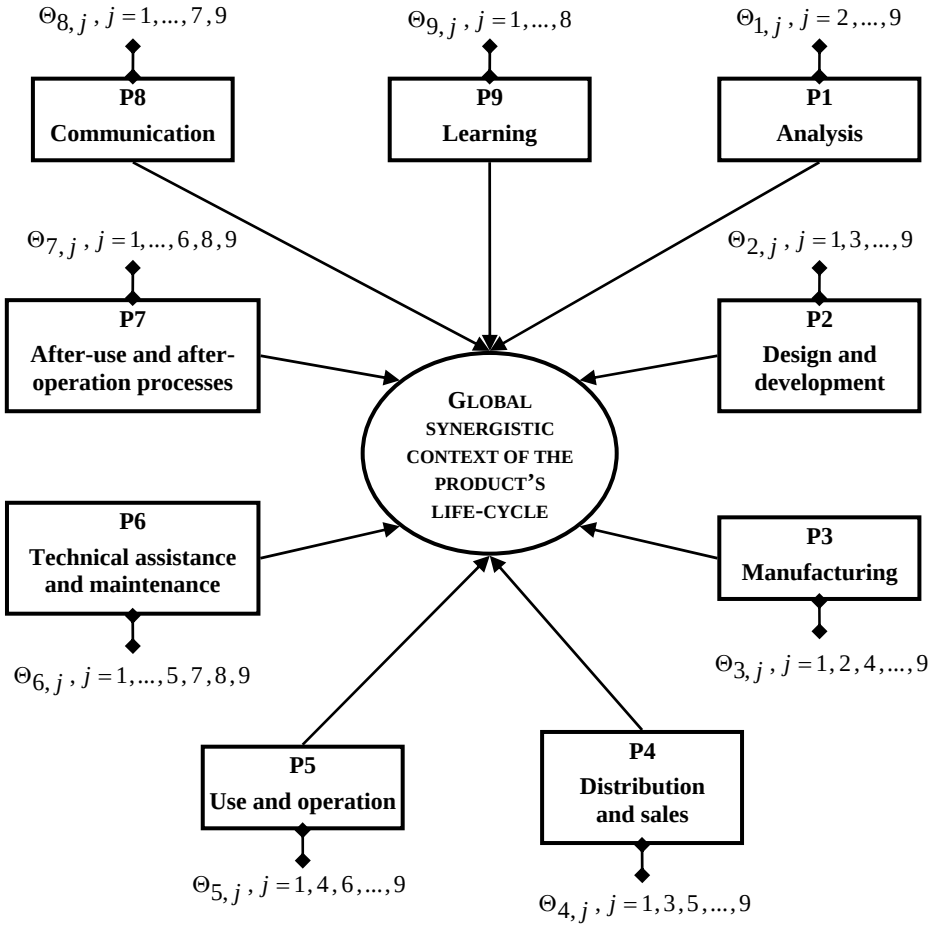

Figure 3. The synergetic oriented process map of the product's global life-cycle.

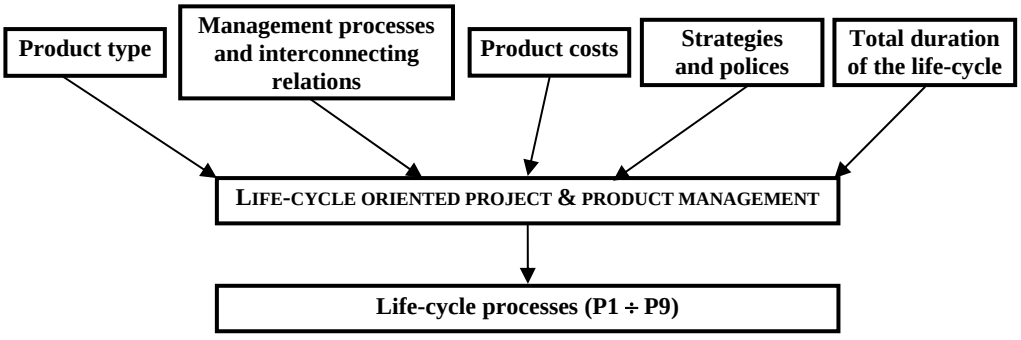

Figure 4. The life-cycle oriented project & product management framework.

It is important to be highlighted that the implementation of the product's life-cycle concept and of the product & project management is possible only when it is based on research (e.g., scientific, technical, economical etc.) that solves in a proactive manner all the aspects regarding product development, long before to the life-cycle initiation and elaboration.

4. The services life-cycle

An important tendency of the present is the transfer of product property towards the exclusively beneficial of its functions as a service.

This phenomenon already produces effects in information and communication technologies (IT&C), for example in mobile communications, cloud computing, transportation etc., and represents a solution for the implementation of high level technologies and costly equipment, as robots destined to individual and social use. Also, the number of services is already large, corresponding to different areas of economy, finance, health, education etc.

For all types of services, the following dimensions can be identified:

1. the ***intangible or insubstantial dimension***, that integrates:
- the service's global goals,
- the operations or activities performed in order to attain the global goal,
- the associated rules, that define the procedure according to which the service is performed at the internal (i.e., enterprise) and external (i.e., clients and external environment) levels;

2. the ***tangible or substantial dimension***, that integrates the material support (i.e., equipment, facilities, material resources etc.) for executing and deliver the service.

Applying in this context the life-cycle definition, results that two categories of processes must be determined according to the above dimensions. Thus, the dominant constitutive processes of the services life-cycle are two-dimensionally developed for the soft life-cycle (*FLC*) and respectively, the hard life-cycle (*HLC*) levels, as in table 2.

Table 2.

The two-dimensional process identification for the services life-cycle.

Services life-cycle (*SLC*) processes	
Soft life-cycle (*FLC*) processes	**Hard life-cycle (*HLC*) processes**
Processes of the intangible or insubstantial dimension	Processes of the tangible or substantial dimension
F1 - requests, necessities, and opportunity analysis **F2** - procedural service design (i.e., goals, activities, and rules) **F3** - procedural implementation **F4** - service delivery to clients and operation at order or request **F5** - user procedural support and assistance **F6** - service evolution regarding goals, activities, and governing rules **F7** - communication processes **F8** - learning processes	**H1** -requests and necessities analysis **H2** -material support design **H3** -material support implementation **H4** -material support use and operation **H5** -technical assistance and maintenance **H6** -after-use and after-operation processes **H7** -communication processes **H8** -learning processes

Each dimension of the services life-cycle is built as the synergistic integration of the corresponding processes, as in figure 5.

In these conditions, the global context of the services life-cycle (*SLC*), considering the two soft- and hard- dimensions, are synergistically integrated as in figure 6, such that the service's life-cycle is defined as the synergistic integration of its *FLC* and *HLC* dimensions as:

$$SLC = FLC \bigcup_{sg} HLC \bigcup_{sg} I_{SLC}, \tag{7}$$

with $I_{SLC} = \{\Sigma_{F_1,H_1}, \Sigma_{F_2,H_2}, \Sigma_{F_3,H_3}, \Sigma_{F_4,H_4}, \Sigma_{F_5,H_5}, \Sigma_{F_6,H_6}, \Sigma_{F_7,H_7}, \Sigma_{F_8,H_8}\}$ representing the informational set of the service's life-cycle (*SLC*), and each dimension represents a synergistic integration of its processes, as following:

$$\begin{cases} FLC = \mathbf{F1} \bigcup_{sg} \mathbf{F2} \bigcup_{sg} ... \bigcup_{sg} \mathbf{F8} \bigcup_{sg} I_{FLC} \\ I_{FLC} = \{\Phi_{k,j} : k = \overline{1,8},\ j = j(k),\ j(k) = 1, ..., k-1, k+1, ..., 8\}; \end{cases} \tag{8}$$

$$\begin{cases} HLC = \mathbf{H1} \bigcup_{sg} \mathbf{H2} \bigcup_{sg} ... \bigcup_{sg} \mathbf{H8} \bigcup_{sg} I_{HLC} \\ I_{HLC} = \{\Xi_{k,j} : k = \overline{1,8},\ j = j(k),\ j(k) = 1, ..., k-1, k+1, ..., 8\}, \end{cases} \tag{9}$$

where I_{FLC} and I_{HLC} are the informational sets of the *FLC* and respectively, *HLC* dimensions.

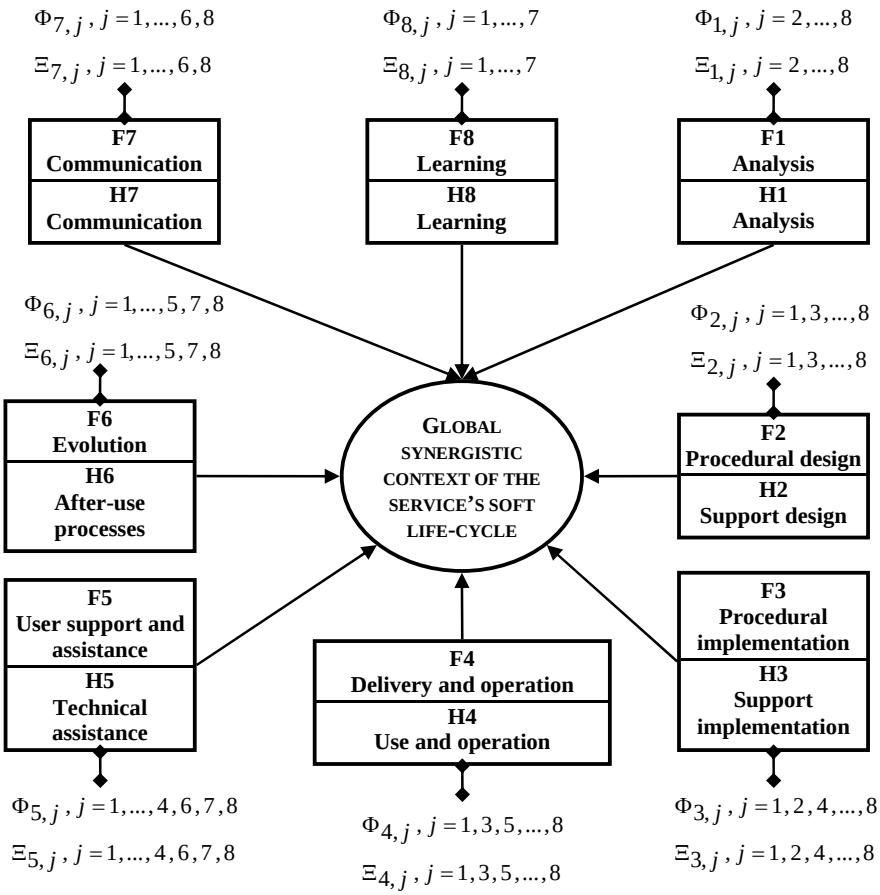

Figure 5. The synergetic oriented process map of the service's soft life-cycle (*FLC*).

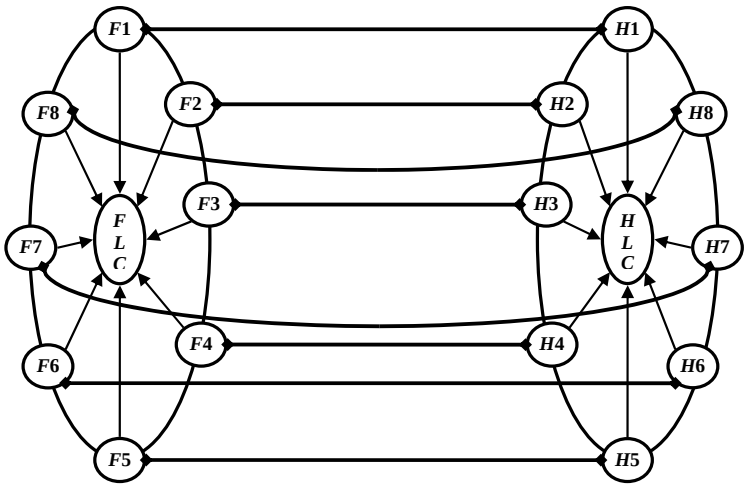

Figure 6. The services life-cycle as the synergistic integration of the tangible and intangible dimensions.
FLC – the soft life-cycle dimension; *HLC* – the hard life-cycle dimension.

According to the synergistic structure of the *SLC* results:

1. the development of services industry requires a corresponding development of products industry as material support;

2. the development of products industry is supported and grows according to the correlated designed services that use the products as support.

Thus, each industry is an engine for the other one, as long as they offer utility and are correctly developed related to the clients, society, environment, and market.

5. The life-cycle oriented development of the enterprise

Considering the products and service life-cycle context, the enterprise's developments polices will have new dimensions determined by the necessity to specify, design and implement the corresponding life-cycle processes.

Thus, to design a product or a service will involve designing synergistically the corresponding processes of their life-cycle in order to attain competitiveness at each level, fact that will influence the evolution of the enterprise and its policies, according to the framework of figure 7.

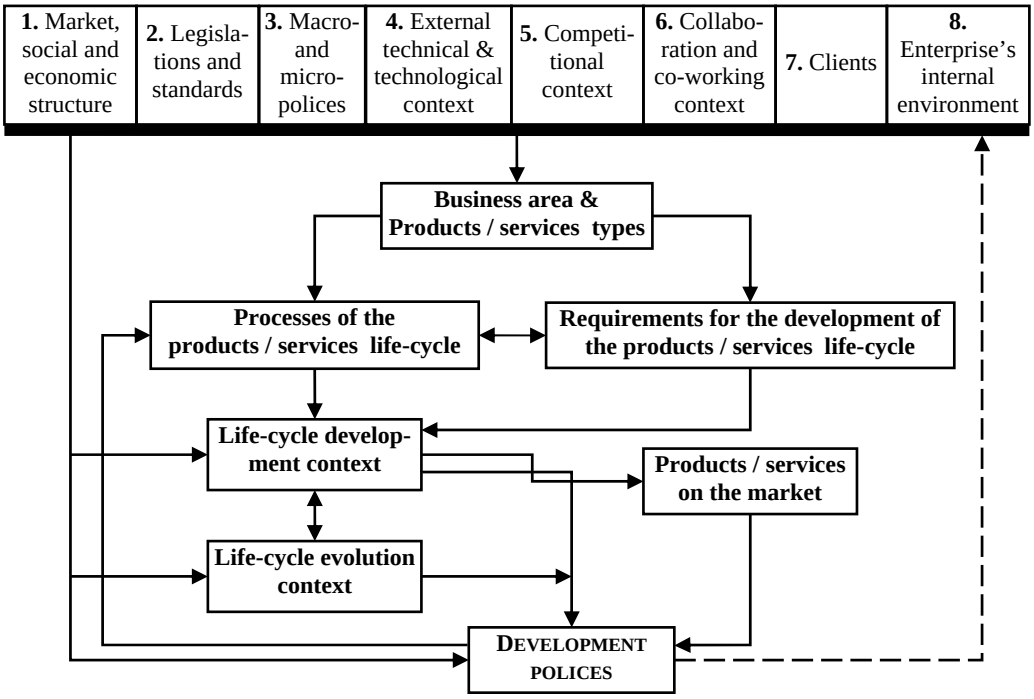

Figure 7. The framework of the life-cycle oriented development of the enterprise.

In these conditions, the enterprise's competitiveness will be evaluated related to its products and services life-cycle, such that the concept of the ***integrated dynamic quality of the enterprise*** will be determined as its capability to:

- perform research and to propose, under utility conditions, specific products and services;
- define, develop and implement the corresponding life-cycles for the products and services, under time, quality and safety conditions;
- define and represent a specific, positive point in the competitiveness space, by satisfying the conditions of compatibility with the users, non-users, and natural environment;
- develop a proactive and ethical behaviour related to its internal and external environments.

6. Conclusions

The development and evolutions of the enterprises are mainly oriented towards economic and financial goals obtained according to their activities in specific business areas.

The present paper highlights the fundaments and support to attain these goals, as being the products and/or services that the enterprises offer to the market, and, more generally, to their external environment. From this point of view, products and services are defined considering their complexity perspective, and the life-cycle concept is developed as the unifying solution between the enterprises and its products and services in creating competitiveness (see Figure 8).

Thus, the processes of the products and services life-cycle are identified in a synergetic context, and accordingly the framework of the life-cycle oriented development of the enterprise is determined.

In these conditions, the integrated dynamic quality of the enterprise is defined as the generating context for the enterprise's evolution.

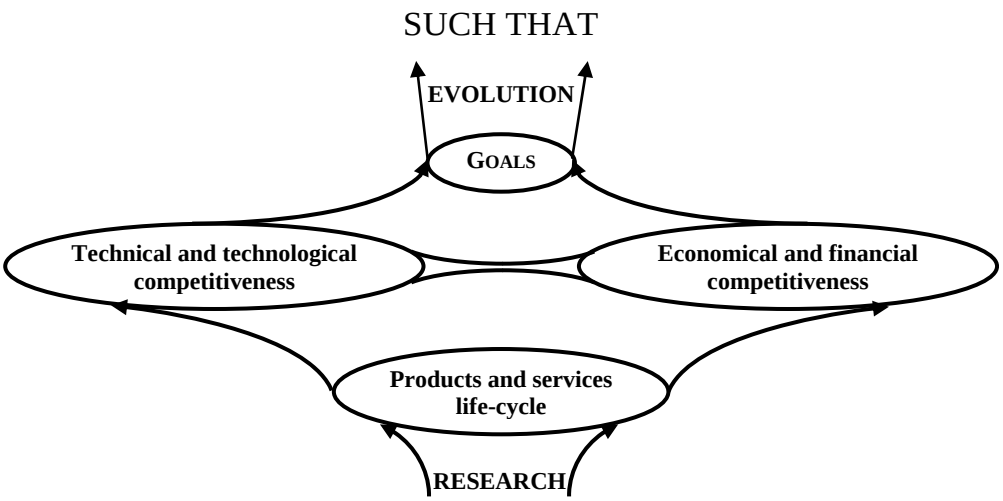

Figure 8. The evolution of the enterprise.

REFERENCES

[1] I. Armas, *Mechatronic Products and Services* (in Romanian), Editura AGIR, Bucharest, 2009.
[2] W. R. Wessels, *Practical Reliability Engineering and Analysis for System Design and Life-Cycle Sustainment*, CRC Press, 2010.
[3] I. Armas, *Design for Mechatronics and Robotics* (in Romanian), Editura AGIR, Bucharest, 2011.
[4] P. A. Hsiung, et. al., *Reconfigurable System Design and Verification*, CRC Press, 2009.

IMPACT OF FISCAL POLICY ON ECONOMIC DEVELOPMENT IN UKRAINE

Iryna LUKIANENKO*, Mariia SYDOROVYCH

Abstract. *The paper aims on assessing the impact and efficiency of fiscal policy instruments for gaining macroeconomic goals in transition economies. For this reason the macroeconomic model of the Ukrainian budgetary sector is developed. The model represents interrelations between the budgetary sector and real economy reflecting the way financial resources are redistributed through the budgetary system. The model serves as a basis for scenario analysis of different types of fiscal policy and their effect on economic indicators. It also allows for forecasting budgetary revenues in order to compose an efficient budget estimates. The results of the research prove that in transition economy such as Ukrainian the channel of tax rate on personal income tax is more efficient from fiscal point of view than the one on enterprise profit tax, while reallocation of resources between budgetary transfers to population and R&D is more efficient than increasing budget expenditures. Since increase in expenditures influences higher increase in GDP, the model proves Keynesian effects in the economy of Ukraine.*

Keywords: *fiscal policy, macroeconomic model of budgetary sector, simultaneous equation modelling, economic development, transition economies, Ukraine.*

1. Introduction

The debates on a role of the government in economic development held during the recent century are permanently heated by the serial crises and the new challenges they send to the political and economic leaders of the world. The question they raise is what is more efficient for economic performance: government intervention in the market through implementing economic policy (fiscal in particular) or non-intervention. Government intervention is justified by market failures resulting from market imperfections – a concept developed by Henry Sidgwick [1] and Francis

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Bator [2], while non-intervention – by the principle called '*laissez-faire*' of Classical economics and by the concept of *government failure* of Public choice school.

Buchanan and Musgrave [3] responding to the position of public choice theory, argue that "defining the good fiscal system should be our primary task, not because there are no policy failures but because first things come first. If we do not know how to do it well, how do we know what failures to correct?" [3, p. 82]. At the same time Krugman, Stiglitz, and Sachs proclaim the entry of the modern economics into "a period in which orthodox views are openly questioned, creating an atmosphere characterized by a crisis of confidence" [4, p.160]. This stance of questioning makes the scientists opened for the discussion of any possible theories and results.

There are plenty of empirical studies validating Keynesian theory and criticizing it. The results are mostly diverse; many assumptions still need to be relaxed; and there is still no common position prevailing. As a result, policy implications are rather ambiguous. Moreover, there are few studies applying recently developed theories to the transition economies, Ukrainian in particular. There is scarcity within those researches explaining whether Keynesian theory is applicable to transition economies, such as Ukrainian one, which instruments of fiscal policy are significant and efficient for providing future sustainable economic development. The problem of specifying which economic theory is appropriate to explain recent economic processes and, thus, can be the basis of government decision making for the sake of economic development, nowadays has more empirical rather than theoretical character.

Various techniques of econometrics that have been developed during recent seventy years such as simultaneous equations (SEM), error-correction (ECM), structural vector autoregressive (SVAR), dynamic stochastic general equilibrium (DSGE) and computable general equilibrium (CGE) modelling give plenty of opportunities to model the behaviour and interrelations within the economy. For this aim the peculiarities of the economy under investigation should be explored thoroughly.

Carmignani [5], in his study of the impact of fiscal policy on private consumption and social outcomes in Europe and CIS with the focus on transition countries, employs simultaneous equations modelling. This technique allows for developing a long-term macroeconomic relationship between fiscal policy and economic growth, using real per-capita consumption growth as the dependent variable.

Carmignani, following Giavazzi and Pagano [6], separates the effects that fiscal policy has in normal times from those that it has during periods of large structural changes, using a dummy variable to pick episodes of sizeable fiscal adjustments or expansions. Thus, Carmignani [5] comes to the conclusion that fiscal policy has Keynesian effects in transition countries; non-significant effects in high-income OECD economies in normal fiscal times, and non-Keynesian effects outside normal times. For several developed countries this research partly correlates with previously held investigation of Alesina and Perotti [7], which comes to conclusion that fiscal adjustments mostly do not cause recessions. They address governments to cut expenditures, in particular transfers and government employment expenditures, rather than increase taxes. In addition, Carmignani [5] proves that government expenditures for public health and social protection are efficient in terms of social outcomes. The results suggest that fiscal adjustment in transition economies tend to be contractionary, while fiscal stimulus is expansionary. It also means that there is scope for a counter-cyclical usage of public spending, as long as fiscal policy is used symmetrically over the business cycle. On the contrary, for high-income OECD economies, the findings suggest that fiscal stabilizations might be expansionary, albeit the multiplier is likely to be small.

Lukianenko [8] develops a systematic model of Ukrainian economy decomposing it into sub-systems of real, monetary, external, budgetary sectors, and labour market. In the research she is employing SEM approach to model long-run relationships within the system. The model serves as the basis for scenario analysis, as well as provides analysis of multiplier effect of government expenditures and money supply. The findings of this study prove Keynesian effects in Ukrainian economy since decrease in tax pressure leads to increase in GDP.

Though the results stemming from the large volume of empirical investigations on the question about the contribution of fiscal policy to economic development are hard to summarize, as they are often incompatible and ambiguous, it is more common for transition and developing economies to bear Keynesian effects, while recent evidence from the developed economies in policy implications is mostly opposite and controversial. This, in turn, may be the evidence of higher efficacy of fiscal policies in transition countries.

Monetary policy is thought to have more rapid impact on the economy because of short inside lags. However, in transition economies it may appear less efficient than fiscal policy because of low level of

financial market development, which leads to longer outside lags in monetary policy and poor transmission effect. Thus, fiscal policy, because of its being built-in in the transition economies deeper than monetary one, needs to be investigated more thoroughly.

There are considerably more studies exploring the transmission of monetary policy shocks than of fiscal policy ones. In addition, applications of the theories and methodologies are mostly focused on the sample of high-income and/or developed countries rather than developing and transition economies in particular. As Carmignani [5] emphasizes, studies that are specifically focused on transition economies are “still in its infancy”. Since the history of transition is relatively short, time series are short and data quality is under question, which is constraining empirical analysis.

Ukrainian market along with other national markets in the world is thought to be inefficient. However, this inefficiency has peculiarities that are to be discovered. Explanation of how the Ukrainian economy works, and in turn, how taxation system should be designed, which policy instruments should be used by governments to foster economic development in the transition economy based on the contemporary data of Ukrainian economy is the gap that this research strives to fill in.

The main purpose of the research is to discover interrelations between fiscal policy and macroeconomic state and development based on the way how financial resources of the economy are redistributed through the budgetary system. This paper aims to explore the case of the Ukrainian economy in order to contribute to the body of knowledge in the policy making process for the sake of economic development of transition economies.

To support the efficiency of public finance, relevant instruments of fiscal policy should be developed. Macroeconomic model of budgetary sector should serve as a basis for an efficient fiscal policy reflecting interactions between policy variables and target variables. The problem of development of macroeconomic model of budgetary sector of Ukraine is important nowadays because of the necessity of reforms in the budgetary sector of Ukraine and of the methodological framework for further efficient fiscal policy. Decision making process in the budgetary sector needs to be transparent, and the government should be provided with the instruments for efficient fiscal policy. Moving from *ad hoc* approach in decision making in budgetary allocation and taxation policy to coordinated budgetary policy is necessary for gaining public policy goals.

2. Experimental

This research is applying simultaneous equations modelling approach, which is useful for gaining its purpose. *Simultaneous equations modelling (SEM)* is a structural type of models containing multiple equations and variables. It is a system of a class of complete systems that have as many equations as there are endogenous variables. In these models it is chosen to regard variables as "simultaneous" or "jointly determined", in the sense, that they are related but no causal ordering can be assigned [9, p. 72]. Thus, simultaneous equations framework requires an a priori division of the variables into endogenous and exogenous categories. This kind of distinctions between variables imposes restrictions on the short-run dynamics of the model in order to achieve identification [9, 10, 11]. SEM can be applied to efficiently investigate long-run relations within the system – budgetary sector and real economy in particular, thus it is chosen as a method for this research. Macroeconomic model developed within the framework of SEM allows for developing various scenarios of fiscal policy which may serve as a basis for choosing the most efficient policy and design of tax system.

Applying SEM to the data of Ukrainian economy serves to investigate interrelations between the budgetary and real sector of Ukraine that illustrate the structure of the economy. It is important for design of tax system and fiscal policy implications. The system of equations contains 6 regression equations and 3 identities. It can be presented as follows:

$$PIT = f\{SAL\_BUDG,\ P,\ (SAL\_BUDG(-1) - SAL(-1))*TR\_PIT(-1),\ DUMMY\_CR\}; \quad (1)$$

$$EPT = f\{GDP(-1),\ d(INNOV\_GOV(-2)),\ P(-1)*TR\_EPT(-1), Q1*PPI,\ DUMMY\_2010\_3\}; \quad (2)$$

$$VAT = f\{CONS - IMP,\ IMP,\ EX/GDP,\ TR\_POPUL/EXPEN,\ DUMMY\_CR1,\ DUMMY\_2010\_3\}; \quad (3)$$

$$EXCISE = f\{IMP,\ CONS\text{-}IMP,\ DUMMY\_09\}; \quad (4)$$

$$REV\_OTH = f\{GDP,\ DUMMY\_11\}; \quad (5)$$

$$GDP = f\ \{EXPEN,\ EX,\ VAT*(1+TR\_VAT)/(TR\_VAT)*(1-EXPEN/CONS),\ DUMMY\_08,\ DUMMY\_2010\_3\}; \quad (6)$$

$$REV = PIT+EPT+VAT+EXCISE+REV\_OTH; \quad (7)$$

$$EXPEN =$$

$$= INNOV\_GOV + TR\_POPUL + SAL\_BUDG + EXPEN\_OTH; \quad (8)$$

$$BD = REV - EXPEN, \quad (9)$$

where *PIT* – personal income tax (mln UAH); *EPT* – enterprise profit tax (mln UAH); *VAT* – value added tax (mln UAH); *EXCISE* – excise on import and export (mln UAH); *REV_OTH* – other revenues of the consolidated budget (mln UAH); *REV* – revenues of the consolidated budget (mln UAH); *EXPEN* – expenditures of the consolidated budget (mln UAH); *BD* – deficit of the consolidated budget (mln UAH); *INNOV_GOV* – expenditures of the consolidated budget on innovations (Expenditures on research-and-development and Expenditures of state and regional significance) (mln UAH); *TR_POPUL* – transfers to population from the consolidated budget (mln UAH); *SAL_BUDG* – salaries to employees of institutions financed from the budget (mln UAH); *EXPEN_OTH* – other expenditures of the consolidated budget (mln UAH); *TR_PIT* – tax rate of PIT (%); *TR_EPT* – tax rate of EPT (%); *TR_VAT* – tax rate of VAT (%); *SAL* – salaries fund (mln UAH); *P* – profits of enterprises before taxes (mln UAH); *GDP* – gross domestic product (mln UAH); *CONS* – consumption (mln UAH); *IMP* – import (mln UAH); *EXP* – export (mln UAH); *PPI* – index (%); *Q1* – dummy-variable for the first quarter; *Q4* – dummy-variable for the fourth quarter; *DUMMY_CR and DUMMY_CR1* – dummy-variables that indicate the impact of financial and economic crisis on budgetary and macroeconomic indicators during the period from 4th quarter of 2008 to 4th quarter of 2009, and 1st quarter of 2010 respectively; *DUMMY_08* – dummy-variable that indicates the impact of financial and economic and debt crises on budgetary and macroeconomic indicators starting from 4th quarter of 2008; *DUMMY_09 and DUMMY_11*- dummy-variables that indicate the impact of changes in the Ukrainian legislation concerning rates of excise taxes in 2009 and system of taxation in 2011 respectively.

In order to estimate the system of equations, it should be over identified or exactly identified. For this, there are sufficient condition for consistency (order condition) and necessary condition (rank condition for identification). After testing for both conditions, the system of equations proved to be over identified, so it can be assessed with the use of Two-stage least square (2SLS) or Three-stage least square (3SLS) Method.

This study was conducted in the framework of the official statistics published by the State Statistics Committee of Ukraine, State Treasury of

Ukraine, National Bank of Ukraine, and other state institutions [12, 13 and 14]. Data collection of the main budgetary and economic indicators of Ukraine is a hard procedure, as there is still lack of transparency in governmental institutions. The data on performance of consolidated budget from 2000 to 2011 derives from the state reports provided by the Institute of Economic Research and Political Consultancy. The budgetary data from 2004 to 2011 derives from the Treasury reports. The data of the real economy derives from public reports of the State Statistics Committee of Ukraine. All the time series explored in the model are quarterly data starting from 2000 to 2011.

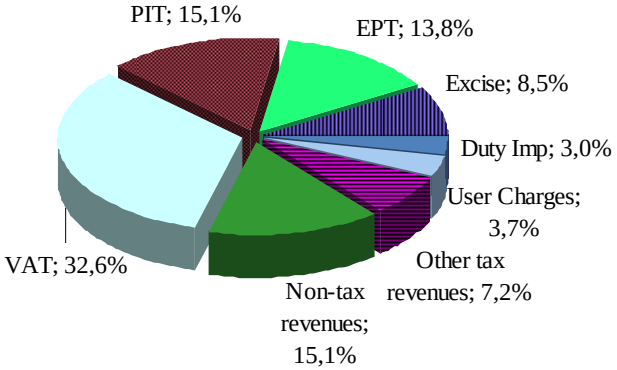
3. Results Section

3.1. Statistic Analysis of the Data

The main components of budget system are revenues, expenditures, and budget deficit. They are interconnected through real economy and economic policy that is the reason for exploring these relations more thoroughly.

3.1.1. Characteristics of the revenue side of consolidated budget

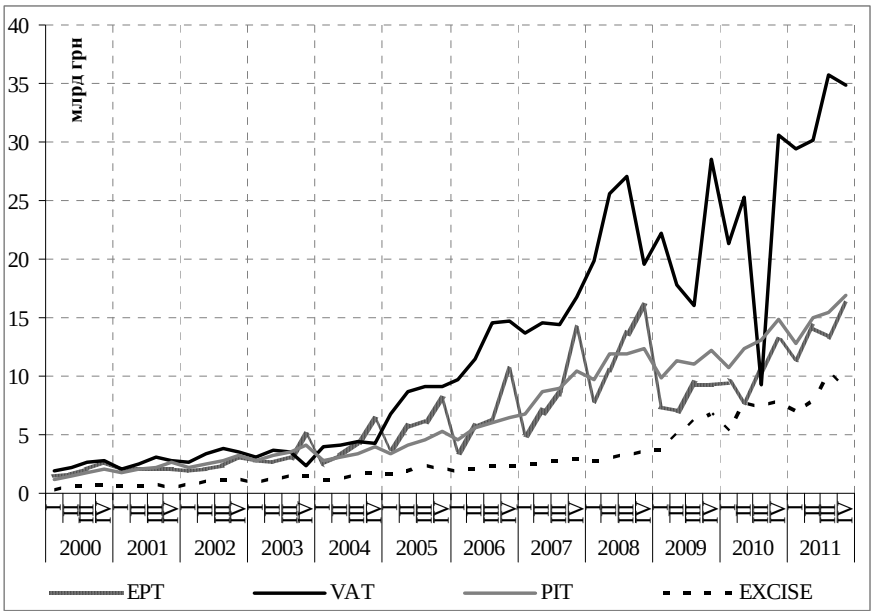
Budget revenues of Ukrainian budgetary system by 84% consist of tax revenues (2011). The main tax revenues of the consolidated budget are value added tax (VAT) (32,6% of revenues in 2011), personal income tax (PIT) (15,1%), enterprise profit tax (EPT) (13,8%), and excise (8,5%) (Graph 1). Since in transition economies indirect taxes, such as value added tax (VAT) and excise, are harder to evade, their comparatively high share in Ukraine (amounts to more than 40%) makes the revenue side of the budget system more stable.



Graph 1. Structure of the revenues of consolidated budget of Ukraine in 2011.

Since there are 4 taxes that comprise 70% of budget revenues of Ukraine, for the sake of modelling budget system, revenue side of the budget is divided into 5 components: VAT, PIT, EPT, excise, and the rest of the revenues (other revenues). Yearly and quarterly dynamics of the main revenue components of consolidated budget that are VAT, PIT, EPT and Excise during 2000-2011 is illustrated in Graphs 2. Their shares constituted from 53,2% in 2000 to 66,7% in 2008, and 70,1% in 2011 in the total revenues of consolidated budget with an increasing tendency.

During 2000-2004 revenues from PIT, EPT and VAT were equal. Though being the hardest to administer, starting from 2005 VAT had constituted significantly higher share in the revenues of consolidated budget. VAT revenues started to dominate in budget revenues with an active government campaign against fake reimbursement of this tax from the budget (see Graph 2).



Graph 2. Quarterly dynamics of the main tax revenues during 2000-2011.

Redistribution of financial resources has followed a change in the system of taxation of incomes of individuals. Since the reform was aimed at stimulating legalization of incomes in 2004, the state proposes a new way of taxation – introduction of a flat rate of 13% instead of progressive scale. As a result of transition to the new system of taxation comparing to the previous dynamics of budget revenues to the budget in short term the budget under levied a lump sum of resources. This is demonstrated in the

dynamics of PIT in 2004 comparing to 2003 and comparing to the amount of the other taxes levied (Graph 2).

Also Graph 2 illustrates that in the past years until 2008 the amount of PIT and EPT overtook each other depending on the quarter. The reason for those changes in the dynamics are seasonality, that is inherent to both of the taxes, deriving from the specifics of the tax base, as well as of tax legislation that influences the dynamics of levying the taxes. Since recent financial and economic crisis caused decrease in the level of profits of enterprises, starting from 2009 the amount of PIT levied has predominated the amount of EPT.

3.1.2. Characteristics of the expenditure side of consolidated budget

Recently the expenditures on social protection, education, healthcare, and economic activities have constituted more than 70% of expenditures of the consolidated budget. The level of expenditures on economic activities of the state (11,6%-17,9% in 2007-2011) corresponds to the level of financing state functions (10,7%-12,0% in 2007-2011). About half of the latter expenditures finance administrative apparatus, and the other half is debt service. High share of those expenditures serves as an evidence for the necessity of increasing the efficiency of debt policy aimed at decreasing the level of interest rates and the scope of loans, as well as restructuring administrative apparatus.

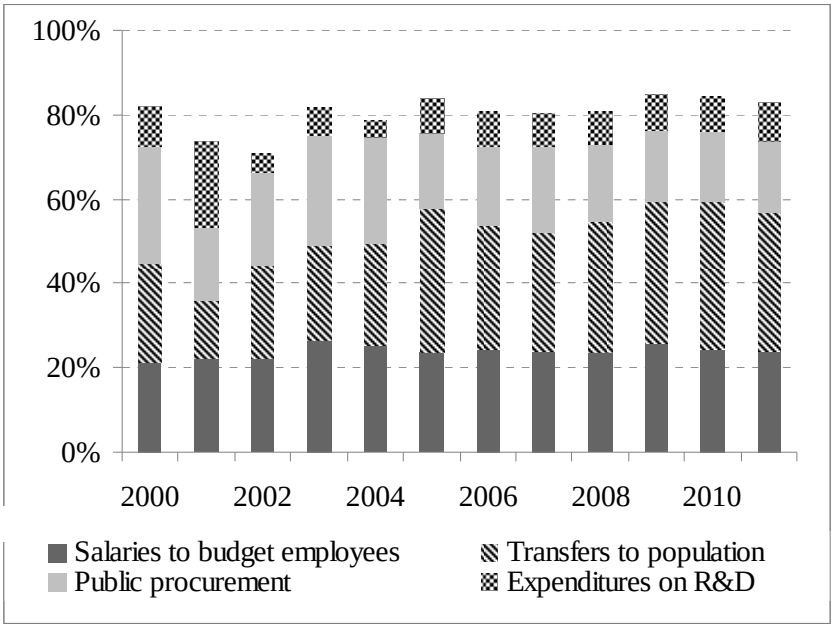
Growing expenditures on social protection and social service simultaneously with shortening expenditures on defence, even though national defence is strategic from the point of view of state integrity, are recent trends of the past decade. It serves as an evidence of non-proportional distribution of resources of the budgets of Ukraine and the necessity of increasing the efficiency of the budget process.

According to economic classification, expenditures of consolidated budget consist of current and capital expenditures, as well as unallocated expenditures and crediting excluding amortization. Growing share of current expenditures and shortening capital expenditures is a recent trend in Ukraine. Thus, in 2011 the share of capital expenditures amounted to 10,1%, ranging between 6,5% (in 2009) and 20,4% (in 2004) [State Treasury of Ukraine]. In the meanwhile it is worth to mention that in transition economy most of institutional changes may be provided with capital expenditures, including expenditures on innovations.

According to economic classification of the budget, the main components of expenditures of the consolidated budget are salaries of the employees of budget institutions, expenditures on investments in innovative programs (expenditures on R&D), transfers to the population and public consumption.

The first group representing salaries of the employees of budget institutions and pensions of former military forces employees (group «Salaries to budget employees» at Graph 3) in 2011 constituted 24,2% of the expenditures of consolidated budget of Ukraine.

Graphic analysis of Graph 3 demonstrates that the share of this group in the expenditures during 2000-2011 was the most constant and during this period ranged between 21,3% and 26,7%. From the point of view of fiscal policy efficiency this group of expenditures is a significant constituent of incomes of individuals, influencing the amount of PIT levied in the future periods. The second group of budget expenditures is expenditures on research and development (R&D) and expenditures of the state or regional significance (group «Expenditures on R&D» at Graph 3), since a part of those expenditures are long term, they influence innovative constituent of the state economy. In 2011 this group of expenditures amounted to 15,7% of budget expenditures. During 2000-2010 the share of this group in the expenditures of consolidated budget constituted from 3,7% (in 2004) to 9,7% (in 2000) (excluding 20,4% in 2001), being constant during 2005-2010 (around 8%), and increased almost twice in 2011. The second group of budget expenditures is expenditures on research and development (R&D) and expenditures of the state or regional significance (group «Expenditures on R&D» at Graph 3), since a part of those expenditures are long term, they influence innovative constituent of the state economy. In 2011 this group of expenditures amounted to 15,7% of budget expenditures. During 2000-2010 the share of this group in the expenditures of consolidated budget constituted from 3,7% (in 2004) to 9,7% (in 2000) (excluding 20,4% in 2001), being constant during 2005-2010 (around 8%), and increased almost twice in 2011. Expenditures on R&D stimulate innovative activities of the enterprises, which under the efficient allocation of the resources allow increasing incomes in the economy, and consequently, the amount of EPT levied in the future periods.



Graph 3. Dynamics of expenditure constituents of the consolidated budget in Ukraine during 2000-2011.

The third group of expenditures representing their influence on the VAT levied, consists of 2 parts: current and capital transfers to the population, charges on the salaries of the employees of budget institutions (subgroup «Transfers to population» at Graph 3); and the materials, inventory and services provided to the state (subgroup «Public procurement » at Graph 3). Current and capital transfers to the population are assigned in the form of targeted aid to some categories of population (e.g. low-income groups), charges on salaries are redistributed through Pension fund and funds of social insurance (assurance, unemployment et.al.), which afterwards is mostly used for consumption by the low-income group of population, influencing the amount of VAT. Subgroup of transfers to the population in 2011 amounted to 32,4% of the expenditures of consolidated budget, while expenditures on goods and services of the state – 17,3% (Graph 3). During 2000-2010 the share of transfers to the population in budget expenditures ranged between 13,5% (in 2001) and 32,4% (in 2011) with an increasing tendency, expenditures on goods and services of the state – between 17,3% (in 2011) and 28,2% (in 2000) with a decreasing tendency in the past years (Graph 3). Since expenditures on goods and services of the state are not that volatile, but also influences on the amount of VAT levied, for the sake of modelling they are joined to the transfers to

the population (in the model the third group is called transfers to the population).

Regression Analysis
Modelling and assessing regression equations of budget revenues and GDP

Modelling of 6 separate regression equations of endogenous variables of the model of the budgetary sector (1)-(6) based on theoretical background employing least squares (LS) method to the real data of Ukrainian economy with E. Views packet has given the following results.

1. Estimation of PIT equation

Modelling budget revenues from personal income tax (PIT) as the regression (1) of the model in a whole with respect to the designations of the variables mentioned in the above gave the following results. Dependence of PIT on the exogenous variables can be presented in the following equation (t-statistics is shown in the brackets):

$$PIT = -98{,}96 + 0{,}347 * (SAL(-1) - SAL\_BUDG(-1)) *$$
$$\text{(-0,60)} \qquad\qquad \text{(5,27)}$$
$$) * TR\_PIT(-1)/100 + 0{,}356 * SAL\_BUDG + 0{,}023 * P +$$
$$\text{(8,94)} \qquad \text{(3,32)}$$
$$+ 748{,}4 * DUMMY\_CR + \varepsilon, \qquad (10)$$
$$\text{(1,72)}$$
$$\hat{R}^2 = 0{,}982, dW = 1{,}962.$$

This regression explains PIT revenues of the consolidated budget with the factors of salary, tax rate, profit, salary of budget sector employees and crisis impact on the economy during the period from the 4th quarter of 2008 till the end of 2009 on the level of 98,2%. From the coefficients of the equation above, the following conclusions can be drawn. Under 1 mln UAH growth of tax burden on PIT on incomes of budget sector employees, other conditions being equal, PIT revenues grow by 347 ths UAH, which is 34,7% of anticipated revenues from the tax, while under 1 mln UAH growth of salary of budget sector employees, PIT revenues grow by 356

ths UAH, which is 35,6% of the tax base. However, since under increase of the tax burden and worsening of macroeconomic conditions propensity to shadowing incomes in transition economies is rather high, during the periods of economic crisis incomes are getting shadowed and IT is levied proportionally on the budget and non-budget sectors employees. Thus, tax rate is the most influential regulator for tax revenues.

Under 1 mln UAH growth of profits of enterprises, other conditions being equal, PIT revenues grow by 23 ths UAH. The scope of the profit is important for modelling PIT revenues, since enterprises are the tax agents of this tax. Thus, the level of legal incomes and the taxes paid accordingly versus shadow economy depends on the level of profits of the enterprises. Though profit cannot serve as a policy regulator for PIT, the latter is an important macroeconomic indicator for tax modelling.

2. Estimation of VAT equation

Modelling budget revenues from value added tax (VAT) as the regression (2) of the model in a whole with respect to the designations of the variables mentioned in the above gave the following results. Dependence of VAT on the exogenous variables can be presented in the following equation (t-statistics is shown in the brackets):

$$VAT = -2429{,}6 + 0{,}135 * (CONS - IMP) - 3178{,}9 * EX / GDP + 0{,}121 * IMP +$$
$$\text{(-0,85)} \qquad \text{(6,32)} \qquad \text{(-0,55)} \qquad \text{(11,46)}$$
$$+ 6608{,}0 * TR\_POPUL / EXPEN - 2460{,}9 * DUMMY\_CR1 - 19052{,}3 * DUMMY\_2010\_3, \tag{11}$$
$$\hat{R}^2 = 0{,}957, dW = 2{,}048.$$

This regression equation explains VAT revenues of the consolidated budget with the factors of internal consumption, export, import, transfers to the population from the budget, and impact of financial and economic crisis on the level of 95,7%.

From the coefficients of the equation above, the following conclusions can be drawn. Under 1 mln UAH growth of import of goods, other conditions being equal, VAT revenues grow by 121 ths UAH, which is equivalent to 12,1% of the tax base, which is lower than basic rate on VAT, which amounts to 20%. The reason is preferential taxation, which is widely used for export operations.

The coefficients of the equation demonstrate that increasing transfers to population makes positive impact on the rise of VAT revenues since

mostly they are transferred to sustain the level of low income groups who use this aid for consumption of the goods of first need.

3. Estimation of EPT equation

Modelling budget revenues from enterprise profit tax (EPT) as the regression (3) of the model in a whole with respect to the designations of the variables mentioned in the above gave the following results. Dependence of EPT on the exogenous variables can be presented in the following equation (t-statistics is shown in the brackets):

$$EPT = 515{,}2 + \underset{(1{,}44)}{0{,}039} * GDP(-1) + \underset{(15{,}0)}{0{,}244} * P(-1) * \underset{(4{,}54)}{TR\_EPT(-1)/100} +$$

$$+ \underset{(1{,}07)}{0{,}141} * D(INNOV\_GOV(-2)) - \qquad (12)$$

$$- \underset{(-3{,}12)}{571{,}6} * Q_1 * PPI + \underset{(1{,}48)}{1064{,}4} * DUMMY\_CRI,$$

$$\hat{R}^2 = 0{,}917, dW = 2{,}170.$$

This regression equation explains EPT revenues of the consolidated budget with the factors of tax rate on EPT, enterprise profits, GDP, increase of decrease of expenses on research-and-development, and seasonal peculiarities of tax administration on the level of 91,7%.

The coefficients of the equation demonstrate that EPT tax burden is 3,9%. Under 1 mln UAH growth of tax burden on EPT, other conditions being equal, in the following quarter EPT revenues grow by 244 ths UAH, which is less than increase in PIT revenues under change of tax burden on the latter tax. This misfit of tax rates efficiency is caused by the differences in book-keeping accounting and tax accounting according to which taxes are levied. Another reason is the amount of non-justified rebates and tax exemptions on EPT. Thus, there is need to shorten their amount and to shorten the disparities.

Though state expenses on innovation which are common in transition economies for facilitating innovative processes and creating innovative economics are still low and statistically insignificant, they start to influence on EPT revenues with positive impact.

Seasonality of the first and fourth quarters is the result of the norms and order of paying EPT. It causes uneven tax burden on the enterprises that in its turn may cause shadowing of incomes.

4. Estimation of Excise tax equation

Modelling budget revenues from excise tax (Excise) as the regression (4) of the model in a whole with respect to the designations of the variables mentioned in the above gave the following results. Dependence of excises on the exogenous variables can be presented in the following equation (t-statistics is shown in the brackets):

$$\underset{}{EXCISE} = \underset{(0,90)}{147,0} + \underset{(4,28)}{0,018} * (CONS - IMP) + \underset{(10,28)}{0,019} * IMP + \underset{(9,04)}{2726,3} * DUMMY\_09, \quad (12)$$

$$\hat{R}^2 = 0,962, dW = 1,739.$$

This regression equation explains excise revenues of the consolidated budget with the factors of internal consumption, import and impact of increase in excise tax rates in 2009 on the level of 96,2%.

For modelling VAT and excise revenues that are indirect taxes consumption is the main factor of influence. That is why these taxes are sensitive to the phases of economic cycles. Excise consists of excise on import and on export. After accession of Ukraine to WTO in 2005 import started to prevail on export.

From the coefficients of the equation above, the following conclusions can be drawn. Under 1 mln UAH growth of consumption of the goods produced in the economy, other conditions being equal, excise revenues grow by 18 ths UAH, or 1,8% of the tax base, which can be interpreted as an efficient (real) rate of excise levied on the goods of internal production. Under 1 mln UAH growth of import of the goods produced in the economy, other conditions being equal, excise revenues grow by 19 ths UAH, or 1,9% of the tax base, which can be interpreted as an efficient (real) rate of excise levied on the imported goods.

Test diagnostics by LM-test confirms correct specification of the regression equation.

5. Estimation of regressions for other revenues

Modelling other budget revenues as the regression (5) of the model in a whole with respect to the designations of the variables mentioned in the

above gave the following results. Dependence of other budget revenues on the exogenous variables can be presented in the following equation (*t*-statistics is shown in the brackets):

$$REV\_OTH = \underset{(2,23)}{1337,7} + \underset{(26,70)}{0,100} * GDP - \underset{(-3,16)}{4748,7} * \text{DUMMY\_11} \quad (13)$$

$$\hat{R}^2 = 0,950, dW = 2,473.$$

This regression equation explains other revenues of the consolidated budget through the scope of production in the economy and impact of changes in the tax legislation on the level of 95,0%.

Most of other tax revenues that are of the higher share in other budget revenues depend on the level of production of the goods in the economy since mostly they depend on the peculiarities of economic activities. From the coefficients of the equation above it can be concluded that under 1 mln UAH growth of production of goods and services, other conditions being equal, other budget revenues grow by 100 ths UAH, or 10,0%. Also as a result of the changes in tax legislation and amendments of the Tax Code of Ukraine other revenues of the consolidated budget decreased.

6. Estimation of GDP equation

Modelling GDP as the regression (6) of the model in a whole with respect to the designations of the variables mentioned in the above gave the following results. Dependence of GDP on the exogenous variables can be presented in the following equation (–-statistics is shown in the brackets):

$$GDP = \underset{(4,23)}{17169,2} + \underset{(7,70)}{1,037} * VAT / (TR\_VAT / 100) * (1 + (TR\_VAT / 100)) * (1 - EXPEN / CONS) +$$

$$+ \underset{(7,42)}{1,587} * EXPEN - \underset{(0,83)}{0,126} * EX - \underset{(-1,37)}{11029,6} * \text{DUMMY\_08} + \underset{(-6,28)}{102728,3} * DUMMY\_2010\_3, \quad (15)$$

$$\hat{R}^2 = 0,980, dW = 2,079.$$

This regression explains the amount GDP with the factors of the amount of budget expenditures, export, absorption of private sector

(consumption and investments), as well as crisis impact on the economy on the level of 98,0%.

From the coefficients of the equation above, the following conclusions can be drawn. Under 1 mln UAH growth of budget expenditures on consumption and investments the amount of GDP grows by 1,59 mln UAH. Thus, the multiplier of budget expenditures amounts to 1,59, which stands for increase of GDP with higher growth rate than budget expenditures. This is the justification for the Keynesian effects in the economy of Ukraine.

Under 1 mln UAH growth of the amount of the absorption of private sector, which includes consumption and investments within the state from the side of the private sector, the amount of GDP grows by 1,04 mln UAH. Less than 1 mln UAH growth of export of goods and services, the amount of GDP grows by 126 ths UAH. Thus, external sector holds an important role in the economy, however, with the disproportions in the trade balance it is not the basic regulator of the economic state of the country. There is also evidence for negative impact of economic crisis on GDP.

Test diagnostics, LM-test in particular, confirms correct specification of all regression equations.

The justification of the main assumptions of econometric modelling within which are absence of autocorrelation, heteroskedasticity, multicollinearity, and correct specification of the equations is done with the help of internal econometric tests. Validity and reliability of the modelled equations are justified with the tests of Darbin-Watson, Fisher's criteria, LM-test of Breush-Godfrey, for some regression equations if needed adjustment for Heteroskedaskity White test was done. Conclusion on absence of multicollinearity can be gained from the analysis of separate regression equations, since most of economically forecasted interrelations are justified with the results of assessment of equations. Correlation between the factors of separate equations is less than 80%, and the number of statistically insignificant variables is small.

Assessing the system of equations and solving simultaneous model

Comparison of the results of assessment of the parameters of the simultaneous equation model (1)-(6) as whole with two-stage least squares

(2SLS) and three-stage least squares (3SLS) showed that 3SLS gives more accurate results for more consistent parameters, since determinants of covariance matrices of errors of 3SLS (1,74E+38) are less than of 2SLS (2,16E+38). Thus, 3SLS is applied for further assessment of the model.

Estimated model with 3 additional identities can be presented in the following form:

$$PIT = 71{,}6 + 0{,}31*(SAL(-1) - SAL\_BUDG(-1))*TR\_PIT(-1)/100 + 0{,}39*SAL\_BUDG + 0{,}02*P + 191{,}7*DUMMY\_CR \quad (16)$$

$$VAT = -1289{,}9 + 0{,}13*(CONS\text{-}IMP) + 0{,}12*IMP - 5868{,}4*EX/GDP + 6311{,}0*TR\_POPUL/EXPEN - 2052{,}1*DUMMY\_CR1 - 18353{,}7*DUMMY\_2010\_3 \quad (17)$$

$$EPT = 530{,}1 + 0{,}04*GDP(-1) + 0{,}24*(TR\_EPT(-1)*P(-1)/100) + 0{,}17*d(INNOV\_GOV(-2)) - 624{,}8*Q1*PPI + 1135{,}9*DUMMY\_CR1 \quad (18)$$

$$EXCISE = 200{,}1 + 0{,}020*IMP + 0{,}016*(CONS - IMP) + 2741{,}5*DUMMY\_09 \quad (19)$$

$$REV\_OTH = 1147{,}0 + 0{,}102*GDP - 5548{,}5*DUMMY\_11$$

$$GDP = 20270{,}8 + 1{,}54*EXPEN + 0{,}006*EX + 1{,}177*(VAT/(TR\_VAT/100)*(1+(TR\_VAT/100))*(1-EXPEN/CONS)) - 6281{,}2*DUMMY\_08 + 103377{,}5*DUMMY\_2010\_3 \quad (21)$$

$$REV = PIT + EPT + VAT + EXCISE + REV\_OTH \quad (22)$$

$$EXPEN = INNOV\_GOV + TR\_POPUL + SAL\_BUDG + EXPEN\_OTH \quad (23)$$

$$BD = REV - EXPEN\ .$$

The results of estimation demonstrate that the model defines PIT revenues with the factors of salary, tax rate, profit, salary of budget sector employees on the level of 98,1%. It explains VAT revenues of the consolidated budget with the factors of internal consumption, export, import, transfers to the population from the budget on the level of 95,5%. The model explains EPT revenues of the consolidated budget with the factors of tax rate on EPT, enterprise profits, GDP, increase or decrease of expenses on research-and-development, and seasonal peculiarities of tax

administration on the level of 91,6%. It explains excise revenues of the consolidated budget with the factors of internal consumption, import and impact of increase in excise tax rates in 2009 on the level of 96,1%. It also explains other budget revenues through the scope of production in the economy and impact of changes in the tax legislation on the level of 94,6%. The model explains GDP with the factors of the amount of budget expenditures, export, absorption of private sector (consumption and investments) on the level of 97,9%, as well as factor of financial and economic crisis that makes impact on each endogenous component of the model.

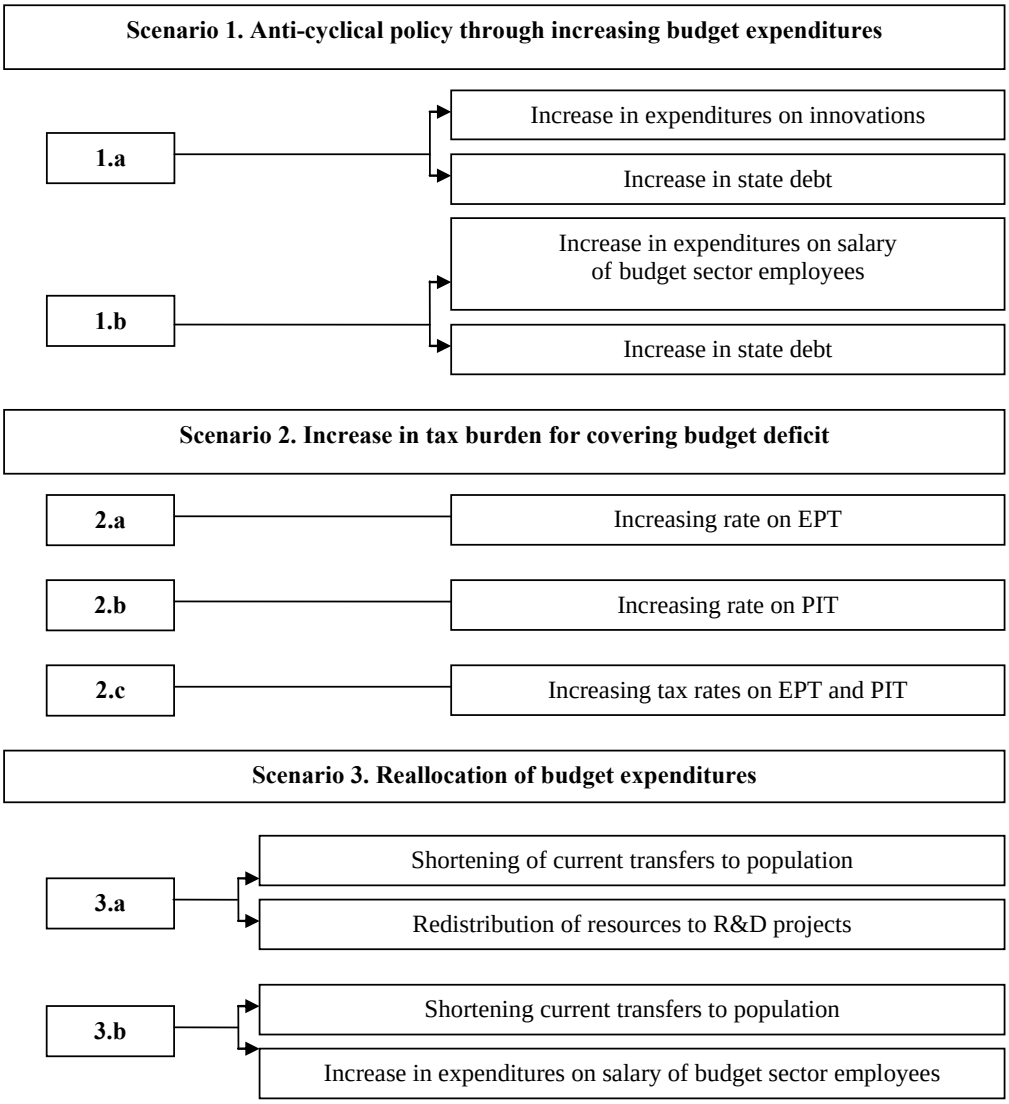
Assessment of the simultaneous model of 3 identities and 6 regressions applying Dynamic-Deterministic Simulation reflects that the series converge, which means that the model is steady and can be applied for forecasting, simulations, and developing scenarios of fiscal policy. The graphs also demonstrated that most of simulated data of endogenous variables accurately follow the real data.

The analysis of the forecasting capabilities of the model demonstrates that they are satisfactory for applying the model for forecasting. It is supported with the indicator of forecasting capability of the separate equations (MAPE), which is a relative indicator of accuracy of the model. According to it forecasting capability of PIT is good (the value of MAPE is 0,0897), and of EPT (0,1504), VAT (0,1754) and Excise (0,1541) taxes is satisfactory. Hence, forecasting capability of the model is satisfactory. The model is simulating all of the turning points of the data at a sufficient level and it is stable to insignificant changes in the parameters.

Scenario Analysis

In order to compare the efficiency of certain instruments of fiscal policy in the economy of Ukraine, different scenarios of fiscal policy were simulated based on the model of the budget system. They are the scenarios that simulate anti-cyclical policy through increasing budget expenditures; the increase in tax burden for covering budget deficit, including changes in the tax rates of EPT and PIT; reallocation of budget expenditures through changing the proportions of budget expenditures, including shortening current transfers to the population; redistribution of the

resources to R&D projects or to salaries of the employees of budget institutions. (Graph 4)



Graph 4. Scenarios of fiscal policy in frames of macromodel of budgetary sector of Ukraine.

Graphic results of estimations of the scenarios have demonstrated that in the group of the scenarios implying the increase in the revenue side of the budget, scenario 1, which proposes to increase budget expenditures, is the most fiscally efficient. Increasing budget expenditures on salaries of the employees of budget institutions by 10% is the most efficient scenario out

of scenario groups 1, 2, and 3. The next efficient scenario is redistribution of budget resources from current transfers to the population (30% of the basic level) to salaries of the employees of budget institutions, and the next one is the alternative of applying the instrument of the tax rates. Under use of tax channels, changes of the tax rate of PIT allow gaining bigger changes in the budget revenues rather than changes of the rate of EPT. It is worth to mention that every scenario of the groups 1-3 is more efficient than the basic one.

Scenarios 1.b and 3.b that imply increase in budget expenditures and reallocation of budget expenditures from transfers to the population to salaries of the employees of budget institutions are equally efficient from the point of view of increase in revenues of PIT.

Scenarios 1.a and 2.a that imply increase in expenditures to R&D (1.a) in the first half of the forecasting period, and increase in tax rate of EPT (2.a) in the second half of the period respectively are more efficient from the point of view of increasing revenues of EPT. Overtaking temps of increase of revenues of EPT at the end of the forecasting period as a result of undertaking scenario 2 serves as an evidence that the instrument of tax rates on this tax is more efficient than increasing in budget expenditures on innovative projects absorbed in the economy.

Reallocation of expenditures of salaries of the employees of budget institutions (scenario 3.b) can be efficiently applied in order to decrease the difference in revenues and expenditures (comparing to the basic scenario); applying the instrument of tax rates, in particular rate of PIT (scenario 2.b) is also rather efficient.

From the analysis of scenario realizations the following conclusions can be drawn. Three alternative fiscal policy directions were proposed. Reallocation of budget expenditures from transfers to population to salaries of the employees of budget institutions, as well as changes in the tax rates are the most efficient instruments from the point of view of the resources mobilized. The highest fiscal efficiency of changing tax rate of PIT serves as evidence that PIT is the most sensitive in a sense of changing the scope of budget revenues. It is worth to mention that the comparison of the alternatives depends on the scope of resources redistributed and the size of changes in the tax rates.

The choice of combinations of fiscal policy instruments might have multiple options. Besides changing budgetary and tax components, scenarios might imply different macroeconomic conditions in the forecasted period. Constructing scenarios involves elaborating optimistic and pessimistic scenarios of economic conditions and the alternative

packages of macroeconomic indicators. Thus, scenario analysis allows assessing the efficiency of certain fiscal policy instruments under given economic conditions, as well as forecasting the variance of the future budget revenues which serves to assess possible risks related to under-levied revenues caused by slow recovery or worsening of economic conditions in the state.

4. Discussion

The results of the research, among other conclusions, confirmed Keynesian type of effects in the Ukrainian economy at its current transformational stage of development. Recently after financial and economic crisis, many researchers have confirmed the same effects in other economies, both in developed and transition economies, which make the conclusions of this research stand in line with international evidence and theoretical implications.

However, to apply fiscal policy instruments in order to attain economic effect, a number of reforms have to be undertaken to support economic system of Ukraine with better functioning of transmission mechanism.

It is worth mentioning, that this research is aimed at modelling budgetary sector of Ukraine. Thus, it is modelling interrelations exclusively between budgetary sector and real economy and among their components. The model does not take into consideration interrelations with other sectors, monetary and external in particular. Modelling the system of all interrelations within the economy could contribute to the accuracy of coefficients mentioned above. However, this model would be more complex, and not all the interrelations could be possible to model, which may worsen the accuracy of the impacts modelled.

Also the state of the world or local economy that influence budgetary sector, as well as the other exogenous variables that may be necessary for the model proposed in the research, are to be estimated and forecasted with the use of external models that are subject of another research.

With the use of the model proposed in the research, it is possible to replicate the research with the data of the other economies. Replication of quantitative estimation of the parameters of interrelations in the budgetary sector primarily would need conducting the analysis of qualitative

interrelations, based on the peculiarities of tax and budgetary system of a particular country.

5. Conclusions

Economic fluctuations in the developed and transition economies during the recent century, as well as recent sovereign debt crisis of the leading world economies has proved a necessity of continuous improvement and adjustment of fiscal policy to current macroeconomic conditions in the economy. Hence, it is highly topical to focus on investigating actual interrelations between fiscal policy and the state of the economic system of Ukraine, which is still being on the stage of transformation. Conducting efficient economic policy demands considering the features that are inherent to the economic system, as well as exploring the mechanisms involved in the economic processes of Ukraine. They should serve to explain the way tax and budgetary systems of the state should be arranged, as well as to suggest a justified choice of economic policy instruments applied that support economic development.

Providing further development of methodological principles of conducting fiscal policy is important for making balanced fiscal policy decisions. However, there is insufficient use of economic-mathematical tools among the researches on the budgetary sector of Ukraine, which would allow estimating quantitative and qualitative characteristics of the budgetary sector and corresponding interrelations between budgetary and economic indicators that serve to estimate the efficiency of different instruments of fiscal policy. In order to achieve the aims of socio-economic development of the state, conducting effective fiscal policy has to involve complex and regular evaluation of interrelations in the budgetary and economic systems, including macroeconomic indicators. Methodology of macroeconomic modelling, in particular simultaneous equations modelling (SEM), allows providing the assessments.

Budget revenues as an instrument of fiscal policy accomplish important fiscal and regulative functions. They are measured with capability of the budgetary system to levy taxes and duties in order to provide accomplishment of the fiscal function by the state, and at the same time to regulate economic processes in the state by changing tax rates, tax base, tax preferences and the character of administering tax system. Thus, the character of the tax preferences employed may stimulate the

development of certain industries, which at the same time understate budget revenues. The choice of optimal rates for each tax is a key task in implementing fiscal function of the state, simultaneously serving as a regulator of the economic processes and redistribution of financial resources in the state. At the modern stage of development of the Ukrainian economic system, the efficiency of tax policy should include the expansion of the tax bas, which is possible under conditions of decline of the level of shadow economy, as well as tax preferences should be economically grounded in order to encourage investments and innovations.

Investigation of interrelationships in the budgetary system of Ukraine suggested the following results. There are tight relationships inherent to budgetary sector and real economy of Ukraine. They were subject of analysis based on applying SEM approach. Results of assessment of interrelations reflected that Keynesian effects of fiscal policy on economic development are justified in the economy of Ukraine. Thus, in Ukraine with economy at transformation stage, fiscal adjustment tends to be contractionary. This result is coherent with Carmignani's [5] investigations.

Performance of personal income tax depends on the following macroeconomic factors: the amount of salaries in the economy, enterprise profits and the level of the tax rate. Macroeconomic factors that influence the performance of enterprise profit tax are the amount of enterprise profits, gross domestic product, expenditures on innovation activities that are absorbed in the economy, the level of the tax rate and the administrative issues of levying the tax. Performance of value added tax depends on the following macroeconomic factors: the amount of internal consumption, import, export, transfers to the population from the budget. Macroeconomic factors that determine the performance of excise are the amount of internal consumption and import, and of the other budget revenues — the amount of goods and services produced in the country. Financial and economic crisis has negatively influenced the amount of the taxes levied.

Scenario analysis has justified that tax rates are an efficient instrument in mobilizing financial resources. From fiscal point of view tax rate of personal income tax is more efficient than the rate of enterprise profit tax. Low level of real tax rate of the enterprise profit tax comparatively to the nominal one serves as an evidence of the necessity to reform the system of income taxation. Preferential tax treatment of enterprise income should be transparent and justified based on an objective

demand of support of certain industries and enterprises. Determining the tax base of income tax should be done based on the universal principles of taxation. Improving tax administration should serve to remove the schemes of avoiding income taxation and obtaining budgetary compensations of fake tax credit of VAT. Determining the level of tax rates and tax base of excise tax should be done based on the financial and regulatory rationale.

Budget revenues and expenditures are highly interrelated with macro-economic indicators. In the structure of GDP of Ukraine, consumption remains the main constituent of the economy. However, investments should be basic for providing economic development, which should be supported with different activities and stimulus, including fiscal ones. Negative dynamics of trade balance of Ukraine is caused by losing a part of target markets and recent decreasing prices on the basic export products of Ukraine on the world markets, combined with the lack of competitive import substitutive products produced in the economy. Growth of per capita incomes comparatively to the world indicators and the level of declared income of enterprises is still low, which stipulates low base of taxation on the majority of taxes. Thus, in order to achieve sustainable economic development, coherent tax system should serve to improve trade balance and stimulate investment activities (which constitute a tax base of excise tax and duty), as well as to raise incomes of population and enterprises, and thus, of product consumption (which constitute a tax base of taxes on incomes and consumption).

Under current economic conditions in Ukraine budget expenditures may stimulate economic activities, this being confirmed by the Keynesian effects in the economy of Ukraine at the present stage of economic development. In order to promote long-term economic development, a part of budget expenditures should be effectively allocated in research-and-development, which stimulates economic development. Changing the structure of budget expenditures implies reducing budget transfers to the population and enterprises. Hence, pension system should be reformed in order to provide autonomous functioning of Pension Fund of Ukraine.

However, a huge scope for further research remains. The data used within the analysis of this paper does not give an answer on the impact of shadow economy within this model. Also, further research can be conducted to apply the approach suggested to modelling the interrelations in other transition economies and compare the results attained. Finally, further development of the study can be done through dynamic modelling combining SEM with ECM.

REFERENCES

[1] Medema, S. (2007), *Sidgwicks Utilitarian Analysis of Law: A Bridge from Bentham to Becker?* American Law and Economics Review, vol. **9(1)**, pp. 30-47.

[2] Cowen, T. (1988), *The Theory of market failure: a critical examination* (Fairfax, Va. Lanham, MD: George Mason University Press; Distributed by arrangement with University Pub. Associates).

[3] Buchanan, J. M., and R. A. Musgrave (2001), *Public finance and public choice: two contrasting visions of the state* (Cambridge, Mass.: MIT Press).

[4] Nell, E., and M. Forstater (2003), *Reinventing functional finance: transformational growth and full employment* (Northhampton, MA: Edward Elgar Pub.).

[5] Carmignani, F. (2008), *The impact of fiscal policy on private consumption and social outcomes in Europe and the CIS*. Journal of Macroeconomics.

[6] Giavazzi, Francesco and Marco Pagano (1996), *Non-Keynesian Effects of Fiscal Policy Changes: International Evidence and the Swedish Experience*. Swedish Economic Policy, vol. **3**, May, pp. 75-111.

[7] Alesina, A., and R. Perotti (1995), *Fiscal expansions and adjustments in OECD countries*. Economic Policy, **10(2)**, 207-248.

[8] Lukianenko, I. (2004), *Systematic modeling of indicators of budgetary system of Ukraine: principles and instruments* (Kyiv: Pub. House KM Academy).

[9] Davidson, J. (2000), Econometric theory. Malden, MA: Blackwell Publishers.

[10] Mills, T. C. (1999), *The econometric modelling of financial time series* (2nd ed.). Cambridge, U. K.; New York: Cambridge University Press.

[11] Garratt, A., Lee, K., Pesaran, M. H., & Shin, Y. (2006), Global and National Macroeconometric Modelling: A Long-Run Structural Approach. Oxford: Oxford University Press.

[12] State Statistics Committee of Ukraine: official portal: http://ukrstat.gov.ua/

[13] State Treasury of Ukraine: official portal: http://treasury.gov.ua/.

[14] National Bank of Ukraine: official portal: http://bank.gov.ua/.

APPLICATION OF THE FEASIBILITY STUDY IN PROJECT FINANCE ON THE BASIS OF A SELECTED INVESTMENT PROJECT

Teresa SZOT-GABRYŚ[1]

Abstract. *The paper presents a selection of aspects of a practical application the feasibility study in the evaluation of investment projects for implementation based on the project finance structure. The feasibility study is an analysis on the basis of which owners make decisions whether to implement or abandon a given project. Feasibility studies are particularly important when they investigate investment projects that are totally new and which will be built up from scratch, which is quite typical for project finance method. One advantage of project finance is that it creates a new business entity in order to carry out an intended investment project. It facilitates the analysis of conditions of project performance and its financial effectiveness; whereas, legal and economic isolation from its owners limits project-related investor risk.*

The paper includes a case study of a selected investment project for a better understanding of the project finance method and the role the feasibility study plays in the assessment of principles of a planned investment.

Keywords: *feasibility study, investment project, project finance.*

JEL Classification: *M21 – Business Economics.*

1. Introduction

Carrying out investment projects in an enterprise is a typical indication of corporate development strategy. In general, one may invest into real resources (tangible goods) or intangibles, or financial assets. Numerous corporate investors, amongst them private entities, undertake to set up project finance companies, i.e. new business entities established to build up a business idea from scratch. Performance of said business undertaking begins with drawing up an investment project as a newly

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created business unit must be equipped with adequate fixed assets (buildings, constructions, machines, devices, equipment etc.). In economic practice, two methods of investment financing and completion have been developed: *project finance* and *corporate finance*[2]. *Corporate finance* involves taking investment decisions by enterprises that are already functioning on the market as the subsequent stage of their development. *Project finance* involves creating a *special purpose vehicle,* the only purpose of which is to manage the undertaken project on behalf of investors in the investment stage, then in the stage of exploitation of its current, operational activity. The *project finance method* is utilised mainly for the purpose of extensive, complex and capital-intensive investments. Typical examples of said investments are industrial plants or infrastructural investments (such as a construction of motorways, energy industry). However, *project finance* may also be employed when carrying out some smaller and relatively simple investments, such as building a hotel or a medium-sized production plant[3].

This study portrays the application of *project finance* to the development of a project finance idea involving the construction and maintenance of a spring water bottling plant. The second key issue depicted herein is the role of the *feasibility study* as a document containing a multifaceted feasibility analysis of an investment project that is to be implemented. One of the aspects of an investment project examined in the *feasibility study* is its financial effectiveness, sources of finance and financial liquidity of a project at the investment and operational activity stages formed as part of facility investment. Additionally, there is also an analysis of the structure of investor contributions and prospective benefits of project implementation from the point of view of said investors.

Particular emphasis was placed on demonstrating the practical applications of *project finance* and the range of *feasibility study* analyses included therein. Therefore, the study employs the *case study* method and presents a case of a new undertaking involving the construction of a spring water bottling plant.

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[2] W. Rogowski, *Rachunek efektywności inwestycji [Investment effectiveness statement],* Wolters Kluwer business, Kraków 2008, pp. 32-37.

[3] K. Brzozowska, *Finansowanie inwestycji infrastrukturalnych przez kapitał prywatny na zasadach project finance [Infrastructural investment finance with private capital on the basis of project finance*], Cedewu, Warszawa 2005, pp. 78-79.

2. Principles of *project finance*

Carrying out lucrative business projects in practice requires adopting suitable organisational and legal forms for the planned project and seeking proper, externally supplied finance, as typically primary investor resources prove insufficient. A convenient method of financing new business projects involving building up a company from scratch is *project finance.* In order to implement a prospective investment project, a new business entity is established, referred to as a *special purpose vehicle,* which usually is largely financed with external investment loans. This way the project is separated from the up-to-date activities of its owners, which reduces liability risk of said investors for their prospective debts, if any such undertaking fails. The *project finance* method is often defined as the method of funding a self-contained business entity (isolated investment), in which a creditor is originally interested in monetary flow and profits generated by said entity as a source of loan and interest leverage and in entity's assets as a source of loan collateral[4].

Project finance is based on predictions regarding future outcomes of a newly created company. Such company is usually separated from an already existing institution or it is made from scratch. Hence, as a consequence there is no possibility to assess current financial standing of the entity. The sole object of analysis is the project itself and the size of risk associated with it. The underlying loan collateral is an asset, which is the outcome of project completion. The key moment for the project is the start-up and the early phase of project performance[5].

Therefore, an investment project is seen from the angle of cash flows related to the project and analyses focus on whether the undertaking maintains financial liquidity throughout its life-cycle. The project life-cycle is a complex and multidimensional process, which begins with expenditure associated with investment organisation and implementation, and ends with proceeds from the sale of a facility being closed down. We ought to examine the matter due to monetary resource movement related therewith. Generally speaking, the cycle consists of the following phases[6]:

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[4] P. K. Nevitt, F. J. Fabozzi, *Project financing,* Euromoney Books, London 2000, p. 1.

[5] K. Czerkas, *Project Finance w polskiej praktyce. Zastosowanie w działalności deweloperskiej [Project finance in Polish practice – property development activities]*, Biblioteka Bankowca, Twigger, Warszawa 2001.

[6] A. Wojewnik-Filipkowska, *Project finanse inwestycjach infrastrukturalnych [Project finance in infrastructural investment]*, CeDeWu, Warszawa 2008, pp. 22-23.

1) Collecting financial resources (capital), i.e. the capital accumulation phase. In the case of new enterprises, the process is always of an external nature.

2) Spending monetary resources on indispensable economic means – the assets.

3) Returning the cash spent based on the sales of goods and services.

4) Reinvestment of retained profits, developing the next turnover of investment resources

Looking at an investment project from the point of view of cash flows is a practical approach because maintenance of financial liquidity, i.e. synchronisation of inflows and outflows at all times during the project life-cycle, along with generation of net cash in the exploitation period, are key for its success and survival.

All in all, we may enumerate the following main features of *project finance*[7]:

- the subject of financing is a legally and economically self-contained investment project in a form of a specially developed project company referred to as a *special purpose vehicle*, whose only business activity is managing said undertaking;
- capital is usually raised for a new investment rather than an already established activity;
- there is usually a high level of debt (loan liabilities) within the structure of the total capital engaged in project funding;
- creditors have no guarantees from primary investors – the so-called *non-recourse finance;*
- creditors' decision to finance an investment project is based on the analysis of future, anticipated profits, as a newly established project company has no history of activity and has no assets.
- the principal source of loan collaterals are assets created in the course of undertaking implementation (licenses, proprietary rights to land, buildings and constructions, machines and equipment);
- project finance debt should be repaid by the end the life-cycle of a given investment project.

Due to above determinants of *project finance*, this structure needs to be thoroughly and comprehensively analysed and assessed at the stage of

[7] E. R. Yescombe, *Principles of project finance,* Academic Press, San Diego 2002, s. 7-8, za: W. Rogowski, *Rachunek efektywności inwestycji,* Wolters Kluwer business, Kraków 2008, s. 33-34.

concept development for an investment project to be implemented. The examination and evaluation of feasibility and cost-effectiveness of a project is conducted in a form of the *feasibility study*.

3. The meaning of the *feasibility study* as a document encompassing an analysis of project idea

Pursuant to the adopted concept, which is in effect in the field of corporate finance management, the main long-term objective of an enterprise is to increase its owners' assets[8]. Establishment of a new company to conduct a business project is recognised as one of the available forms of investing said capital. It is obvious that investors expect that capital invested in the company will generate financial return. A decision to invest capital in setting up a new undertaking should be preceded by drafting of a *feasibility study* regarding such project.

When referring to investment projects, including but not limited to projects involving establishment of a new company for the purpose of a business undertaking, one may talk about the so-called *project-implementation cycle.* This approach is done according to the commonly adopted methodology of the project course- and resources-planning. *The project-implementation cycle* is a period which begins when research and analyses are undertaken, and which lasts until operation activities end. The structure of the *project-implementation cycle* may be presented as follows[9]:

Pre-implementation stage:

I. Research and analysis phase
II. Project scenario and resources planning phase
III. Underlying design phase
IV. Undertaking implementation process design phase

Implementation stage

V. Implementation phase
IV. Operational phase

[8] K. Jajuga, T. Jajuga, *Inwestycje,* Wydawnictwo Naukowe PWN, Warszawa 2008, s. 335.

[9] A. Stabryła, *Zarządzanie projektami ekonomicznymi i organizacyjnymi [Economic and organisational project management],* Wydawnictwo Naukowe PWN, Warszawa 2006, p. 100.

Investment projects that involve establishing a new company from 'zero' are highly complex. Any decisions related thereto must be preceded by a multisided analysis of feasibility of a given project, not only in terms of its financial profitability but also with reference to legal, organisational, technological, or market feasibility, etc. UNIDO provides a comprehensive methodology in the field. UNIDO is a specialist organisation, operating as part of the UN system, which supports industrialisation processes and helps developing countries and states in the period of economic transition[10].

According to UNIDO, the life-cycle of an investment project consists of the following stages:

- pre-investment stage,
- investment stage,
- operational stage.

Table 1 set out a detailed list of the phases of each individual stage of carrying out investment projects.

Table 1.
Investment project cycle according to UNIDO methodology

Investment project	Pre-investment stage	Identification, possibility study
		Preliminary selection, pre-feasibility study
		Ancillary studies
		Feasibility study
		Evaluation and performance report
	Investment stage	Negotiations and concluding agreements
		Technical design
		Formulation (construction)
		Pre-production marketing
		Training
		Acceptance and start-up
	Operational stage	Exploitation
		Extension

Source: W. Behrens, P. M. Hawranek, *Poradnik przygotowania przemysłowych studiów feasibility [Manual for formulation of industrial feasibility studies],* UNIDO, Warszawa 1993, p. 10.

[10] http://www.unido.pl/

The pre-investment stage in particular is well developed in the presented methodology. The feasibility study plays an essential role here as it is a document composed of an analysis of legitimacy and viability of a project. The objective of the *feasibility study* is to conduct a multi-faceted analysis of conditions for carrying out a project; thus, to include the following: market and economic and financial feasibility analysis, within the context of adopted principles, environmental impact and local conditions analysis, and macro-surrounding conditions, i.e. formal and legal, political and social conditions analysis. Such comprehensive analysis leads either to project recommendation or project rejection. Therefore, it serves as the basis for decision-makers to implement a project.

UNIDO methodology is adequate for large industrial projects where companies are often built up from scratch. However, Polish economic reality typically demonstrates entities from the small and medium enterprises sector which carry out projects on a much smaller scale. Nevertheless, the general UNIDO methodology, in particular that of financial analysis involving discount methods of investment project evaluation, is also suitable for typical investment projects carried out in the Polish reality by small and medium-sized enterprises[11]. Said methodology is highly suitable for analysing investment projects intended to be implemented in an enterprise. This issue will be developed further on in the study in the form of a case study of a spring water bottling plant.

Principles and objectives of the construction of a spring water bottling plant project – a case study[12]

Project background

The feasibility study has been prepared on initiative of potential investors' group interested in the undertaking that consists in spring water plant start-up. The company's activity will be based on existing and

[11] T. Szot-Gabryś, *Projekty inwestycyjne infrastrukturalne i biznesowe. Aspekty teoretyczne i praktyczne [Investment, infrastructural and business projects. The methodological and practical aspects]*, Difin, Warsaw 2011, pp. 47-48.

[12] This case study was developed on the basis of: T. Szot-Gabryś, project dossier: *Feasibility study for spring water bottling plant,* unpublished materials. Due to limited space of this chapter the presented case study omits an analysis of macro-environment determinants and a marketing analysis and strategy, which are included in the feasibility study of the presented project. What is more, the financial analyse presented has also been limited.

documented water intake, located in K. locality, F. commune in Poland. The intake is on a plot owned by Ms. T.D. The spring water present in that location has undergone numerous expertises and analyses that proved its beneficial physical chemistry characteristics and high quality of the water. Assumptions made for this feasibility study foresee the operations of spring water bottling enterprise in the form of Limited Liability Company.

The feasibility study addresses the economic, financial and marketing assessment of undertaking that consists in setting and operating activity of mineral water bottling plant. The enterprise will operate in the legal form of Limited Liability Company.

Preparing the feasibility study following studies and analyses were used:

- „Hydrogeology documentation determining exploitation resources and the project of protected area of subterranean water intake localized on the plot owned by Ms. T. D., in K. locality, F. commune, Lubelskie Voivodeship, Poland";
- „Hydrogeology opinion concerning the presence possibility of terminal waters and waters suitable for bottling as spring water in K. locality";
- „Assessment of water holding capacity in K. locality, F. commune";
- Report on water tests;

The aforesaid studies and expertises unequivocally prove that:

- water present in the intake located on Ms. T. D.'s plot is characterised by high quality parameters in respect of physics chemical characteristics;
- water meets bacteriological standards of spring waters;
- resources of aquifer are large and present high ability to naturally renew, what in a prospect of long-time exploitation guarantees that no shortage in raw material for the plant will occur;

The confirmation of water high quality constituted a preliminary condition to undertake further studies concerning profitability of exploitation, bottling and distribution.

The feasibility study is aimed to analyse the economic and financial feasibility of investment undertaking represented by construction and

operation of spring water plant. 2 stages of enterprise operations have been planned:

- investment phase – building-up the enterprise;
- phase of operating activity.

Carried out analyses are aimed to indicate conditions for success of planned undertaking, on the assumptions made in relation to the value of capital employed by investors and indication of its profitability from the potential investor's point of view.

4.1. Phase of enterprise founding and organization

The planned investment undertaking consists in construction and set-up of spring water plant's activity. In this connection the projection of the enterprise has been divided into two stages:

- investment phase – enterprise build-up – means carrying out organising activities and collecting fixed assets that constitute basis for company operations;
- phase of operating activity carried by the enterprise.

The investment phase consists of following activities:

I. Registration of the Company

From the moment of registration the Company will become a legal person that will start the process of enterprise organisation and collecting its tangible assets. Share capital of the Company contributed by the owners in cash by shares subscription will amount to PLN 100 000.

Table 2.
The structure of share capital

Shareholders	Value of shares in PLN	Percentage in total capital value
Ms T.D.	50 000	50.0%
H.N.V. B.V.	48 000	48.0%
A.J. H.	2 000	2.0%
Total	100 000	100%

Source: T. Szot-Gabryś, project dossier: *Feasibility study for spring water bottling plant,* unpublished materials.

Additionally, the shareholders will grant the Company following loans:

- H.N.V. B.V. – PLN 552 000, 9% annual interest rate;
- Ms. T.D. – PLN 500 000, 9% annual interest rate.

Totally, indicated financing means will allow collecting assets amounting to PLN 1 152 000, of which PLN 100 000 of share capital will be recognized in balance-sheet equity capital and liabilities as Equity Capital and total amount of loans, i.e. PLN 1 052 000 as Long-term Liabilities.

II. Purchase of land with spring water intake

The first step of investment process is purchase by the Company from Ms. T.D. of land where spring water intake is localized, against amount of PLN 600 000. Before starting the activity on the basis of the purchased land, the destined use of land needs to be reclassified in local development plan from the destination for agriculture use to the business activity. To avoid tax on that transaction, it is important that the Company buys the land classified as agricultural land.

III. Construction design and permits:

- Order to execute the construction design of bottling plant and its infrastructure;
- Obtaining a construction permit;
- Purchase of aquatic legal survey;
- Water qualification by the Spa Institute in Poznań.

Issuance of opinion on water by the Spa Institute in Poznań constitutes a condition to accept the water for production and distribution.

IV. Construction of bottling plant's building (manufacturing shop, office and social rooms).

V. Purchase of machines and devices, equipment, means of transport

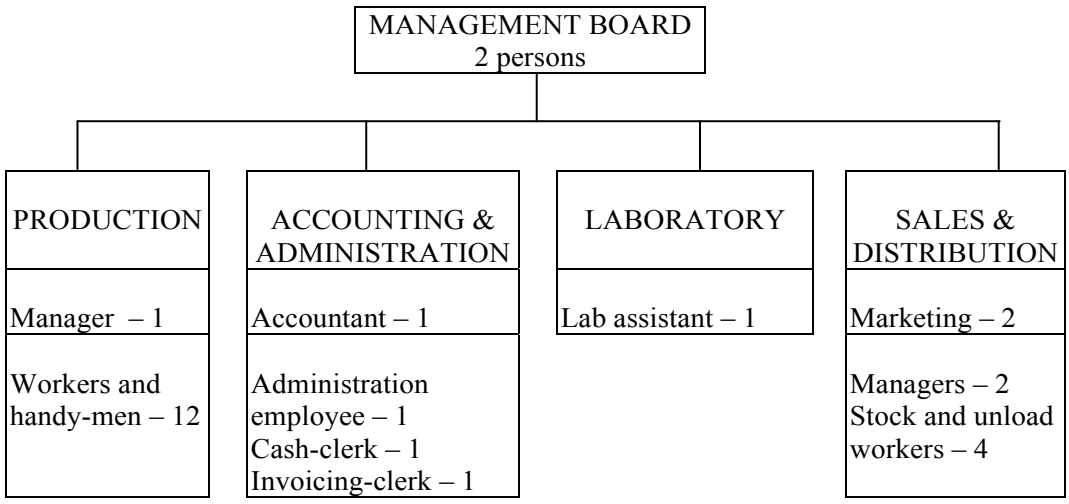
The key element of this phase is the acquisition of process line for bottled water's manufacturing. It is assumed that the cost of purchasing the process line will amount to PLN 300 000, line capacity 3 000 litres per hour, what means 2 000 pieces of 1.5 litre bottles per hour.

VI. Enterprise organisation

Simultaneously to the investment process, the works connected with enterprise organisation in order to prepare the Company to conduct its operating activity should be carried out. They include employment planning. 4-5 persons per one shift should operate the process line of spring water production.

Table 3.

Company organization structure



Source: T. Szot-Gabryś, project dossier: *Feasibility study for spring water bottling plant,* unpublished materials.

Employment of 28 persons is the minimum employment at planned activity scale of spring water bottling plant.

4.1. Investment costs

Carrying out the operating activity requires the use of certain tangible assets. The shareholders, in the project assumptions have foreseen the collected capital value in amount of PLN 1 152 000.

Table 4.

Investment needs

Assets component	Value in PLN	Notes
Purchase of land on which the water intake is localized	600 000	The land will by bought by the Company from Ms. T.D.
Purchase of process line for bottled water manufacturing	300 000	The purchase of line with 2 000 pieces of 1.5 litre bottles per hour capacity is planned. On the assumption of two shifts work, i.e. 15 h and 25 working days a month, manufacturing of 9 000 000 pieces of bottles a year will be possible.
Production shop with stock room, social facilities, offices and infrastructure (including design documentation).	250 000	Building's parameters should be defined in the way ensuring that indicated functions can be fulfilled. Necessary stock rooms need to be estimated so that in the winter season, when the demand for portable water decreases, inventories could be accumulated in the plant. To have available specified number of products in the high season, it is planned to maintain steady production level in course of the whole year. Such policy is recommended in connection with low capacity of the bottling line.
Forklift	80 000	Forklift is necessary to work in the store and to reload products in the pant and to load pallets on lorries.
Truck with trailer	150 000 60 000	Industry specificity is products' transportation to receivers by own means of transport. Hence means of transport need to be bought or transportation services need to be contracted. Transportation services costs amount to net PLN 1,35 per 1 km.
Equipment	30 000	The item includes purchasing of office furniture and equipment, etc.
Total	1 470 000	

Source: T. Szot-Gabryś, project dossier: *Feasibility study for spring water bottling plant,* unpublished materials.

From estimations made it results that in the relation to the capital defined by the investors for organising of the enterprise in amount of PLN 1 152 000, the minimum investment needs are still higher of at least PLN 318 000. Moreover, the Company should also have available additional capital securing correct level of operating capital and covering

expenditures to be incurred due to VAT tax on investment purchases. These needs are estimated in the following amounts:

- PLN 120 000 to finance the VAT tax;
- PLN 30 000 to finance purchase of enterprise equipment;
- PLN 1 200 000 to finance operating activity of the enterprise.

The company is starting its operating activity in February and initially will manufacture products designated for stocking in order to secure correct number of products to be sold in the high season.

From performed analysis it results that the capital in the amount of PLN 1 152 000 proposed by the investors is not sufficient to collect assets necessary for enterprise operations.

One of solutions is to use machines or transportation in leasing. It is because the Company has limited possibility to get investment credit in its initial phase of operations. There is a greater chance to get operating credit that would allow correct securing of operating capital. Leasing represents the financing form that is more available than a credit, especially to companies in their initial phase of operations. But the costs of leasing burden the financial result. It should be mentioned that the Company in connection with loans granted by its shareholders in amount of PLN 1 052 000 will be charged with monthly costs totalling PLN **7** 890 and from credit capacity viewpoint will have unfavourable capital structure.

Table 5.

Capital structure

Equity capital and liabilities component	Value in PLN	% in total
Company share capital	100 000	8.68
Loans from shareholders	1 052 000	91.32
Total	1 152 000	100

Source: T. Szot-Gabryś, project dossier: *Feasibility study for spring water bottling plant,* unpublished materials.

4.2. Operating activity

The operating activity planning concerns forecast of revenues, costs, inventory, receivables and liabilities management. Amount of operating expenses results from applied production technology.

Process technology of spring water bottling plant consists of several steps:

- water uptake from the intake;
- filtering, water tests;

- carbonation;
- bottles preparation and filling-up;
- labelling;
- distribution;

The feasibility study focuses on spring water manufacturing profitability and its distribution analysis. Thus the projection of revenues and costs must be carried out. One 1.5 litre bottle of water has been applied as a unit cost.

Table 6.

Direct materials costs

No.	Material type	Net costs in PLN
1.	Packaging – PET 1,5 l bottle	0.3669
2.	Ring cap	0.1085
3.	Bottle neck label	0.0086
4.	Central label	0.0180
5.	Adhesive	0.0001
6.	Heat-shrinkable film	0.0023
7.	Water – output fee	0.0003
	Total	**0.5047**

Source: T. Szot-Gabryś, project dossier: *Feasibility study for spring water bottling plant,* unpublished materials.

4.3. Financial analysis conclusions

1. From the conducted analysis it results that the capital proposed by investors in the amount of PLN 1 152 000 is insufficient to collect assets potential necessary for enterprise operations.

2. The minimum level of operating credits on operating activity is following:

- PLN 120 000 to finance VAT tax;
- PLN 30 000 to finance purchase of enterprise equipment;
- PLN 1 200 000 to finance operating activity of the enterprise;

It should be also taken into account that due to its disadvantageous capital structure (share capital and loans granted to the company by its owners that are included in liabilities) and no credit history, the Company can have difficulties to be granted the operating credit.

3. From the finance projection it results that the enterprise can have temporary finance liquidity problems, especially in dead season months. Then other operating credits will have to be incurred or another form of current activity financing will have to be sought.

4. At the production scale resulting from manufacturing capacity, the company will not be able to carry out the production strategy based on returns to scale. If the product is placed successfully on the market (i.e. on the assumption that the whole production will be sold), the undertaking will be profitable.

5. From the economic finance analysis carried out in the paper it results that:

- planned investment undertaking is profitable, provided that indicated assumptions as to enterprise financing are met and the company places the product on the market;
- IRR and NPV ratios are outstanding;
- return on investments calculated with application of simple rate of return method will be realised in the fifth year of investment's operations;
- the Company is able to earn profits to handle liabilities and to pay out dividends to shareholders (after 5 years);

The financial analysis proves that the investment project, on made assumptions as to revenues and costs, is profitable and maintains financial liquidity, i.e. shows capacity to pay back the credits incurred for its performance. Thus the investment planned by the Company is justified from market and economic financial point of view.

5. Conclusions

The task of the *feasibility study* is to provide a comprehensive analysis of an investment project, including analysis seen from the angle of its financial effectiveness and potential barriers and obstacles and risk of failure. One advantage of the *feasibility study* is that it is prepared within the idea stage of a project, before some concrete steps are undertaken in reality. Hence, the *feasibility study* is an analysis on the basis of which owners make decisions whether to implement or abandon a given project. It is a document used by owners and investors, who want to invest their capital in the analysed project, for the purpose of decision-making. On the other hand, analyses included in the *feasibility study* may be employed in the process of acquiring external funds.

The *feasibility study* plays a key role whenever it investigates investment projects that are completely new and that will be built up from scratch, which is quite typical for *project finance*. In such situation there are no historical data which would serve as the grounds for planning future outcomes. Implementing *project finance* makes it possible to distinguish the legal and economic aspects of a given undertaking from its current activities. The above facilitates economic and financial evaluation and reduces investor financial risk, as it concerns capital entrusted to finance that project alone.

REFERENCES

[1] Brzozowska K. (2005), *Finansowanie inwestycji infrastrukturalnych przez kapitał prywatny na zasadach project finance [Infrastructural investment finance with private capital on the basis of project finance*], Cedewu, Warszawa.

[2] Czerkas K. (2001), *Project Finance w polskiej praktyce. Zastosowanie w działalności deweloperskiej [Project finance in Polish practice – property development activities]*, Biblioteka Bankowca, Twigger, Warszawa.

[3] Jajuga K., Jajuga T. (2008), *Inwestycje,* Wydawnictwo Naukowe PWN, Warszawa.

[4] Nevitt P. K., Fabozzi F. J. (2000), *Project financing,* Euromoney Books, London.

[5] Stabryła A. (2006), *Zarządzanie projektami ekonomicznymi i organizacyjnymi [Economic and organisational project management],* Wydawnictwo Naukowe PWN, Warszawa.

[6] Szot-Gabryś T. (2011), *Projekty inwestycyjne infrastrukturalne i biznesowe. Aspekty teoretyczne i praktyczne, [Investment, infrastructural and business projects. The methodological and practical aspects]*, Difin, Warsaw.

[7] Szot-Gabryś T., project dossier: *Feasibility study for spring water bottling plant,* unpublished materials.

[8] Rogowski W. (2008), *Rachunek efektywności inwestycji [Investment effectiveness statement],* Wolters Kluwer business, Kraków.

[9] Wojewnik-Filipkowska A. (2008), *Project finanse inwestycjach infrastruktu-ralnych [Project finance in infrastructural investment]*, CeDeWu, Warszawa.

[10] Yescombe E. R., *Principles of project finance,* Academic Press, San Diego, 2002, s. 7-8, za: W. Rogowski, *Rachunek efektywności inwestycji, [The bill of the investment efficiency],* Wolters Kluwer business, Kraków, 2008.

[11] http://www.unido.pl/

INTEGRATING FLEXIBLE BEHAVIORS INTO THE ECONOMIC TRANSMISSION MECHANISMS[1]

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Abstract. *The paper suggests a method for the analysis of the economic transmission mechanisms based on directional graphs.*
The relations between the economic variables are defined based on a vector of conditional probabilities which reflect a mix of specific economic behaviours. At the same time, the conditional probabilities are defined for complex events which reflect economic behaviours characterized by: a) different triggering factors; b) different frequencies and patterns.
This approach facilitate the integration of a variety of economic behaviours in one economic transmission mechanism make it a more flexible and realistic instrument for the economic analysis.

Keywords: *economic transmission mechanisms, conditional probabilities, flexible behaviours, functionalities, complex, systems.*

Jel classification: C15, C63.

1. Introduction

The economy is a dynamic system with specific trajectories for the variables of interest. This rises a challenge to define the probability for elementary events associated with a given states of one or more variables. Taking the classical example of coin tossing as a metaphor for the economic system, each tossing of a coin, in the case of the economic system each economic event doesn't have the same initial conditions. This is equivalent with saying that we cannot define a set of elementary events having the same set of conditions. This suggests that the sequence of relative frequencies may not converge, thus the limit of the relative frequency may not exist.

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Having this is mind the paper has three objectives.

The first objective is to analyze the probability of complex events associated with the phase space of the economic dynamic system.

The second one is the analysis of the stability of the relative frequencies of elementary events.

The third one is the characterization of the economic transmission mechanisms using conditional probability.

The paper is structures in six chapters.

The second chapter focuses on the literature review of the main applications of the dynamic systems in economics as well as on describing the place of the paper in relation to these contributions.

In the third chapter the paper presents the impact of structural factors on the economic variables of interest. The focus is on inflation due to the characteristics of the analyzed structural factors, namely the downward rigidity of wages which is a nominal variable strongly connected with inflation.

The fourth chapter presents the model focusing on the blocks of the model and on the specific system of differential equations.

The fifth chapter presents the results of the model's simulation focusing on the three objectives enumerated above.

The sixth chapter is a synthesis of the main findings of the paper.

2. Systems of differential equations – applications in economics

The analysis of the economic processes using systems of differential equations underlines the complex behaviour of the economic system: bifurcations, critical points, possible chaotic behaviours. The main economic problems analyzed using the dynamic framework are diverse indicating the versatility of the approach: economic cycles (see Goodwin 1967, Desai 1973, Wolfstetter 1982, Sportelli 1995, Purica and Caraiani 2009), the dynamic theory of oligopoly (see Yoshida 2011), demographic problems and its application in economics (see Lucas 2002, Galor and Moav 2002, Hansen and Prescott 2002, Jones 2001), economic growth (see Kaldor 1956, Pasinetti 1962, Samuelson and Modigliani 1966 as well as unified economic growth theory developed by Galor 2011), the problem of stability of the economic equilibrium (see Woodford 1990, Bewley 1998, Bohm and Kaas 2000).

From the perspective of the variables used in the model the differences in comparison with the cited approaches are not significant.

These approaches are based on the fundamental relations in economics between demand, offer, inflation, wages etc. The current paper introduces no innovative new relations. The main differences are related with the hypothesis of a unique market. The large majority of the models follow the tradition set by the dynamic stochastic general equilibrium models (DSGE) working with a single market, thus not taking into account the relation between economic sectors and the impact of these relations. Due to the nature of the problem studied in this paper, namely the impact of structural factors, the model builds a market with two sectors with all the implications that derive from this approach. The main implications are the capacity to capture the relation between sectors and the effect of the modifications of one of the sectors on the others.

From the perspective of the behaviours of the solutions there are some differences in „the logic" of the approaches which should be underlined. We are going to focus on those approaches which are relevant to the current analysis.

The approach which is steadily gaining ground is based on the use of non-linear systems of differential equations in describing and analyzing economic processes (see Rand 1978, Day 1982, Puu 2000, Nishimura and Yano 1995, Vosvrda 2001). Admittedly there is a richness of behaviours of these systems but also artificiality in defining the non-linear behaviours of the economic variables. One approach is to build trigonometric functions to reflect the relations between variables (see Hommes 1997) or to use equations known for their non-linear behaviours as in the case of the Faigenbaum equation (see Jensen and Urban 1984) or Van der Pol equation (see Bouli 1999). The use of these equations is made without clearly presenting their capacity to reflect the specific economic mechanisms. In this context it is not clear if the economic variables exhibit a chaotic behaviour or if there is a selection of those equations which allow the chaotic behaviour to manifest itself. In these cases the analysis of the chaotic behaviours is made in the logic of Thom 1989 and Zeeman 1977 (interestingly these papers are not analyzing economic processes), namely analyzing the behaviour of the systems for different intervals of the coefficients. In this context, the perturbations on the system are described by the modifications in these coefficients. These modifications affect the stability and the behaviour of the system's solutions only if they generate a change of the above mentioned interval of variation (see Goodwin 1990 for a presentation of the problem). The current paper investigate the processes which are behind these approaches, namely it focuses on understanding the

connection between the trajectories of the system's solutions and the characteristics of the economic transmission mechanisms specific to the analyzed system. The problem is presented in chapter 5.

3. The impact of structural factors on the economic indicators

The different ways to quantify and theoretically define structural changes led to different approaches to analyze the structural component of the economic variable of interest. Dobrescu (2009) analyzes structural inflation from the perspective of the relation between the modifications of the weights of different sectors in the total production and the changes in the prices calculated as a ratio between the sectorial price index and the total aggregated price index. Balke and Wynne (1996) showed that the sectorial technological changes are reflected in the transversal distribution of price changes. Sheedy (2005) analyzed the impact of a shock (changes in the oil price) on the firm's costs which led to price adjustments with different lags. In this approach the shocks that affect the economy are structural because they reflect the structure of the firms in the economy and the differences in their behaviours.

The paper follows the logic of Sheedy (2005) in the sense that the changes in the key variables due to structural shocks are a result of the structure of the economy and its characteristics. The idea that we want to underline in this paper is that the economic system through its structure (the relation between the components of the system, the feed-back structure, behaviour characteristics) facilitates the understanding of the relations between the trajectories of the prices and the characteristic of the economic transmission mechanisms and their relations.

4. The model's presentation

The chapter will focus on presenting the system of differential equations and the algorithm for running the model. The mathematical relations will be presented for a model with n components (consumers, firms). This version of the model uses an accommodative monetary policy. In the context of a constant money velocity, the monetary mass varies to equate the product of the quantity of goods in the economy and the prices of these goods.

4.1. The system of differential equations

The system of differential equations models the variation of prices, wages, demand and offer in the context of the interaction between producers and consumers on two markets. The symbols used stand for p^{ri} – the price of the economic good, for the sector i, c^{ri} – demand for the good produced by the sector i, o^{ri} – offer for the good produced by the sector i, w^{ri} – nominal wage for the sector i, e^p – expected inflation, i^s – the wage index, a^s – wage adjustment index, r^s – downward rigidity of wages index, a_{prod} – production adjustment index, $profit^{ri}$ – the profit of the firm in sector i calculated as the difference between income and costs. The relations that model the evolution of prices, wages, demand and offer are described below:

$$p_t^{r_i} = p_{t-1}^{r_i} * \frac{C_{t-1}^{r_i}}{O_{t-1}^{r_i}} * e^p; \tag{1}$$

$$w_t^{r_i} = w_{t-1}^{r_i} * \frac{p_t^{r_i}}{p_{t-1}^{r_i}} = w_{t-1}^{r_i} * \frac{C_{t-2}^{r_i}}{O_{t-2}^{r_i}} * e^p * i^s; \tag{2}$$

$$i^s = \begin{cases} a^s & \frac{C_{t-1}^{r_i}}{O_{t-1}^{r_i}} > 1 \\ r^s & \frac{C_{t-1}^{r_i}}{O_{t-1}^{r_i}} < 1 \end{cases}; \tag{3}$$

$$O_t^{r_i} = O^{r_i}{}_{t-1} * a_{prod} \quad ; \tag{5}$$

$$a_{prod} = \begin{cases} a_{prod}^1 & \text{Profit}^{r_i} > 0 \\ a_{prod}^2 & \text{Profit}^{r_i} < 0 \end{cases}; \tag{6}$$

$$C_t^{r_i} = \frac{w_t^{r_i}}{p_t^{r_i}} = \frac{w_1^{r_i}}{p_1^{r_i}} * \frac{C_0^{r_i}}{O_0^{r_i}} * \frac{O_{t-1}^{r_i}}{C_{t-1}^{r_i}} \ . \tag{7}$$

The system of differential equations reflects the following economic behaviour mechanisms.

The variation in prices is proportional with the ratio between demand and offer and depends on the inflation expectations. The prices are increasing if the demand is higher than the offer, remain unchanged in the case of equality and decrease otherwise (see relation 1). The variation in wages depends on the variation of prices. An increase in prices generates pressures in the direction of an increase in wages. The wage adjustment is captured by the wage index which can be an index higher than unity (the wages increase more than the prices) equal to unity (the wages increase is equal to the price increase) and lower then unity (the wage increase is lower than the price increase). By algebraic manipulation of the relation (9), the wages can be written as a function of demand and offer with lag two (t–2), the inflation expectations and the wage index. The output variation depends on the differences between output and demand and on the production adjustment capacity (which can be seen as the elasticity of output to demand changes). This capacity is endogenously defined and it depends on the specificities of the production process and on the characteristics of the market (see relation 4). There are two possible values of the adjustment indicator, which corresponds to the case of positive profits or negative ones (see relation 5). The demand at the moment t depends on the relation between income and prices. By algebraic manipulation of relation 13, the demand can be written as a function of the demand and offer with one lag ($t - 1$), of the demand and offer at time t_0 and of the ratio of income and price and time t_1 (see relation 6).

4.2. The algorithm of the model

The model has the following theoretical framework. The market is described by representative producers from two sectors. The producers use row materials (intermediate consumption) and labour force to produce the output. The relations between the sectors are reflected by the technological coefficients. The consumers income depend on the number of hours worked and the hourly cost of the labour force in the sector of work. The demand for the goods in the economy reflects the consumers' choices taken into account their disposable income and preferences. The algorithm is based on the following steps:

Step 1. At the beginning of the simulation there are defined: 1) a set of prices for the goods produced; 2) a set of prices for the labour force; 3) the technological coefficients matrix; 4) the consumer preferences. The above mentioned variables are randomly defined but they respect a set of constraint that confers economic consistency. In this respect the

technological matrix reflects the ratio between the weight of the intermediate consumption and the labour force in the production process. The set of prices is chosen as not to be smaller than the marginal costs.

Step 2. The output at the sectoral level is the solution to the problem of profit maximization. The demand is the solution to the problem of utility maximization.

Step 3. Prices increase if $C > O$ and decrease if $C < O$. The changes are proportional with the difference between demand (C) and offer (O) (see relation 8). The process illustrates the price adjustment mechanism of the output to the demand.

The wages vary proportionally to the changes in prices taken into account the difference between the demand and offer (see relation 2). A supplementary constraint is added, namely the wages cannot decrease below the minimum wage in the economy.

If the profit is negative after the price adjustment then the firms adjust their output to match demand. The adjustment is made with a lag reflecting the production characteristics of the firm, market strategies, the characteristics of the labour market etc.

Step 4. Steps 2 and 3 are repeated for a number of periods (45 in the case of the simulations in the paper).

5. Running the model

The simulation investigates the limits of describing the evolution of a dynamic economic system using probabilities. To make the arguments more transparent some clarifications are in order.

There are different types of events, specific to the economic system, for which we want to know their probability. Three types of events are of a particular interest.

First of all, we want to know what is the probability that at time t_i we have event A = {an increase in demand is accompanied by an increase in offer}. If the system is deterministic, as in the case of the model, the answer is 1. If the dynamic system can have more than one initial conditions then $P(A) = \lim_{n \to \infty} \frac{n(A)}{n}$ and the values is a direct result of the probability of occurrence of the initial conditions and of the trajectories of the system specific to those initial conditions.

Second of all given a deterministic system and the trajectories of the key variables we want know what is the probability that event A will occur

at a random chosen time period given that no relevant random events will take place. The probability will be then defined by the relative frequency of the event A given the deterministic evolution of the system.

Third of all we want to know how the probability of the event *A* changes if one of the variables of the system changes it value in an interval ε.

Having this in mind, we propose the following procedure.

Step 1. We run the model described by equations 1 to 6 for 45 periods obtaining the economic variables of interest for the two sectors.

Step 2. We apply the Granger causality test between the series demand and offer, prices and offer, prices and demand for the two sectors.

Step 3. Based on the results of the Granger causality we define the events: $A_1 = \{d(O_t) < 0$ given that $d(C_{t-i}) < 0\}$, $A_2 = \{d(P_t) > 0$ given that $d(O_{t-i}) < 0\}$ and $A_3 = \{d(P_t) < 0$ given that $d(C_{t-i}) < 0\}$. We calculate the probability that event A_i will occur at a random chosen time period given that no relevant random events will take place. As mentioned before, the probability will be defined by the relative frequency of the events A_i given the deterministic evolution of the system.

Step 4. We run the model 100 times. At each run the structural variables downward rigidity of wages r^s is perturbed by a random term, where $r^s \in [1.01\ 1.01 + \varepsilon]$ and ε is the random term $\varepsilon \in [0\ 0.01]$.

Step 5. The output of the model give us 4 key series of length 100, namely r^s and $P(A_i)$ where $I = 1$ to 3.

Step 6. We defined the functions $f_i : S_{r^s} \rightarrow S_{P(A_i)}$ where S_{r^s} is the set of all values obtained for the 100 runs of the model of r^s and $S_{P(A_i)}$ is the set of all values obtained for the 100 runs of the model for $P(A_i)$. The functions are then analyzed to see if they were monotonic.

Step 7. An interval of variation is defined for $P(A_i)$, namely $P(A_i) \in [\min(f(r^s))\ \max(f(r^s))]$.

The application of the procedure generated the following results (see Table 1). The relevant statistics can be found in the appendix of the paper.

The probability $P(A_i)$ describes the behaviour and stability of the economic transmission mechanism regarding demand and offer, offer and prices, demand and prices. The most stable mechanism to the fluctuations in the downward rigidity of wages is the one described by event A_1 between demand and offer and the least stable the one described by A_3 between demand and prices.

Table 1

Synthesis of the simulation's results

Relevant indicators	Interval of variation for the indicators – sector 1	Interval of variation for the indicators – sector 2
r^s	(1.010 1.019)	(1.010 1.019)
$P(A_1)$	(0.962 0.969)	(0.942 0.88)
$P(A_2)$	(0.52 0.83)	(0.5 0.77)
$P(A_3)$	(0.32 0.54)	(0.5 0.77)

Source: Own computations

The function f_1 is weakly decreasing for both sectors, thus it preserve the order. The functions f_2 and f_3 are not monotonically. This indicates that we are in a case of demand driven economy. The decrease in prices is not accompanied by a decrease in wages of the same magnitude. This suggests that there are situations in which an increase in demand is not accompanied by an increase in prices and an increase in offer is not accompanied by a decrease in prices. Both mechanisms are possible due to the different lags between them.

A way of looking at the results in Table 1 is to look at the economic transmission mechanism as functionalities of the economic system and to ask in what conditions (perturbation of r^s) are the functionalities preserved. To make the argument more clearly we are going to analyze a more complex transmission mechanism, namely a decrease in demand generates a decrease in offer which generates an increase in price. This mechanism can be seen as a directional graph with the nodes demand, offer and prices. The event specific to this mechanism is A_4 = {a decrease in demand generates an increase in price given that a decrease in demand generates a decrease in offer}.

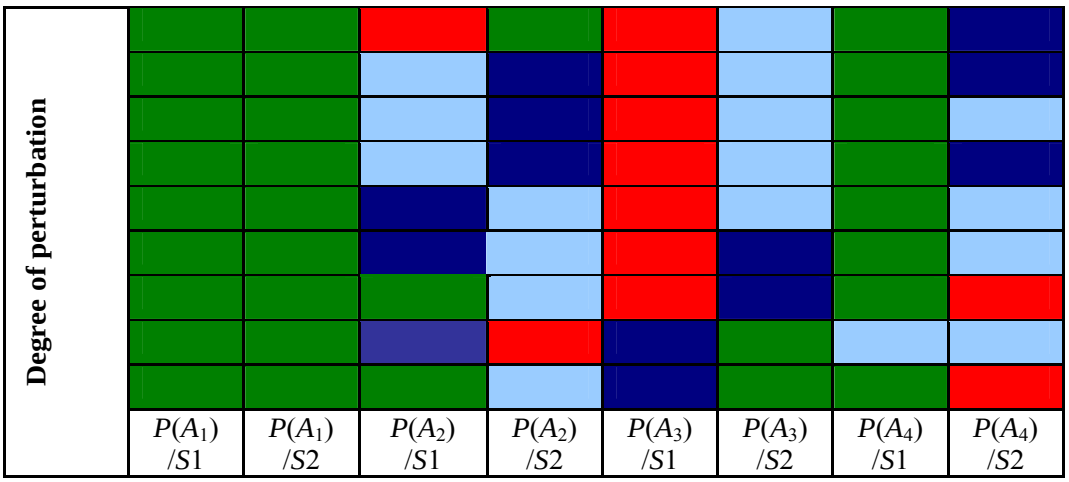
The simulation of the model for 100 runs indicates the following results for the mechanism presented above $P(A_4)$ is constant for sector 1, its value being 0.5 and $P(A_4)$ varies in the interval (0.2 0.33) for the second sector.

The robustness of functionalities to perturbation in r^s is analyzed by looking at the correspondence between the increase in the value of r^s in comparison with the starting value of 1.01 and the change in the probability of the events A_i. The approach is similar with the one proposed by Kitano (2007). As a starting point we defined intervals for the probabilities $P(A_i)$. The intervals reflect if and in what degree the functions of the system are kept. The functions correspond to the events A_i. Red colour indicates that the system fails to have/maintain its function, different

blue colours show the level of degradation of the function and the green colour indicates the system maintains the functionality, white colour indicates that the indicator of robustness could not be calculated. A system is considered more robust if it better preserves it functionalities.

Table 2

The robustness of the functionalities to perturbations in r^s



Legend: $f_i(r^s) \geq 0.75$, $f_i(r^s) \geq 0.65$, $f_i(r^s) \geq 0.5$, $f_i(r^s) < 0.5$

Source: Own computations

The analysis of Table 2 suggests an interesting question: "Are the complex functionalities more robust then simple ones?". In the case of our analysis the complexity of functionality is easy to be defined, and it corresponds to events which are the combination of other events. In this respect, $A_4 = A_2 \setminus A_1$ where "\" should be read as "given". In the case of the analysis the robustness of A_4 is a direct consequence of the robustness of A_2 and A_1 the last having the lowest performance of those two. In this respect the increase in complexity leads to a decrease in robustness.

6. Conclusions

The paper theoretically analyzed the probability of complex events associated with the phase space of an economic dynamic system. It proposed a procedure for the analysis of the stability of the relative frequencies of elementary events and the characterization of the economic

transmission mechanisms using conditional probability. The procedure allows for the definition of the interval of variation for the probability of different economic transmission mechanisms.

In the context of the analysis of the probability associated with complex transmission mechanisms the paper discussed the robustness of different economic transmission mechanisms to perturbations in r^s and finds that complex functionalities are less stable in comparison with simpler functionalities.

REFERENCES

[1] Böhm, V. & Kaas L. (2000), *Differential savings, factor shares, and endogenous growth cycles,* Journal of Economic Dynamics & Control, **24,** 965-980.

[2] Bouli, S. (1999), *Feedback Loop in Extended Van der Pol's equation Applied to an Economic Model of Cycles*, International Journal of Bifurcation and Chaos, **9**(4), 745-756.

[3] Colceag, F. (2011), *Fractal completeness philosophy of the alive universe, Lattice Automata*, available at http://www.sustainability-modeling.eu/media/Fractal_ completeness_philosophy _of_the_alive_universe.pdf, [Accessed on 04.11.2011].

[4] Day, R. (1982), *Irregular Growth Cycles*, American Economic Review, **72,** 406-414.

[5] Desai, M. (1973), *Growth cycles and inflation in a model of the class struggle*, Journal of Economic Theory, **6**(6), 527-545.

[6] Galor, O. (2011), *Unified Growth Theory*, Princeton University Press.

[7] Galor, O. & Moav, O. (2002), *Natural selection and the origin of economic growth*, Quarterly Journal of Economics **117,** 1133-1192.

[8] Goodwin R. M. (1967), *A Growth Cycle, in Socialism, Capitalism and Economic Growth*. Cambridge: Cambridge University Press.

[9] Hansen, G., Prescott, E. (2002), *Malthus to Solow,* American Economic Review **92,** 1205-1217.

[10] Hommes, C. H. (1991), *Chaotic Dynamics in Economic Models: Some Simple Case-Studies*, Groningen:Wolters-Noordhoff.

[11] Jensen, R. V. & Robin Urban (1984), *Chaotic price behaviour in a non-linear cobweb model,* Economics Letters **15,** 235-240.

[12] Jones, C. I. (2001), *Was an Industrial Revolution inevitable? Economic growth over the very long run*, Advances in Macroeconomics, **1,** 1-43.

[13] Kaldor, N. (1956), *Alternative theories of distribution*, Review of Economic Studies **23,** 83-100.

[14] Kitano, H. (2007), *Towards a theory of biological robustness,* Molecular Systems Biology 3:137.

[15] Lucas, R. E. (2002), *The Industrial Revolution: Past and Future,* Harvard University Press, Cambridge, MA.

[16] Nishimura, K. & Yano M. (1995), Nonlinear Dynamics and Optimal Growth: An Example, *Econometrica*, **63,** 981-001.

[17] Pasinetti, L. L. (1962), *Rate of profit and income distribution in relation to the rate of economic growth*, Review of Economic Studies, **29,** 267-279.

[18] Purica, I. & Caraiani, P. (2009), Second Order Dynamics of Economic Cycles, *Journal for Economic Forecasting*, **6**(1), 36-47.
[19] Puu, T. (2000), *Attractors, Bifurcations, and Chaos*, Springer-Verlag.
[20] Rand, D. (1978), *Exotic phenomena in games and duopoly models,* Journal of Mathematical Economics, 173-184.
[21] Samuelson P. A. & Mondigliani F. (1966), *The Pasinetti paradox in neoclassical and more general models*. Review of Economic Studies, **33,** 269-301.
[22] Sims, C. A. (1980), *Macroeconomics and Reality,* Econometrica, **48,** 1-48.
[23] Sportelli M. (1995), A Kolmogoroff generalized predator-prey model of Goodwin's growth cycle, *Journal of Economics,* **61**(1), 35-64.
[24] Stănescu D. & Chen-Charpentier B. (2009), *Random coefficient differential equation models for monod kinetics,* Discrete and continuous dynamical systems, Supplement, 2009, 719-728.
[25] Thom, R. (1989), *Structural Stability and Morphogenesis*. Redwood City, California, AddisonWesley Publishing Company.
[26] Vadasz, V. (2007), *Economic motion: An economic application of the Lotka – Volterra Predator-Prey model,* available at http://dspace.nitle.org/bitstream/-handle/ 10090/4287/ Vadasz.pdf?sequence=1, [Accesed at 28.10.2011]
[27] Vosvrda, M. S. (2001), Bifurcation Routes and Economic Stability, *Computing in Economics and Finance* nr. 132 .
[28] Wolfstetter, E. (1982), *Fiscal Policy and the Classical Growth Cycle*, Journal of Economics, **42,** 375-393.
[29] Woodford, M. (1989), *Imperfect financial intermediation and complex dynamics, in Economic Complexity: Chaos, sunspots, Bubbles, and Nonlinearity,* Cambridge University Press, Cambridge.
[30] Yoshida, H. (2011), *Chaotic fluctuations in mathematical economics*, Journal of Physics: Conference Series 285, 012021.
[31] Zeeman, C. (1977), *Catastrophe Theory,* New York: Addison-Wesley.

APPENDIX 1

Statistical tests

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 15:22
Sample: 1 45
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
OFERTA1 does not Granger Cause CERERE1	44	14.0765	0.00054
CERERE1 does not Granger Cause OFERTA1		71.8107	1.5E-10

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 15:22
Sample: 1 45
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
OFERTA2 does not Granger Cause CERERE2	44	11.3302	0.00167
CERERE2 does not Granger Cause OFERTA2		9.63126	0.00346

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 15:18
Sample: 1 45
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
OFERTA1 does not Granger Cause IPC1	43	3.58946	0.06539
IPC1 does not Granger Cause OFERTA1		0.99193	0.32526

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 16:50
Sample: 1 45
Lags: 3

Null Hypothesis:	Obs	F-Statistic	Probability
IPC2 does not Granger Cause OFERTA2	41	4.78036	0.00694
OFERTA2 does not Granger Cause IPC2		3.12014	0.03867

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 15:52
Sample: 1 45
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Probability
IPC1 does not Granger Cause CERERE1	42	9.74905	0.00040
CERERE1 does not Granger Cause IPC1		6.75465	0.00316

Granger Causality Tests
Date: 04/30/12 Time: 15:24
Sample: 1 45
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
IPC2 does not Granger Cause CERERE2	43	0.69833	0.40831
CERERE2 does not Granger Cause IPC2		2.16304	0.14919

ECONOMIC GLOBALIZATION – A FACTOR OR AN EFFECT OF FOREIGN DIRECT INVESTMENT FLOWS?

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Abstract. *The globalization phenomenon is a strong subject that has been making a lot of ink to flow. Tumultuous economic life challenged the scientists to search for explanations in order to present the negative events and their forms in the economy. Economic foundation has suffered many changes and all these took place in order to find a solution to the various challenges of the economy. From the point of view of many scientists, the last events that happened in the global economy show that economic fundamentals must be revised modified, because they have proved to be wrong in some ways.*

It is often observed that Economics doesn't own any more the theoretical and practical means or analysis tools for intervention in the economy. The already existing theories and applications are inadequate and obsolete and the way of creating a new unifying principle or a generalized concept takes very much importance in order to systematize current and forecast economic phenomena at micro and macroeconomic level also for transnational companies conducting foreign direct investments (FDI).

The concept of economic globalization highlights the changes made and the transformations that the global economy has been facing in recent years. Transfers of capital, free movement of capital flows, mergers and acquisitions globally made between various multinational or transnational companies are the elements that characterize the concept of "economic globalization". But can we state that FDI has been driven by economic globalization phenomenon or has this phenomenon been born because of these capital exchange?

Keywords: *foreign direct investment flows, transnational companies, globally integrated companies, international investment strategies.*

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1. Introduction

One of the features of the world's economy in the last 20 years has been the impressive growth of inflows of foreign direct investment (FDI). It is important to state that the expansion of FDI flows has not been done in a spontaneous manner, but was stimulated over time by important reforms, such as opening economies to attract capital flows, changes in economic policies, and by implementing the privatization processes of companies.

The invested capital in developed countries has always been abundantly, a clear example to sustain this statement is the United States, where the best investment opportunities have been fully exploited. The developing countries have had and still have great opportunities for investments, but they still remain unexplored. We are witnessing a very interesting economic phenomenon; many owners of capital transfer it and perform a reorientation of FDI flows from developed countries to developing countries, in order to take advantage of the opportunities created by the emerging countries. In addition, the profitability of these FDI placed from one country to another is very high, because of the differences between countries, and we can call them "cross differences".

This exchange of capital flows shall eventually have an important impact on host countries, especially in terms of investment (foreign investment that replace domestic investment), and the balance of payments (outflows to finance projects of foreign investment and subsequent capital inflows in the form of repatriated profits). Similar arguments have been made to explain differences in the cost of labour and other inputs, and differences in statements regarding tax rates, tariffs and other governmental interventions (Horst 1971, Caves 1996).

2. Summary

We are witnessing an "economic movement" of major international businesses, because the phenomenon of globalization has led transnational companies to create new types of international investment strategies and in this way to become globally competitive, integrating into the world economy.

In this way, we talk less of multinational or transnational, but more of globally integrated companies. The literature in this domain is extremely rich on this topic. We will review the specialists' main ideas on this economic phenomenon, in order to illustrate the evolution and main elements that have stimulated great expansion of FDI flows over the past decades.

Richard Lipsey believed that technology development, the trend of innovation in companies and the desire to reduce costs represent items that the phenomenon of economic globalization has strengthened. The appearance of these three elements has led in time to internal organizational changes for the companies and they have become more decentralized and flexible, with great changes in their chain value (Lipsey, R. G., Chrystal, K. A., (1995), *An Introduction to Positive Economics,* Oxford University Press, 8th Edition).

Economic globalization should be seen as a global integration. Frank and Beeman believe that the new dimensions of the global economy are the technological part, thanks to the progress in communication and transport, and the economic part, due to the reduction or elimination of international flows barriers of goods, services and capital (W. J. Beeman, I. Frank, *New Dynamics in the Global Economy*).

International connections, such as international trade cover some distances in order to conduct to such relations, while the global connections, like satellite communications, for example, happen instantaneous and are decoupled from space. They can propagate simultaneously and instantaneously from one place to another, "outside of time", so that may be deemed "over-territory phenomena"[1]. Therefore, at present, international relations coexist with global ones, the contemporary world being an international and globalizing world.

The international investment strategies from the past were based on export and sale of products on the other countries markets. This time, the new strategies combine the full range of cross-border activities: export and supply internal/external, foreign investment and international alliances. Those companies that succeed in using these strategies are profitable due to

[1] Scholte, J. A. (1997), *The Globalization of the World Politics,* Oxford.

a higher level of coordination, operations and local diverse implantation[2]. We can identify a process of "internationalization of capital" which seeks capitalization in different geographical areas of the world, depending on available resources and markets.

Economic globalization has allowed the development of network companies that have increased the capital accumulation process and the decentralization of activities, offering in the same time, the penetration of capital across countries. International investment flows made by transnational companies through mergers and acquisitions processes have led to a consolidation and a high concentration of global capital. From an economic perspective, globalization has allowed the development of strategic partnerships between large multinational corporations especially in the technologic domain.

Economic globalization is based on the increasing interdependence of states, which determines the creation of a system of interdependent markets. Due to these factors, the technology news spread quickly, the lower costs of production of a country affect all other countries products, the position of a company on a particular market can attract another market reputation, etc. So globalization of markets requires their integration, firms having no possibility to separate decisions. The interdependent markets will force them to adopt a "comprehensive strategy" aimed at the entire world market.

Joseph Stiglitz, in his book "Globalization and Its Discontents"[3], believes that globalization has huge potential, still unrealized, in order to eradicate poverty and stimulate economic growth in a country. In recent years, the International Monetary Fund (IMF), World Bank (WB) and World Trade Organization (WTO) have promoted world financial stability, prosperity and free trade; these institutions have been in the middle of economic events and contributed by all their actions to globalization in the last decades. As international organizations it was very easy to direct the global economy to economic globalization phenomenon.

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[2] Ideas taken from the O.C.D.E. study: « *Performances de filiales etrangeres dans les pays de l'OCDE* », Paris, 1994.

[3] Stiglitz, J. (2003), *Globalization and its discontents*, New York: Norton Paperback.

Stiglitz's book seems to be a critique of the IMF and it accuses that although it has been more than 50 years after its creation, its main objectives were not accomplished and the IMF mission failed. According to Stiglitz's theory wrong policies have been adopted, such as the Fund denial of economic aid to countries in economic decline, or the creation of economic instability in the capital markets through forced liberalization too early begun. The mistake is that the IMF interferes improperly on macroeconomic issues at countries levels; there are plenty of examples, our country, Romania being among them. Moreover, aid is granted only to those states that adopt less expansionary policies, rather policies for restricting economic activity by lowering the budget deficit or raising taxes, or increase the interest rate, typically restrictive policies.

Stiglitz blames "market fundamentalism", which holds that a "free" market solves all problems flawlessly. He requires that public institutions should reform and become more transparent. This study contributes significantly to the globalization debate and provides an analytical model of the process of assisting countries facing the challenges of economic development and transformation.

Liberalization of capital markets is an aspect of globalization that Stiglitz long fought about, because its benefits have been remarkable only in the developed countries, developing countries suffer from foreign financial flows, which changed direction and have affected banking systems and have collapsed real coins. Most common examples are those in Asia and Latin America. These crises have only served to worsen the poverty and create chaos at the country level.

Four years after showing the increased interdependence challenges of countries, Stiglitz returns with another study on the topic of globalization, named "Making Globalization Work"[4], which seeks to provide inventive solutions for a range of problems facing developing countries development, international fiscal instability, and worldwide pollution. It also emphasizes the need to reform global financial institutions, reviewing commercial agreements and laws on intellectual property in order to make them better and to meet the growing disparities between countries. Now,

[4] Stiglitz, J. (2007), *Making Globalization Work*, New York: Norton and Company.

more than ever before, globalization has gathered the peoples of the world into one community, bringing with it a need to think and act at global level.

John H. Dunning, in the studies, "The Advent of Alliance Capitalism"[5] and "The New Globalism and Developing Countries"[6], provides further explanations on economic globalization. Thus, it highlights a process of globalization based on multiplication of connections and interconnections between states and societies that are currently in the global system. It includes the process that events, decisions and activities in a certain part of the world have significant consequences for individuals and communities located at great distances from one another.

A global company has many branches in different countries, being engaged in alliances and business networks in different parts of the world. Inputs from favourable regions and the products and services produced will be sold on different markets. Countries also have various commercial relations, financial and investment ones and the value added tax represents a significant part of GDP. Thus arises a universal tendency whereby firms and countries of the world are economically integrated, and this fact characterizes the new current global economic structure, different from that of previous generations.

Thus, the development of financial and investment transactions leads to a substantial increase in the value of output produced by foreign companies, financed by foreign direct investment growth capital and strategic alliances that consist of a superior form of internationalization actual trade in goods. Dunning believes that major businesses in the global economy have changed the mentality and ways of operating, adopting a systemic view on the implications of their actions, according to new market demands.

Friedman believes that the unifying feature of globalization system is the integration. The world has become an interconnected place where opportunities, but also threats are propagated in the direction you are

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[5] Dunning, J. H. (1997), *Alliance Capitalism and Global Business*, New York: Taylor & Francis Ltd.

[6] Dunning, J. H. (1997), *The New Globalism and Developing Countries*, New York: United Nations Univ.

connected to. The motive idea for globalization is free market capitalism; if the markets have more skills and if the national economies are opened to free trade and competition, the economy will be more efficient and prosperous. Unlike the Cold War, globalization has its own dominant culture, which according to Friedman's opinion represents the cause of its homogenizer action ("*The Lexus and the Olive Tree: Understanding Globalization*").

The economic globalization process has been stimulated by transnational companies, which contributed and still contribute to geographic mobility of financial resources; we can conclude that this fact has led to a global market. Thanks to advanced technology and innovation, the world has changed in a record time and rhythm, the space barriers have become terms. We are witnessing a continuous shift and relocation of production to different parts of the world. This creates "transglobal relationships"[7] that are actually technical and economic relations based on the enlargement of the production process[8].

Porter and Dunning mention that a transnational company should be able to create its competitive advantage and to strengthen its position on long terms in order to succeed at global level. The both specialists are combatants of short-term strategies. Success can be achieved whether the companies are able to exploit these advantages created through maximum utilization of production capacities (strategic or individual with other companies) and if they locate productive activities so that they meet the overall objectives of the company.

We are aware that resources are scarce and that, at some point in the future, the resources that are now available in the world will become insufficient. Until recently a country was considered strong according to its natural resources. Now, we are witnessing an evolution, which means that a country is considered to be competitive if it owns technology, information, labour, market structure, institutions, but also cultural value. So if a country owns more of such components, it shall have the possibility

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[7] Harvey, D. (2010), *The Enigma of Capital: And the Crises of Capitalism*, London: Profile Books Ltd., pp. 106.

[8] Idea taken from the book *„The Enigma of Capital: And the Crises of Capitalism"*, author David Harvey, Chapter-Capital Goes to Market, Profile Books Ltd., 2010.

to create powerful competitive advantages. We could call this theory, the theory of "relative advantage"[9], a concept that is often found Michael Porter's and J. H. Dunning's books.

We can affirm that, after applying this continuous globalizing phenomenon, the winners should be those countries that have relied on the state's role in the economy, states which haven't based upon free market and upon its total capacity of self-adjustment and self-regulation[10]. Developing countries should manage properly their budgets, should eliminate protectionist barriers that are causes for high prices, should introduce strict regulations for protecting the economy in order to obtain success[11]. From all the statements we have presented before, we can state that nothing can be efficient if the countries are run by inefficient, undemocratic, unresponsive, corrupt and non-transparent governments. The countries that didn't succeed in adapting this system failed because of these factors.

3. Conclusions

We are talking about two different concepts, namely the concept of a global economy and the concept of an internationalized economy[12]. The globalized economy is characterized by an integration of national economies into a global system through all the international processes. In other words, any company that adopts a strategy should think of it not only at national, but also at international level. In this way, the economic system acquires a new dimension, namely an internationalized one, and it becomes autonomous at global level. The opposite term of this globalized economy is the internationalized open economy, where the main actors are national

[9] The term comparative advantage was seen as "the principle of comparative advantage" in the book *"Economic Growth: Science and Technology and Institutional Change in the Global Economy"*, written by Richard G. Lipsey, published by the Canadian Institute for Advanced Research, Toronto in 1991.

[10] Stiglitz, J. (2003), *Globalization and its discontents*, New York: Norton Paperback, pp. 73-74.

[11] Stiglitz, J. (2003), *Globalization and its discontents*, New York: Norton Paperback, p. 14.

[12] Margulescu, S, 2010, *New strategies of transnational companies in globalization and functional specialization*, Bucharest: Student Book publishing House, pp. 38-39.

economies, they preserve their autonomy and they are no longer integrated into the global economic system, according to the theory of the global economy.

The relations between national economies are much closer, thanks to the transactions and international processes, such as international trade and investment flow. We can state that we are witnessing such internationalized economy because we can see that trade relations have been replaced with **FDI flows**. When we talk about their development, it is absolutely necessary to maintain a strong expansion of multinational companies, which are still regulated at national level, by keeping their national foundation. The autonomy of national economies is again emphasized in this way. In other words, the open internationalized economy is nothing else but a conglomeration of national autonomous systems based on international economic relations. So, foreign capital flows have been influenced by economic globalization, but they represent also an amplification factor of this economic phenomenon.

REFERENCES

[1] Beeman, W. J., Frank, I. (1988), *New Dynamics in the Global Economy*, New York, Committee for Economic Development.

[2] Dunning, J. H. (1997), *Alliance Capitalism and Global Business*, Taylor & Francis Ltd, New York.

[3] Dunning, J. H. (1997), *The New Globalism and Developing Countries*, United Nations Univ., New York.

[4] Dunning, J. H., Lundan, S. M. (2008), *Multinational Enterprises and the Global Economy*, Edward Elgar Publishing Limited, Northampton, USA.

[5] Friedman, Th. L. (2000), *The Lexus and the Olive Tree: Understanding Globalization*, Anchor Books Edition, New York.

[6] Gilpin, R., Gilpin, J. (2001), *Global Political Economy: Understanding the International Economic Order*, Princeton University Press.

[7] Joia, R., Huidumac, C. E., Babonea, A. (2011), *China's Strategic Investments and its Relations with Developing Economies*, 6th International Conference on European Integration – Realities and Perspectives, EIRP Proceedings, volume **6,** 2011, pp. 395-401, Danubius University Publishing, Galati.

[8] Lipsey, R. G., Chrystal, K. A. (1995), *An Introduction to Positive Economics,* Oxford University Press, 8th Edition

[9] Porter, M. (1998), *The Competitive Advantage of Nations*, The Free Press, Simon and Shuster Inc., New York.
[10] Porter, M., *On competition* (2008), Harvard Business School Press, United States of America.
[11] Scholte, J. A. (1997), *The Globalization of the World Politics*, Oxford.
[12] Stiglitz, J. (2003), *Globalization and its discontents*, Norton Paperback, New York.
[13] Stiglitz, J. (2007), *Making Globalization Work*, Norton and Company, New York.

REDUCING NATIONAL ECONOMIC SKIDDING

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Abstract. *World economies evolve along with expanding globalization. The traces that this phenomenon leave in national economies must be considered more seriously and thus to adapt continuously to the economics, social and political international mechanisms. The general process of globalization has also negative effects. One is the one we live now, namely the financial crisis. Now, more than ever, the state should intervene in economic and financial markets to try reducing skidding. All these actions require financial resources to achieve macroeconomic stability. Therefore, taxes are an important source of government revenue. Always, tax evasion was active and ingenious, because the tax burden permanently affected taxpayers, reaching the most sensitive point of their pecuniary interest.*
This article examines a small part of the factors causing this phenomenon, that in order to understand and develop some methods to combat, to highlight the adverse effects of the phenomenon and methods of eradication. The article also brings together analysis and research focused mainly on taxation and fiscal policy in Romania to identify the characteristic features of the tax system. Important issue of concern to economists today, but also policy makers, is the problem of optimizing the tax system. Increase, decrease or introduction of a new tax is a very delicate issue because of different effects. This study aims to find appropriate fiscal policies in Romania, according to the European standards.

Keywords: *shadow economy, tax evasion, tax policies.*

JEL Classification: H26; E62.
REL Classification: 11Z

1. Introduction

One of socio-economic phenomena of the utmost importance, facing all states today, in greater or less is tax evasion. Eradicate this phenomenon

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is impossible. All you can do is limit this phenomenon. Taxpayers always tried to evade tax obligations that laws impose, because the tax touches one of the most sensitive points that are monetary interest. Tax evasion is found both in the "surface" economy, especially in the informal economy, where the phenomenon is a generalized form. This part of the economy came to the attention of specialists from the 60s, appearing in the literature under various names: the occult economy, secondary, parallel, immeasurable, etc. Underground economy exists in all countries, in a lesser or greater measure.

Experts in the field have made an assessment of the effects of underground activities have on the economic and social life and their classification as follows: the effect on markets, influence markets this economy through several channels of transmission (supply and demand imbalance official markets through purchases of goods and services not reappear in the tender or delivery of goods and services that are not matched by demand, labour market imbalance by using informal labour, creating parallel markets for some goods and services, which leads to the formation of parallel prices for the same good or service, a situation likely to lead to unjustified reallocation of resources); effect on economic behaviour occurs by changing the nature and investment saving function of public and private sector; effect on monetary equilibrium, is influenced by the occult economy through the following channels of transmission (parallel prices for both goods and services as well as employment for local currency); diversion of significant monetary and thus it disrupts report cash / coin scriptural; Waiting creating effects in relation to the main macroeconomic variables (interest rates, inflation and exchange rates, amplifying speculative transactions), the effect on the balance of payments, effect on general government.

One of the worst effects induced economy is the increasing distances between various social groups. Economic differences are significant in that some social groups participating in underground economic activities can get huge benefits while the company experienced a worsening of living standards.

2. Fiscal consolidation

Romania must take measures for fiscal consolidation by reducing expenses and increasing revenues but these measures are hampered by tax evasion and tax fraud, and the share of 29.6% of GDP in 2011 to economy estimated by the European Commission shows the problem. The EU

average is 19.5%, only Bulgaria has a value above Romania i.e. 33% of GDP, but it began a comprehensive restructuring program and computerization of tax administration, Romania has, likely "great place to exchange with Bulgaria" if not take firm action in this regard. Problem is not just one of income but also of fairness, that's because those affected by increases in taxes are honest taxpayers. Companies in Romania require excessive controls made by the competent authorities with overzealous. They also complain about high taxation that leaves businesses a single chance to survive and that is to make evasion; also unstable legislation prevent the achievement of a viable business plan. Although the debt is not to worry, its growth rate due to the economic crisis, remains worrying. In this respect, combating shadow economy can provide an important source of relief fiscal budget and can remove labour market and the real economy blockage. Authorized institutions must adopt a proactive behaviour by continuing fiscal consolidation measures. Reduced revenues due to the economic crisis, but not at the same pace with spending cuts led to a lack of predictability in the tax area, the state was forced to find resources in a short time regardless of the consequences of the economic future. An example to support this claim is the lack of cost-benefit analysis in the case of fiscal measures, the decision to increase VAT at 24% led to increased evasion tax, especially in the food industry where according to Romanian Employers' Federation of Food, this represents approximately 50-60% of the total market benefit budget was short term but long term negative effects especially for the public. VAT represent a great temptation to evade tax payment, to a decrease in indirect taxes to the previous level of 19% and 5% basic foods would reduce the temptation avoidance and evasion would cost basic foods at an unsustainable level the effect of payment of that taxes. This measure aims to end lower overall taxation, which would lead to a continuous decrease the temptation of tax evasion. State behaviour should change in the fight against tax evasion, especially by punitive sanctions that great design and change taxpayer on the tax burden, the latter can be done by presenting a cost-benefit analysis showing the usefulness and impact fee imposed.

The emergence of the economic crisis increased the tax burden implicit influence on tax evasion. Reducing the tax burden should be a priority, given that, according to official statistics more than 60% of the Romanian monthly income is below 1500 Ron, that are forced to buy only necessities products and services. The major problem in our society should be treated with priority and responsibility of the state, as it limits the state's ability to increase revenue, especially in the current financial crisis and the

desire to reduce the gap with the EU. To reduce tax evasion, legislative and administrative reforms should be supported by close monitoring of the phenomenon and quick response of the authorities.

An increasing fiscal pressure will cause the taxpayer to seek continual breaches in tax law, especially when it coincides with a decline in their real incomes. It is difficult to establish a limit on the tax burden because of intervening factors: economic conditions, international economic context, domestic and international political conditions, age etc.[1]

3. Determinants of tax evasion phenomenon

Evolution of evasion phenomenon is determined by several factors, both internal and external: legislative and institutional size, level of taxation, the real economy dynamic.

According to international tax practices, a modern tax system must consider the education and training prompt taxpayers regarding their tax obligations, and penalties and sanctions that apply to breaches legal provisions. Tax evasions disadvantage the state because state revenues decrease and thus it cannot perform their duties, but also the taxpayer, because taxes are distributed on a mass of low income. To understand the phenomenon of tax evasion in developing and implementing effective control measures, must first consider the term called relative rate of taxation at national level during 2004-2012 (see Table 1).

Table no. 1.

Relative rate of taxation in Romania during 2004-2012

Romania	2004 (%)	2005 (%)	2006 (%)	2007 (%)	2008 (%)	2009 (%)	2010 (%)	2011 (%)	2012 (%)
Tax on profit	25	16	16	16	16	16	16	16	16
Income tax	40	16	16	16	16	16	16	16	16
V.A.T.	19	19	19	19	19	19	24	24	24
Employer contributions	32	32	29.25	27.5	25.2	26.5	26.5	26.5	26.5
Employee contributions	17	17	17	17	15.5	16.5	16.5	16.5	16.5
Total	133	100	97.25	95.5	91.7	94	99	99	99

Source: www.mfinante.ro

[1] Popa, Ş., Cucu, A., *Shadow economy and money laundering*, Expert Publishing, Bucharest, 2000

Relative index of fiscal decreased in 2005 by approximately 33 percentage points, something rarely seen in the economy. The main reason for this decrease was the flat tax set on January 1, 2005 at a level of 16%. This index has a downward trend until 2008, reaching a level of 91.7% (with approx. 41.3 percentage point difference from 2004) and led to growth until 2008. A decision influenced the increase in tax evasion with the advent of the economic crisis in 2008 was to increase contributions paid by the employer and employee. Government has taken various decisions to reduce the fiscal deficit during the crisis by increasing taxes and not taking into account their lower economic crisis. A measure of relaxation by lowering taxes and increasing tax base and reduce tax evasion Default economy. According to a study by Forbes magazine, which takes into account European economies, the country with the lowest level of taxation, the year 2010 is Ireland with an index of tax of 93.4% compared to 164%, the highest score for France.

"Resistance" black market is due to the still very high level of social security, they increased the time that the economic crisis. Despite the optimistic figures given by the authorities in recent years, black labour remains a serious problem in the Romanian economy. The flat has reduced the black labour, and a significant number of employees still working economy. Growth of the number of employees involved in the national economy is much lower than the economic growth rate. At the same time, labour productivity in 2011 registered a growth level below that achieved in 2010. Despite imports of technology, providing increased productivity without requiring an increase in the number of employees, the difference between dynamic indicators emphasize concerns about the underground economy.

4. New national system

The current system of tax collection is characterized by a weak and ineffective administration with excessive bureaucracy, a relatively small tax base with many exceptions and statutory deductions and high tax evasion.

According to an OECD report on tax administration, the administrative efficiency of tax collection is quite low in Romania, this being on the penultimate position in the group of new member states. According to the World Bank report – Paying Taxes 2012, Romania is positioned in 2011 to position 154 of 183 countries worldwide on the ease

of paying taxes, the number of tax payments per year you need to make a company in Romania is of 113. Romania is on last place in this chapter in the countries of Central and Eastern Europe.

Romania has a small share in GDP revenues (tax and non-tax revenues), in 2011 this proportion was only 32.5% of GDP, keeping the big difference compared to the European average in 2011 was 12.1 percentage points of GDP. At existing, the tax revenue to GDP (taxes and social contributions) in Romania was 27.2% in 2011, a slight decrease compared to 2010 by 0.7 percentage points, well below the EU27 average (39.6% – 2011, 39.9% – 2010), the difference was 12.4 percentage points. Structure of tax revenues in Romania differ from the European average is, a higher percentage of revenues from indirect taxes, 46.32% of the total tax revenue from 33.08% EU-27 average, while the share of direct taxes was only 21.32% (EU27 – 31.81%), the share of social contributions revenue was 21.25% (EU27, 35.1%)[2].

The rise of VAT and excise duties increased share of indirect taxes compared to 2010 by approximately three percentage points, while contributions and direct taxes have declined relative importance in terms of their share in GDP declines greater than reduction of total tax revenues to GDP. Can be considered indirect taxes were the main component of fiscal consolidation on the revenue budget.

As an example of poor collection of taxes, Romania collected 8.4% of GDP in tax revenue in 2011, same as Estonia, while the statutory tax rate in Romania is higher than that of Estonia (24% compared with 20%). In addition, Bulgaria, the structure of the economy relatively similar to that of Romania and lawful VAT rate lower (20%), collected more of the value added tax in 2011, 8.6% of GDP. Regarding social security contributions paid by employees and employers revenues collected in 2011 were 8.3% of GDP, much lower than in the Czech Republic (13.1% of GDP) and Hungary (12, 3% of GDP), although statutory rates were relatively similar contributions, Slovenia (13.6%), Estonia (11.4% of GDP), Lithuania (9.5%), Poland (9.1%) and Latvia (8.6%), where statutory social contributions rates were significantly lower than in Romania.

Romania is ranked among first of the highest taxes in the EU-27: 4th place, with the lowest rate of personal income tax, after Bulgaria (10%), Lithuania and the Czech Republic (both 15%) , 6th, taxing profits of firms, after Bulgaria and Cyprus (both 10%), Ireland (12.5%), Latvia and Lithuania (both 15%), 21st place, six states have higher tax burden on

[2] Burghelea, C., Molănescu, A., G., Crăciun, L., *Macroeconomic development*, Transversal Publishing, Bucharest, 2013

labour, namely: Belgium (55.4%), France (49.3%), Germany (49.1%), Austria (47.9%), Italy (46.9%) and Hungary (46.4%), 24th place, 3 European countries tax higher than in Romania Hungary (27%), Sweden and Denmark (25%).

5. Size combat tax evasion

According to calculations Fiscal Council, based on National Statistical Institute evasion in Romania has a high dimension, unless CAS, VAT and income tax it represents 10.3% of GDP in 2010 (see Table 2) but stagnation is observed at the level of 2009. Romania has one of the smallest tax revenues in GDP of EU-27, which is 27.2% of GDP, a gap of 12.4 percentage points of GDP to the European average. If the system of taxes in Romania would be effective, the state would reduce gap the European average. But this system is characterized by a small collection, excessive bureaucracy and high tax evasion. A profound reform of the administration of taxes in Romania targeted towards increasing tax collection is absolutely necessary[3].

Table no. 2.

Evolution of the number of employees

Evolution of the number of employees	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
Average number of employees in the national accounts (1)	5,882	5,786	6,138	5,900	6,408	6,162	6,412	6,436	6,513	6,573	6,633	6,647
Average number of employees reported by employers (thousands) (2)	4,623	4,619	4,568	4,591	4,469	4,559	4,667	4,885	5,046	4,774	4,376	4,297
Average number of employees in underground economy (thousands) (3)=(1)-(2)	1,259	1,167	1,570	1,309	1,939	1,603	1,744	1,551	1,467	1,799	2,257	2,349
Share of employees in underground economy (4)=(3)/(1)	21.4%	20.2%	25.6%	22.2%	30.3%	26.0%	27.2%	24.1%	22.5%	27.4%	34.0%	35.3%

Source: Council Tax calculations based on data from www.insse.ro

[3] Coşea, M., *Romania underground*, Bucharest, Economic Publishing, 2004

Special emphasis will be placed on increasing the number of checks covering personal tax position of individuals in terms of income tax and indirect methods will be enhanced control used to determine the adjusted taxable. Another course of action would be to monitor newly established taxpayers registered for VAT purposes, a field considered to have a high potential for tax evasion. Conditions will be checked and declared them as inactive, respectively, if they have the ability to conduct business for which they were authorized. Control of electronic commerce will be developed to identify risk withholding tax on dealer quality by individuals and businesses. It will then be prioritized checks on companies that have experienced negative returns successive amounts of VAT and no claim, given the frequent cases of tax evasion in this area. Will be developed and implemented software application "COLIBRI" in the process of issuing and use of the documents used to control individuals. Will create fiscal risk indicators to determine the taxpayer's tax risk profile start-ups and risk analysis will be performed in order to identify taxpayer's affiliates that produce structural loss, to encompass their priority control plan. National Customs Authority will implement IT application monitoring movements of excise goods under suspension of excise duty, including intra-Community movements of excise paid in the Member State of dispatch – EMCS Phase 3.

Form filling system will be reorganized in order to lead to a smaller number of forms and deposits will continue building unique service to receive statements. National Agency for Fiscal Administration (NAFA) will continue to strive for more and better information to taxpayers about their rights and obligations in relation to tax administration, including the facilities and the continued growth of the use of electronic access citizens in relation to tax administration.

Until 2014, the services provided by the tax payer will meet the most part, transparent and stringent quality standards. NAFA policy to support the business will continue in the coming years by maintaining a stable system of incentives for taxpayers to pay obligations to temporary difficulty caused by temporary lack of available funds and scheduled payments will continue providing outstanding tax obligations.

For NAFA more complete collection of revenues is an ongoing objective. To this end, measures are in place to have the effect of improving payment compliance. Increased revenues will be achieved by promoting measures to broaden the tax base and improve collection. At the same time, it will focus on increasing computerization of internal activities of NAFA.

Improving organizational NAFA will increase efficiency, i.e. constant reduction in the cost of collection. NAFA next step necessary restructuring, initiated in 2011 will involve strengthening the organizational structure of the territory. Current training system will be improved to better meet the real needs for training in NAFA – own unit and subordinate units, thus contributing to performance and efficiency of the institution. External communication will increasingly illustrate the principle said to increase transparency, aiming not only large taxpayer's information on NAFA activity, but also the prestige of the institution. Communication will be particularly important for mediating and understanding by taxpayers control the actions that will take place in the immediate future following so that they correctly perceive the risk posed to non-compliance with tax regulations.

NAFA will implement a program of measures, which include both short-term actions and medium and long term[4]. Short-term economic activity in the areas NAFA known to have high evasion and trade in products subject to excise duty (tobacco, alcohol, oil) and trade in fruit and vegetables will focus on the measures of preventive and deterrent escapist initiatives. The fight against tax evasion should result in collection of amounts to the budget, not for controls that determine damage with little chance of recovery of tax arrears to be the state to watch them without success, consuming resources unnecessarily. Path tracing direct action is money, and there should be improved collaboration with other institutions including banking and judiciary.

In the long run, it is very important restructuring under the NAFA World Bank. By improving relations with taxpayers and simplifying procedures for declaration and payment of budgetary obligations, the program aims to increase voluntary compliance. It will be easier for taxpayers to pay their tax obligations, which translates to the tax authority in raising revenues and to reduce collection costs. Fines and checks are temporary measures to bring the budget revenues and costs are high. To increase the share of revenues to the European average, the tax payers should provide both information and a simple submission of declarations, obtaining documents and payment.

[4] PRESS Release – referring to reform and action priorities of National Agency for Fiscal Administration, 23 April 2012

6. Conclusions

Tax evasion exists in all countries; the difference between them is share in the total economy. Current economic climate, economic crisis caused by financial worsens public financial balance of Romania, one of the solutions to this scourge as limiting recovery leading to deepening crisis. Combating tax evasion must be a goal, national security "because it kept rising trend in recent years with no signs of decline or limit the phenomenon. With this objective is an important message to convey economic environment, limiting evasion will decrease the tax burden an expected time of economic environment. Due to the economic crisis is affected by frequent changes in the tax code; its stability and predictability are vital conditions for a business plan.

Romanian state must take very strong measures to combat tax evasion, to change its behaviour in the fight against tax evasion large representing approx. 80% of the total. Measures must be supported by legislative changes to provide economic alternative than tax evasion. Evasion legislation leaves room for interpretation because it is brief and there is no clear definition of the concept of tax evasion. Reducing bureaucracy leads to decreased tax evasion and corruption in the system. Training fiscal agent, NAFA has a problem in terms of numbers than even the existing fiscal agents do not have a sufficiently high level of preparedness to cope with complex problems.

Doing Business Advisory Council established in 2012 came up with a series of proposals including some to limit this phenomenon. A proposal concerns the restriction of cash in transactions to 2,000 Euros, according to the model introduced by Mario Monti in Italy. In a country where the economy reaches a level vision board 40% of GDP, cash plays an important role, it circulates freely if no transactions are made through banks, if the cash restriction to 2000 Euros when each transaction exceeding this amount will be justification money in the bank. Another proposal is the creation of Tax Police, which would work in coordination with the prosecutor’s office after the Dutch model, so evasion will arrive soon in court. State should take action against the practice of transfer pricing, we need a system of rules and methods to monitor and detect transfer pricing. The EU Commission has set up a Joint Transfer Pricing Forum and arbitration forcing the fiscal receipt issue and severe penalties for noncompliance. Computerization in tax matters: Bulgaria's fiscal cash registers connected to the GPRS tax administration, this system performed

real-time risk analysis; the benefit of this system is sending control bodies only where risk is reported. Another example would be centralized administration including issuing invoices; bills are issued using a common tax-payer so that each taxpayer information requirements are real-time. Significant short-term costs for this system but medium and long term these costs are amortized to revenue growth and the efficient control system. Total computerization of tax collection system reduces bureaucracy and corruption in the system default as limited personal relations between taxpayers and tax agents.

Regarding excise tax evasion in this area is high even though it is highly regulated and efficient scanners are awarded at exhibitions of inventions produced in the country. In terms of legislation and control system are not much to be done, limiting avoidance can be achieved by applying existing legal provisions.

With regard to "black" work, this problem can be solved with effective controls to detect employers who pay their employees under the table, backed by severe penalties. Should review state social security system, as employers take advantage of certain legislative gaps certain social groups have the privilege to pay less than the normal huge fiscal cost.

REFERENCES

[1] Bîrle, V. (2005), *Tax fraud*, Teora Publishing, Bucharest.

[2] Boicean, D. (2004), *The Institutions Of The European Union. Combating illicit shadow economy phenomena,* Lucian Blaga University, Sibiu.

[3] Burghelea, C., Molănescu, A. G., Crăciun, L. (2013), *Macroeconomic development*, Transversal Publishing, Bucharest.

[4] Craiu, N. (2002), *Altenative variants of financial and economic policy in relation to fraud and tax evasion*, Lucian Blaga University, Sibiu.

[5] Craiu, N. (2004), *Shadow economy between "Yes" and "No",* Economic Publishing, Bucharest.

[6] Corduneanu, C. (1998), *The tax system in the science of finance*, Codecs Publishing, Bucharest.

[7] Voicu, C. (2003), *Financial and banking crime investigation*, Polipress Publishing, Bucharest.

[8] Coşea, M. (2004), Romania underground, Bucharest, Economic Publishing.

[9] Florescu, D., Coman, P., Bălaşa, G. (2005), Taxation in Romania*: doctrine, regulations, jurisprudence*, All Beck Publishing, Bucharest.

[10] Schneider, F. (2005), *Shadow Economies Around the World: What Do We Really Know?* European Journal of Political Economy, Vol. 21/3, September.

[11] Lazăr, *A.* (2004), *Investigation of fraud in the business environment*, LuminaLex Publishing, Bucharest.

[12] Lazăr, A. (1998), *The Underground Economy. A Strategic Scan of the Justice Environment, Department of Justice*, Canada.
[13] Minea, M. Ş., Costaş, C. F. (2006), *Taxation in Europe at the beginning of the 3rd Millennium*, Rosetti Publishing.
[14] Pestieau, P. (1989), *L'économie souterraine,* Editura Pluriel, Inédit, Hachette.
[15] Popa, Ş., Cucu, A. (2000), *Shadow economy and money laundering*, Expert Publishing, Bucharest.
[16] Şaguna, D., Tutungiu, M. E. (1995), *Tax evasion*, Oscar Print Publishing, Bucharest.
[17] Văcărel, I. (2006), *Public finance*, Didactic and Pedagogic Publishing House, Bucharest.
[18] Voicu, C. (1999), *Dirty money laundering*, SYLVI Publishing, Bucharest.