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ECONOPHYSICAL APPROACHES FOR THE DIRECT FOREIGN INVESTMENTS

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Abstract. *In this paper will be applied some principles and methods from econophysics in the case of the direct foreign investitions (D.F.I.), particularised for the Greenfield type, and mixed firms of trade and industrial production (Joint Ventures). To this aim will be used some similarities and parallelisms between the mentioned economic domains and some phenomena and processes from physics, especially from thermodynamics, solid state physics (the grow of crystals and thin policrystalline layers etc.), electromagnetism etc.*

Keywords: econophysics, thin films, direct investments, joint-venture, greenfield.

1. Introduction

Econophysics – science recently appeared – developed especially owed to the interpretations and models created for applications in the financial and investment domains, generally in the domains of industrial economy or goods consumption. Less has been the attempts to find the applicable models of the econophysics in the domains of the trade and marketing and especially in the trade and direct foreign investments domains.

By his nature of exact science which studies the laws of the nature, the physics proved to be the source and support of other sciences and especially of the technical ones (electrotechnics, metallurgy, electronics, civil constructions, machine engineering, thermotechnics etc.) but also for many boundary sciences as biophysics, geophysics, physico-mathematics, biomedical physics and more recenty econophysics and sociophysics.

Solid state physics, deals with the study and applications of the solid materials, especially metals, semiconductors and dielectrics with numerous and extremly important applications in electronics and microelectronics, in optics, in energetics (solar cells) etc., as well as with the plastic materials, polymers etc.

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For the most diverse applications, solid materials (metals, semiconductors, oxides etc.), are used like thin films made by various methods from which the most used is that of evaporation and condensation in vacuum on an adequate solid support (substrate), especially in microelectronics and nanotechnologies.

Between the process of obtaining of thin films by evaporation and condensation from the gaseous phase and the process of the direct investments in the variants „greenfield”, fusions, acquisitions or mixed firm associations (joint ventures), there are more similarities or analogies which shall be exposed and analised in this paper. The examination of these similarities between the two process types can lead to the settlement and/or understanding of some criterions or realisation conditions of some direct investments in other countries like „greenfield” investments on bare spots, or of fusions and acquisitions.

To be able to distinguish and understanding the similarities between the monocrystalline or policrystalline thin films obtaining process and the mode to realise the direct foreign investitions (D.F.I.), further on shall be succintly presented the conditions to obtain, and the main physical and structural characteristics of the thin films condensed in vacuum on plane supports warmed at various temperatures.

2. General considerations about the obtaining of the solid state thin films

To obtain the thin films by evaporation and condensation in vacuum some special devices are used: the evaporation precinct, in form of a glass case (Fig. 1) in which there are the evaporation source in form of a crucible containing the evaporation substance as well as the plane support 3 situated in the path of the atomo-molecular beam of the evaporated substance from the crucible. The support is warmed at various temperatures by dint of a separate oven (Fig. 1).

An important parameter to obtain a compact thin layer is the number of the incident particles on the support i.e. the atomo-molecular flux density produced by the evaporation source that must exced a critical value I_c to have a condensation on the support. There is too a temperature T_c of the support above which the condensation can’t occur because T support is too high [1].

The formation and growth of thin films process can be best explained by the adsorption theory of Frenkel and Langmuir and further developed by others researchers.

According to this theory the incident atoms on the support surface remain on it a time t_s given by:

$$t_s = \tau \cdot e^{-\frac{E_d}{RT}} \tag{1}$$

where E_d is the adsorbtion (desorbtion) energy, and τ the molecular period of the adsorbed molecule (atom) vibrations (≈ 10^{-14} s) on the support. During the time t_s the adsorbed atoms migrate on the support similarity to bidimensional gas, after which they can reevaporate or to settle on the support.

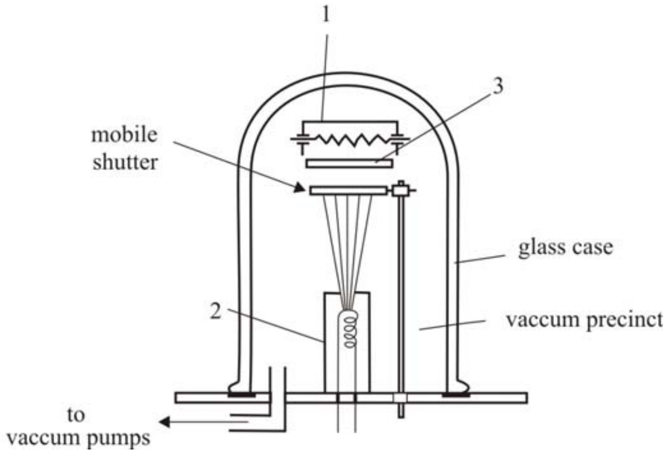


Figure 1. Vaccume evaporation equipment for thin solid films deposition.

The adsorbtion degree depends too on the accomodation coefficient α, defined by the probability of the incident atom (molecule) on the support's surface, to be adsorbed. The adsorbtion coefficients depends on the nature, temperature and surface support condition as well as on the nature and atoms (molecules) incident energy E_i. If E_s and T_s are the energy and the support temperature respectively, and E_{st} and T_{st} the energy and the layer temperature respectively, the accomodation coefficient may be expressed by [1.2]:

$$\alpha = \frac{T_{st} - T_i}{T_s - T_i} = \frac{E_{st} - E_i}{E_s - E_i}. \tag{2}$$

In the case $T_s = T_{st}$ or $E_s = E_{st}$, then $\alpha \approx 1$ that corresponds, for instance, to the metal on metal condensation.

Numerous investigations showed that the nature and the state of the support deposition surface of the thin film presents a main importance for the structure and the layer's physical properties. As function of the nature and support condition may be obtained monocrystalline layers (with some number of defects), policrystalline or even amorphous ones. It is supposed, of course, that the support temperature is high enough to obtain the crystalline phase, the amorphous one should appear in the case of low support temperature and the support surface contains oxides, traces of fatty acids etc. or an appreciable number of defects.

In the case of the supports with insufficiently cleaning or insufficient vacuum degassing or by other procedures, the obtained layers contain crystalls with a high number of structural defects.

It is generally admited that the crystalline structure supports (ionic crystalls, semiconductors, metals, mica etc.) facilitate the apparition of cvasi-monocrystalline layers called epitaxial layers.

In case of autoepitaxy the support must be of the same nature and eventually with the same orientation and network type like the epitaxial layer (for instance, Si on Si, Ge on Ge, Cu on Cu etc.). It is supposed that all other conditions of epitaxial growth are realised (high support temperature, the absence of defects or impurities on the support surface, atom supersaturation of the deposed film etc.).

The amorphous supports like those of plane glass, generally lead to policrystalline or even amorphous layers.

3. Determination of the mass and the thickness of the condensed layers on the support

From the molecular physics it is known that inside a vacuum precinct containing a number N of molecules with a total mass M, the number ν of molecules coming from all directions and hitting the surface unity in time unity is given by:

$$\nu = \frac{1}{4} n\overline{v} \tag{3}$$

in which:

$$\overline{v} = 2\left(\frac{2k_B T}{\pi m}\right)^{1/2} \tag{4}$$

is the arithmetic mean velocity of the molecules at temperature T.

By replacing the Boltzmann constant value k_B and of the other numeric entities, the relation (4) becomes:

$$\nu = 4{,}66 \cdot 10^{24} p(MT)^{-1/2} \tag{5}$$

in which p represents the gas pression inside the precinct, and $M = mN$ is the molecular mass.

The adsorbtion velocity of the molecules deposed on the plane surface support placed on the way of the evaporation source from the inside of the precinct is given by:

$$w_{ad} = \frac{\mathrm{d}n_a}{\mathrm{d}t} = \alpha\nu \tag{6}$$

where α is the accomodation coefficient, defined as the probability of the atom (molecule) incident on a surface to be adsorbed, namely fixed on the support by the Van der Vaals adhesion forces or by other forces of atomo-molecular nature.

Taking into consideration (5), the relation (6) becomes:

$$\frac{\mathrm{d}n_a}{\mathrm{d}t} = \alpha p(2mk_BT)^{-1/2} = 3{,}513.10^{22}\alpha p\frac{1}{\sqrt{MT}},\ [\mathrm{cm}^{-2}, \mathrm{s}^{-1}]. \tag{7}$$

For the deposition of a layer of a substance of mass m it is necessary to establish a (dynamic) equilibrium between the solid phase (or liquid), from the support surface and the gaseous one (of the incident atomo-molecular flux). That is to say that the evaporation velocity w_{ev} should be proportional to the mass m of the evaporated substance as well as to the adsorption velocity w_{ad}, of the particules from the incident atomo-molecular beam given by relation (7):

$$w = m\frac{\mathrm{d}n_a}{\mathrm{d}t} m\alpha_v p_v (2\pi mk_BT)^{-1/2}, \mathrm{g/cm^2 \cdot s} \tag{8}$$

where α_v is the evaporation coefficient (similar to the accomodation one α), and p_v the saturated vapours pression of the evaporated substance. Because $M = mN$, from (8) is obtained:

$$w = 0{,}0583\alpha_v p_{v(\mathrm{torr})}\sqrt{\frac{M}{T}},\ \mathrm{g/cm^2 \cdot s}. \tag{9}$$

For the surfaces considered clean $\alpha_v = 1$.

From (8) and (9) formulas it is observed that the evaporation velocity depends on the substance nature (through the intermediary of the mass M), as well as on the thermodynamic evaporation temperature T.

Supposing a punctual evaporator with spherical simmetry, a sufficiently reduced pression (to have an atomo-molecular beam) and a beam of not to high density (to neglect the shock number between the evaporated substance molecules), in order to calculate the condensated distribution on the receptacle support, the laws of Lambert from the geometric optics can be applied. From the optics it is known that the mean luminous intensity of a spherical punctual source, for the solid angle unity is:

$$I_0 = \frac{F}{4\pi}, \tag{10}$$

where F is the total flux emitted by the source in all directions. Analogously in the case of a punctual source from which a substance is evaporating in all directions with a constant velocity w[g/s], the substance quantity passing through a solid angle $\mathrm{d}\Omega$ in any direction, in time unity is given by the relation:

$$\mathrm{d}m = \frac{w}{4\pi}\mathrm{d}\Omega. \tag{11}$$

It is admited that, after evaporation, the material is condensed on an elementary surface $\mathrm{d}S_2$ whose normal forms the angle θ with the mean beam direction (Fig. 2).

Because the solid angle is given by:

$$\mathrm{d}\Omega = \frac{\mathrm{d}S}{r^2}\cos\theta, \tag{12}$$

it results that:

$$\mathrm{d}m = \frac{w}{4\pi}\frac{\cos\theta}{r^2}\mathrm{d}S_2. \tag{13}$$

In the case of a plane source emitting in the directions above the respective plane only (Fig. 2) the relation (10) it is written:

$$I_0 = F/\pi, \tag{10'}$$

and consequently the formula (18) becomes:

$$\mathrm{d}m = \frac{w}{\pi}\frac{\cos\theta}{r^2}\mathrm{d}S_2. \tag{14}$$

If ρ [g/cm^3] is the condensed substance density, then the thickness d, of the layer deposited in time unity can be found from the relation:

$$\mathrm{d}m = \rho\mathrm{d}V = \rho d \cdot \mathrm{d}S_2. \tag{15}$$

From (14) and (15) results for the thickness d:

$$d = \frac{w}{\pi\rho}\frac{\cos\theta}{r^2}. \tag{16}$$

If the evaporation is from a punctiform source and the condensation is on a plane receiver surface S (Fig. 2), then the layer thickness condensed in the point P will be:

$$d = \frac{w}{\pi\rho}\frac{\cos\theta}{r^2} = \frac{w}{\pi}\frac{h}{r^3} = \frac{wh}{\pi\rho(h^2 + x^2)^{3/2}}. \tag{17}$$

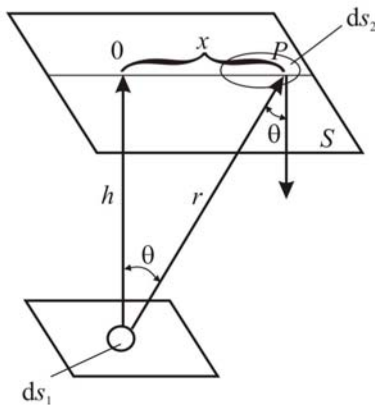


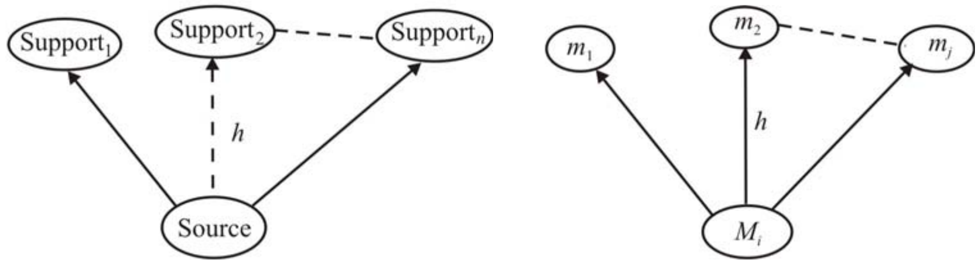
Figure 2. For calculation of the thin films thickness.

The layer thickness in the point 0 ($x = 0$) is given by (see rel. (17)):

$$d_0 = \frac{wh}{\pi\rho}\cdot\frac{1}{h^3} = \frac{w}{\pi\rho}\cdot\frac{1}{h^2}. \tag{18}$$

In the thin films technology, in order to increase the productivity, inside the evaporation precinct can be situated more supports (Fig. 3,a), so from the source of mass M, j layers can condensate of masses m_1, m_2, m_3 … m_j, as it is seen from the figure 3,b.

Obviously, the thicher layers will be obtained on the supports placed just above the source M, at the minimum distance h (Fig. 3) according to the relation (18), whereas the thickness of the other layers should be determined in compliance with (17) relation in which x_j represents the distance from the point 0 (placed above the source) up to the considered support.



a) b)

Figure 3. Deposition of several thin films from a single source.

It is to be mentioned that for the realisation of some compound substances layers or for the controlled impurification as in the case of semiconductor thin films that must be impurified, can be used many evaporation sources with $M_1, M_2 \ldots M_n$ mass tagether with many supports on which can be condensed layers of $m_1, m_2 \ldots m_p$ mass as can be seen in figure 4.

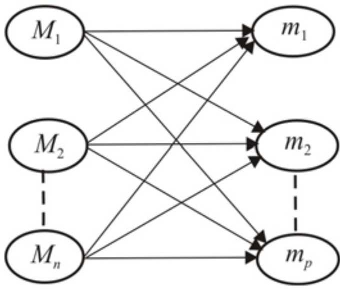


Figure 4. Deposition of several thin films from several evaporation sources.

Between the layer mass m_j and the source mass M_i there is a proportionality relation:

$$m_j = AM_i \tag{19}$$

which in the general case with many sources and many condensation supports cand be written under the form:

$$\left.\begin{aligned} m_1 &= k_{11}M_1 + k_{12}M_2 + \ldots + k_{1p}M_p \\ m_2 &= k_{21}M_1 + k_{22}M_2 + \ldots + k_{2p}M_p \\ m_3 &= k_{31}M_1 + k_{32}M_2 + \ldots + k_{3p}M_p \\ &\vdots \\ m_n &= k_{n1}M_1 + k_{n2}M_2 + \ldots + k_{np}M_p \end{aligned}\right\} \tag{20}$$

or under the form of a functional relation of the type:

$$\left.\begin{array}{l} m_1 = f_1(M_1, M_2, M_3 \ldots M_p) \\ m_2 = f_2(M_1, M_2, M_3 \ldots M_p) \\ \vdots \\ m_n = f_n(M_1, M_2, M_3 \ldots M_p) \end{array}\right\}. \tag{21}$$

Coming again, for simplicity, to the case when we have a single source M and a single plane, finite and regular (circular or rectangular) support placed above the source at a distance h (Fig. 3), the mass m_j of the deposed layer is direct proportional to the thickness d_o of this one, i.e. (see rel. (18)):

$$m_j = kd_o = k\frac{w}{\pi\rho}\frac{1}{h^2} \tag{22}$$

in which it was taken in consideration the evaporation above the plane only, where is situated the evaporation source (Fig. 3). From the relation (22) it is seen that the deposed mass layer depends directly proportional to the evaporation velocity w and inversely proportional to the distance support – source h.

4. The crystalline thin films growth model for the direct foreign investments analysis.

As it was mentioned in § 2, between the thin films obtaining process and the mechanisms of realisation of the direct foreign investments there are more similarities or analogies. Their examination may lead to the establishment or understanding of some specific criterions or realisation conditions of some direct investments types in other countries, markets or regions.

So, the mother-firm, with the economic-financial power (capacity) source M, can be assimililated with the source of mass M_j from which is evaporating the substance that constitues the capital fluxes guided towards various locations j that can be assimilated with the support or with supports of masses m_j, $j = 1, 2 \ldots p$ placed in various locations (countries) j, at „economic" distances h_j toward the source. By economic distance is understood the physical route (way) type as well as the infrastructure and the costs tied to the necessary utilities to realise the capital or equipments, raw materials etc., that can be assimilated with the "trajectory" of the atomo-molecular beam from the vacuum precinct in the case of thin films deposition.

In the case of the direct foreign investments (D.F.I.) the potential investments benefice V_{ij} or the efficiency of the direct investments from the mass central source (the amount of M_i funds of the mother firm), towards her branches or in other investments locations in various countries or regions characterised by "masses" or funds $m_j, \ j = 1, 2, 3 \ldots p$ can be calculated with the general relation:

$$V_{ij} = R_{ij} - C_{ij}. \tag{23}$$

In (23) relation, R_{ij} term represents the actual total value of the incomes expected to be obtained, and C_{ij} is the actual value of the necessary costs to realise the investments m_j in the location (or locations) j (see Figs. 3 and 4).

The mass m_j of the investments realised in the location j characterises the value represented by density ρ (i.e. the receptor region characteristic, from the point of view of the attractivity degree, permisivity for investments), as well as the volume V_j of the investments (the value of each branch) so that:

$$m_j = \rho V_j. \tag{24}$$

On the other hand, as it was shown (see rel. (22)) that "the mass" m_j i.e. the value of the investments realised in location j from other country or region, is directly proportional with the layer thickness d_o i.e. with the dimensions or the volume of the realised investments:

$$m_j = kd_o = k\frac{w}{\pi\rho}\frac{1}{h^2} \tag{25}$$

where h represents the "economic distance" between the mother-firm (source) and the investments (branch) realised on a "plane" support at the location j, and w is the transfer velocity (similar to "evaporation velocity") of the investments funds, equipments and necessary management.

But, as it was mentioned in the previous paragraph, the volume and the value of the realised investments (m_j mass) are conditioned also by the value, the economic-financial power respectively and other characteristics of the mother-firm (source), fact that can be written under the form of the direct dependency (see rel. (19)):

$$m_j = AM_i. \tag{26}$$

In the relation (26) A represents a specific parameter (proportionality constant) to each direct foreign investment type. A represents a managerial decision component at the mother-firm-level establishing the percentage from the total investment fund for a certain target. From (25) and (26) results the necessary value of the transfer velocity of the capital fluxes to location j:

$$w = \frac{\pi\rho}{k} AM_i h^2 = A_o M_i h^2. \tag{27}$$

From the formulae (27) results that the capital flux transfer velocity, equipments etc. from the sourse i to the location j depends directly on the economical (capacity) power M_i of the mother-firm and on the square of the economic distance from the source to the investment location. The remoter the respective location, the higher must be the value and transfer velocity of capital flux to compensate the supplementary expense with the transports that are conditioning also the realised products distribuition at the location j and the delivery of products respectively etc. It may be mentioned, for instance, the case of Nokia firm investments at the "support" industrial park Jucu near Cluj-Napoca from Romania. The selection of the respective locality being in large measure decided because the route distance h is smaller to the great towns from the nordic and central regions of Europe where must be distribuited the products and where from can come the equipments and installations (from Nokia "source") towards the location j, Jucu.

It is proved so that in the case of greenfield investments Nokia from the support Jucu near Cluj-Napoca, has taken in consideration the main conditions and exigences imposed by the realisation of some "layers" which constitutes the investments of mass m_j and namely by the necessary "vacuum" in the work "precinct" characterised by the specific nature of the "support" (region) j having an accomodation coeficient or condensation α (see rel. (9)), symbolised by the cheap manpower attraction and disponible in the region, good conditions to bild the infrastructure (cheap and enough ground plot, construction materials etc.), the distance h relatively small between the source and "support" (j location) and the economic-financial power (capacity) M_i of the mother-firm source as it results from the analyes of (27) formula too.

Similar considerations may be done also for other direct foreign investments categories for which can be detalied the differences of investments type, by the conditions and relations existing between the "source" and "condensation support". This can be done by the introduction of some suplementary, or correction factors, in (22) ÷ (27) formulas and the adaptation of the ratiocination tied to the mother-firm characteristics and support respectively.

So, a joint venture supposes the previous existance in the target country one of the same nature firm (support), eventually of a reduced economic "power" on which a new patrimonial entity is in common realised, a profitable one.

In this case the joint venture investment type can be modell under the form of epitaxial growth[1] of some monocrystalline thin films on a massive monocrystalline support as it is, for instance, the growth of a Si layer on a Si – support or Ge on Ge, Cu on Cu etc. (Fig. 5).

Such mono or policrystalline thin films, named epitaxial, one can be obtained by substance transport from the source *S* up to the support of the some nature like the deposed layer.

The transport of the gaseous substance can be done by dint of on inert gas or, easier, by deposition in vacuum precincts on supports (locations *j*) for which are valid the considerations and the precedent relations, especially (22) ÷ (27) relations.

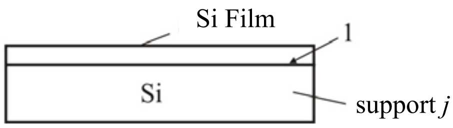


Figure 5. Grow of the epitaxial Si-film on the Si-support.

In figure 5, by 1 was marked the connection plane (joint venture) between the two regions (firms) that even have the same nature and crystalline structure etc., can have also some distinguishable characteristics, in the firm case, that from locality *j* (the film deposed on *j* support) may contain a more diversity of products of the firm *i*, as in the case of the semiconductor thin films that can have an other electronic conduction type,

[1] The epitaxy term is coming from the greek words „επι" meaning „on" and „ταξις" meaning deposing on the same nature support, and was proposed for the denomination of the oriented crystal growts – one over the other, process.

or other characteristics or optical or mechanical properties etc. slightly different from those of the mother-firm or of the main firm (source *i*) [1].

5. Conclusions

In this paper it have been applied some principles and econophysics methods to model some direct foreign investments categories specially for greenfield and joint venture investments types. For that, have been used some similarities and analogies between the mentioned investments types and some processes or phenomena from physics, specially from thermo-dynamics and the solid state physics, with special reference to the crystalline or policrystalline substances growth, by their deposition under vacuum as thin films (layers).

In the paper it is shown that the thickness (as well as the volume) and the thin film deposed mass by vacuum thermal evaporation is determined by the evaporation velocity, i.e. by the evaporator temperature (source), by the mass M_i and by the nature (density ρ) of the evaporating substance, as well as by the distance *h* between the source and the support on which the solid layer m_j is deposed.

By dint of the proposed econophysic model it is established that between the obtaining of the thin films process and the realisation and analysis mechanisms of the direct foreign investments, may be identified more similarities or parallelisms (analogies), the examination of these being able to lead to the establisment or foundation of some specific criterions or realisation condition of some direct foreign investments types.

So, the mother-firm of economico-financial capacity M_i can be assimilated with the “source” of M_i mass from which is evaporating the substance that is constitute of the capital fluxes (similar to the molecular-beam from the solide substances evaporated in vacuum), directed towards the various locations *j* that can be assimilated with the support or supports of m_j, $j = 1, 2 \ldots p$ situated at various locations (countries) at “economic” distances h_j with respect to the source.

In the paper is shown that the mass m_j of the deposed layer, assimilated with the power or economico-financial capacity of the realised investment into the country j is proportional with the mass M_j of source firm. The transfer velocity (similar to the evaporation velocity *w*) of the capital flux from the source M_i to the location *j* depends directly on the economico-financial power M_j of the mother-firm and from the square of the "economic" distance h from the source to the investments location. In other words, the more far is the respective location, the larger must be the volume or the size of capital fluxes devilered by the mother-firm (investmental source) especially for the greenfiled investments types.

For the joint ventures investments type the grow process of thin epitaxial films model can be used, i.e. of a layer realisation (the investments from the location j) on a support of the **same nature** with that of the deposed material delivered (supplied) by the mother-firm that constitutes the 'evaporation" source.

The theoretical considerations regarding the value (size) of the capital fluxes from the source to the locations j are the same to those above mentioned for the greenfield investments type.

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ANALYSIS OF EOQ MODEL UNDER IMPERFECT COMPETITION

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Abstract. *This paper deals with and an important aspect of econo-operations research in which a critical economic analysis is carried out to the inventory model of operations research. Dynamics of market economy is closely influenced not only by the price elasticity of demand but also by the nature of the market structure. The market structures include perfect competitions, imperfect competitions, oligopoly and monopoly etc. In this paper, an attempt has been made to analyze economic order quantity under the market structure of imperfect competition and different analyses have been presented under the specific imperfect structures. Finally, numerical computing of the model has also been added to elucidate the use of the model.*

1. Introduction

The problem of price determination for an EOQ model under imperfect competition is of central importance in the field of inventory control and management, especially such kind of models which study the dynamics of the market economy over the time. Since perfect competition is an ideal situation of market structure and generally is not found in real life. For this reason, imperfect market structure imperatively needs to be investigated to meet the demands of real existing marketing system, for example, vide Robinson [8] and Chamberlin [2] Imperfect structure is a glaring reality in the economy of marketing system and is unanimously described by eminent economists with following features:

- This is the structure where marginal revenue does not equal to price of an item.
- Demand is imperfectly elastic.
- There should not be large number of buyers and retailers in the market for that particular item.
- Firms are not free from joining and escaping from the market.

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- There should not be full knowledge of marketing by buyers and retailers.
- Market should not be near to the production system.
- Production system should not be fully dynamic.
- Only one producer or only one seller of product should be in the market.
- There are no close substitutes of product present in the market.
- More Constraints and many difficulties should be present in the entrance of industry.

There are some particular type of imperfect competitions such as monopolistic competition in which price is higher as compared with that of perfect competition. Monopoly is said to exist when there is only one producer or seller of a product which has no close substitutes or competitive. But when there exists some producers or sellers of same product in the market, this market structure is known as oligopoly. For example, there is only one company manufacturing the toothpaste of Binaca in India but it is not called monopoly because other substitutes are available in the market like Colgate, Pepsodent and Forhans etc. An imperfectly competitive firm cannot sell as much as it wants at the going price. It must recognize that its demand curve slopes down and that its output price will depend upon the quantity of the goods produced and sold. An oligopoly is a structure with only a few producers, each recognizing that its own price depends not merely on its own output but also on the actions of its important competitors in the industry. An industry with monopolistic competition has many sellers producing products that are closed substitutes for one another. Each firm has only limited ability to affect its output price which is briefly presented as

	Perfect Competition	Imperfect Competition		
Characteristic		**Monopolistic**	**Oligopoly**	**Monopoly**
No. of firms	Many	Many	Few	One
Ability to affect price	None	Limited	Some	Considerable
Entry barriers	None	None	Some	Complete
Example	Fruit stalls in Covent Garden	Corner grocer	Cars	De Beers

There are few researchers such as Bonanno and Giacomo [1] analyzed general equilibrium condition under imperfect competition and Hommes and Cars [3] gave the case of the cobweb and illustrated adaptive learning and roads to chaos. Goeree et.al [4] explained about the heterogeneous beliefs and gave new concept about cobweb in the non-linear form of price. Hommes [5] focused on the dynamics of the cobweb model is considered under adaptive expectations and nonlinear supply and demand. Hommes [6] again worked on cobweb dynamics under bounded rationality. Hommes et.al [7] discussed the expectation driven price volatility in an experimental economy under cobweb phenomenon. Maskin et.al [9] defined a theory of oligopoly considering its dynamic nature. Mishra and Mishra [10] emphasized on the price for an EOQ for deteriorating items with the dependency of stock as well as number of selling points in the market under perfect competition. Mishra et.al [11] elaborated a fuzzified deteriorating inventory model with breakdown of machine and shortage cost. Peter [12] performed the testing for an unstable root in conditional and structural error correction in his model.

Steven [13] provided a deep insight of monopolistic competition with outside goods. Schinkel et.al [14] presented exhaustive investigations on imperfect competition laws. Srinivasan et.al [15] in their work suggested the applications of equilibria as well as price normalization in International trade under imperfect market structure. Sonnemans et.al [16] worked on the instability of a heterogeneous cobweb economy in which a strategy experiment on expectation formation. Tuins [17] gave brief concept of oligopoly dynamics with their models and tools and further Baumol [18] also emphasized on the operation analysis and its economic theory. In this paper, an attempt has been made to determine the EOQ under imperfect competition and its specific cases such as cobweb phenomenon, price and production monopoly, Lerner's measure of monopoly power and wage determination under monopsony have been analyzed. Marginal revenue and marginal cost along with price elasticity approach have been employed to determine the EOQ and total optimal cost of the model. Numerical computing and its graphical representation have been presented to gain the deeper perspective of the model. The present work is presumably believed to provide better insight to the inventory managers and economists engaged in this field.

2. Description of model

The theory of imperfect competition as monopoly envisages a large number of quite small firms so that each firm can neglect the possibility that its own decisions provoke any adjustment in another firm's behavior. We also assume free entry and exit from the industry in the long run. In these respects the framework resembles our earlier discussion of perfect competition. In monopolistic competition the long-run tangency equilibrium occurs where each firm's demand curve is tangent to its average cost curve at out-put level at which MC equals to MR. Each firm is maximizing profits but just breaking even. There is no further entry or exit. Imperfect competition refers to all situations in which individual firms believe that they face downward-sloping demand curves. The most important forms of imperfect competition are monopolistic competition, oligopoly and pure monopoly. Monopolistic competitors face free entry and exit to the industry, but are individually small and make similar though not identical products. Each has limited monopoly power in its special brand. In long-run equilibrium, price equals average cost but exceeds marginal revenue and marginal cost at the tangency equilibrium. Oligopolists face a tension between collusion to maximize joint profits and competition for a large share of smaller joint profits. Collusion may be formal, as in a cartel, or informal. Without creditable threats of punishment by other collusive partners, each firm is likely to face a considerable temptation to cheat. One of the most important characteristics of demand function is what is known as its elasticity. According to the law of demand, the change in price and demand are in opposite direction and it is a common experience that price changes affect the demand for different commodities in different degrees. In other words, demand for some commodities is more sensitive to price changes than is the demand for others. For example, demand for necessities decreases very little when their prices rise whereas only and slight increase in the price is known as price elasticity of demand. Price elasticity of demand is defined as the value of the ratio of the relative (or proportionate) change in the demand to the relative (or proportionate) change in the price. Mathematically, we define it as:

$$e_d = \frac{-P_t}{Q_{d_t}}\left(\frac{dQ_{dt}}{dP_t}\right);$$

where, negative sign shows that demand and price move in opposite direction.

Followings are important characteristics which are noteworthy in the context under consideration.

- Price elasticity of demand is always positive.
- Demand is sometimes called over elastic or under elastic according as $e > 1$ or $e < 1$ (If $e = 1$, elasticity is called normal).
- Demand for necessities or conventional necessities are inelastic or less elastic while for luxuries it is elastic.
- Demand for goods having substitutes is elastic.
- Demand for goods having several uses is elastic.
- Goods for which demand can be postponed have elastic demand.

- Demand for jointly demanded goods is comparatively less elastic.
- Elasticity of demand varies with changes in income.
- Elasticity of demand depends on levels of prices.

3. Mathematical Analysis

As we know that in case of instantaneous replenishment of inventory $Q_{d_t} = S_t$.

In this model, it is assumed that $Q_{d_t} = bP_t + a$ and supply at that time $S_t = gP_{t-1} + h.$

From above, it is clear that demand at time t is fulfilled by the supply which is decided under the view of previous price. Hence,

$$bP_t + a = gP_{t-1} + h$$
$$bP_t = gP_{t-1} + (h-a);$$
$$P_t = \frac{g}{b} P_{t-1} + \frac{(h-a)}{b}.$$

After putting $t = 1$ in above, we get:

$$P_1 = \left(\frac{g}{b}\right) P_0 + \frac{(h-a)}{(g-b)} \frac{(g-b)}{b}.$$

Upon taking iteration for $t = 2$:

$$P_2 = \left(\frac{g}{b}\right) P_1 + \frac{(h-a)}{b}$$

$$P_2 = \left(\frac{g}{b}\right)\left\{\left(\frac{g}{b}\right) P_0 + \frac{(h-a)}{(g-b)}\left(\frac{g-b}{b}\right)\right\} + \left(\frac{(h-a)}{(g-b)}\right)\left(\frac{(g-b)}{b}\right)$$

$$P_2 = \left(\frac{g}{b}\right)^2 P_0 + \left(\frac{g}{b}\right)\left(\frac{(h-a)}{(g-b)}\right)\left(\frac{g-b}{b}\right) + \left(\frac{(h-a)}{(g-b)}\right)\left(\frac{(g-b)}{b}\right)$$

For, $t = 3$:

$$P_3 = \left(\frac{g}{b}\right) P_2 + \left(\frac{(h-a)}{b}\right);$$

$$P_3 = \left(\frac{g}{b}\right)\left\{\left(\frac{g}{b}\right)^2 P_0 + \left(\frac{g}{b}\right)\frac{(h-a)}{(g-b)}\frac{(g-b)}{b} + \frac{(h-a)}{(g-b)}\frac{(g-b)}{b}\right\} + \frac{(h-a)}{(g-b)}\frac{(g-b)}{b}$$

$$P_2=\left(\frac{g}{b}\right)^3 P_0+\left(\frac{g}{b}\right)^2\frac{(h-a)}{(g-b)}\frac{(g-b)}{b}+$$

$$+\left(\frac{g}{b}\right)\left(\frac{(h-a)}{(g-b)}\frac{(g-b)}{b}\right)+\frac{(h-a)}{(g-b)}\frac{(g-b)}{b}.$$

Upon taking iteration till t, we get:

$$P_t=\left(\frac{g}{b}\right)^t P_0+\left(\frac{(h-a)}{(g-b)}\right)\left(\frac{(g-b)}{b}\right)\left\{\left(\frac{g}{b}\right)^{t-1}+\left(\frac{g}{b}\right)^{t-2}+\left(\frac{g}{b}\right)^{t-3}+\ldots\ldots\ldots+1\right\}.$$

Let $(g/b)<1$:

$$P_t=\left(\frac{g}{b}\right)^t P_0+\left(\frac{(h-a)}{(g-b)}\right)\left(\frac{(g-b)}{b}\right)\left\{\frac{1-\left(\frac{g}{b}\right)^t}{1-\left(\frac{g}{b}\right)}\right\}$$

$$P_t=\left(\frac{g}{b}\right)^t P_0-\left(\frac{(h-a)}{(g-b)}\right)\left(1-\left(\frac{g}{b}\right)^t\right)$$

$$P_t=\left(\frac{g}{b}\right)^t\left\{P_0+\left(\frac{(h-a)}{(g-b)}\right)\right\}-\left(\frac{(h-a)}{(g-b)}\right)$$

$$Q_{d_t}=b\,P_t+a=b\left[\left(\frac{g}{b}\right)^t\left\{P_0+\frac{(h-a)}{(g-b)}\right\}-\frac{(h-a)}{(g-b)}\right]+a$$

$$S_t=g\,P_{t-1}+h=g\left[\left(\frac{g}{b}\right)^{t-1}\left\{P_0+\frac{(h-a)}{(g-b)}\right\}-\frac{(h-a)}{(g-b)}\right]+h$$

$$\frac{dQ_{d_t}}{dt}=b\left\{P_0+\frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^t\log\left(\frac{g}{b}\right).$$

Since, under cobweb phenomena, demanded quantity is decreasing function with respect to time. So, its first derivative with respect to time will be negative:

$$\frac{dQ}{dt}=-b\left\{P_0+\frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^t\log\left(\frac{g}{b}\right);$$

$$\frac{dP_t}{dt}=\left\{P_0+\frac{(h-a)}{(g-b)}\right\}\left(\left(\frac{g}{b}\right)^t\log\left(\frac{g}{b}\right)\right).$$

Since, under cobweb phenomena, price of item is increasing function with respect to time. So, its first derivative with respect to time will be positive as given above:

$$\frac{dQ_{d_t}}{dP_t} = \frac{dQ_{d_t}}{dt} \times \frac{dt}{dP_t} =$$

$$= \frac{-b\left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^t \log\left(\frac{g}{b}\right)}{\left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\left(\frac{g}{b}\right)^t \log\left(\frac{g}{b}\right)\right)} = -b; \quad \frac{dQ_{d_t}}{dP_t} = -b\,.$$

We can express the following as:

$$\frac{1}{e_d} = -\frac{Q_{d_t}}{P_t}\frac{dP_t}{dQ_{d_t}} = -\frac{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right] + a}{\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right]} \times \frac{-1}{b}$$

$$e_d = \left\{1 + \frac{a}{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right]}\right\}.$$

It is obvious to write that:

$$MR = \frac{\partial TR}{\partial Q_{d_t}} = \frac{\partial(P_t \times Q_{d_t})}{\partial Q_{d_t}} = P_t + Q_{d_t}\frac{\partial P_t}{\partial Q_{d_t}}. \tag{1}$$

Total turn over or total outlay for the commodity is given as:

$$F(P_t) = P_t \times Q_{d_t} = TR\,;$$

$$\partial TR = P_t\, \partial Q_{d_t} + Q_{d_t}\, \partial P_t.$$

Since in case of monopoly ∂P is negative, ∂Q is positive, it is clear from eqn (1) marginal revenue is less than the price of an item.

From equation (2), we get:

$$\frac{\partial TR}{\partial P_t} = P_t\frac{\partial Q_{d_t}}{\partial P_t} + Q_{d_t}\,;$$

$$\frac{\partial TR}{\partial P_t} = Q_{d_t}\left[\frac{P_t\,\partial Q_{d_t}}{Q_{d_t}\partial P_t} + 1\right] =$$

$$= Q_{d_t}\left[1 - e_d\right] = Q_{d_t}\left[1 - \frac{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right]}{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right] + a}\right]$$

$$\frac{\partial TR}{\partial P_t} = Q_{d_t}\frac{a}{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right] + a} = a;$$

$\frac{\partial TR}{\partial P_t} = a = \frac{\partial F(P_t)}{\partial P_t}$; If $a > 0$ then $e < 1$ and $\frac{\partial F(P_t)}{\partial P_t} > 0.$ $F(P_t)$ is increase-ing function of P_t; i.e., the money value of the turnover increases as the price rises.

But if $a < 0$ then $e > 1$ and $\frac{\partial F(P_t)}{\partial P_t} < 0 \Rightarrow F(P_T)$ is decreasing function of P_t, i.e. the money value of the turnover decreases as the price rises.

Further, we can express as:

$$MR = \frac{\partial TR}{\partial Q_{d_t}} = \frac{\partial (P_t \times Q_{d_t})}{\partial Q_{d_t}} = P_t + Q_{d_t}\frac{\partial P_t}{\partial Q_{d_t}}.$$

Price elasticity of demand:

$$e_d = -\frac{P_t dQ_{d_t}}{Q_{d_t} dP_t}.$$

Since $\frac{\partial Q_{d_t}}{\partial P_t} < 0$; in case of monopoly, we have:

$$MR = P_t\left(1 - \frac{1}{e_{d_t}}\right);$$

$$MR = P_t\left(1 - \frac{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right] + a}{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right]}\right);$$

$$MR = \left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right] \times$$

$$\times\left[\frac{-a}{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]}\right] MR = \left(\frac{-a}{b}\right).$$

Moreover, we can have:

$$S_t = gP_{t-1} + h = g\left\{\left(\frac{g}{b}\right)^{t-1}\left(P_0 + \frac{(h-a)}{(g-b)}\right) - \left(\frac{(h-a)}{(g-h)}\right)\right\} + h;$$

$$\frac{\partial S_t}{\partial t} = g\left\{\left(P_0 + \frac{(h-a)}{(g-b)}\right)\right\}\left(\frac{g}{b}\right)^{t-1}\log\left(\frac{g}{b}\right).$$

Since, under cobweb phenomena, supply is increasing function with respect to time. So, its first derivative with respect to time will be positive as shown above. We further have:

$$\frac{\partial P_{t-1}}{\partial t} = \left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^{t-1}\log\left(\frac{g}{b}\right)$$

$$\frac{\partial S_t}{\partial P_{t-1}} = \frac{\partial S_t}{\partial t} \times \frac{\partial t}{\partial P_{t-1}} = \frac{g\left\{\left(P_0 + \frac{(h-a)}{(g-b)}\right)\right\}\left(\frac{g}{b}\right)^{t-1}\log\left(\frac{g}{b}\right)}{\left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^{t-1}\log\left(\frac{g}{b}\right)} = g.$$

Hence, price elasticity of supply can be given as:

$$\frac{P_{t-1}}{S_t}\frac{\partial S_t}{\partial P_{t-1}} = \frac{\left(\frac{g}{b}\right)^{t-1}\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)}{g\left\{\left(\frac{g}{b}\right)^{t-1}\left(P_0 + \frac{(h-a)}{(g-b)}\right) - \left(\frac{(h-a)}{(g-h)}\right)\right\} + h} \times g$$

$$e_s = \frac{\left(\frac{g}{b}\right)^{t-1}\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)}{\left\{\left(\frac{g}{b}\right)^{t-1}\left(P_0 + \frac{(h-a)}{(g-b)}\right) - \left(\frac{(h-a)}{(g-h)}\right)\right\} + \left(\frac{h}{g}\right)}.$$

4. Inventory model under imperfect competition with cobweb phenomenon

In this case, supply and demand at any time t are equal, quantity of supply at time t is decided at time $t-1$ which is based on the price at time $t-1$.

Let $Q_{d_t}, Q_{d_{t+1}}, Q_{d_{t+2}}, ...Q_{d_{t+n}}$ are the demands at time instants, t, $t+1$, $t+2$,..... $t+n$ respectively, quantity demanded at time t is equal to quantity supplied at that time and thus different demands at different times are fulfilled.

Total demand in T time horizon is given as:

$$D = \sum_{i=0}^{n} Q_{d_{t+i}} = \sum\nolimits_{i=0}^{n} S_{t+i} = \sum_{i=0}^{n} g\left\{\left(\frac{g}{b}\right)^{t+i-1}\left(P_0 + \frac{(h-a)}{(g-b)}\right) - \left(\frac{(h-a)}{(g-h)}\right)\right\} + h.$$

Given time horizon, we can find the following costs as:

If q is an EOQ then

Carrying cost = $\frac{C_s qT}{2}$; Ordering cost = $C_0\left(\frac{D}{q}\right)$;

Total Cost = Carrying cost + Ordering cost +Raw Material cost;

$$\text{TVC} = \frac{C_s qT}{2} + \frac{C_0 D}{q} + C_r q;$$

Total fixed cost is sum of setup cost, Labor cost etc.

$$TFC = C_{stp}T + C_w T \ ; \ TC = \frac{C_s qT}{2} + \frac{C_0 D}{q} + C_{stp}T + C_r q + C_w T;$$

From profit optimization, we get:

$$\frac{\partial TC}{\partial q} = MC = MR.$$

This implies that:

$$\frac{C_s T}{2} - \frac{C_0 D}{q^2} + C_r = \left(\frac{-a}{b}\right);$$

where $a < 0$

$$\frac{C_0 D}{q^2} = \frac{C_s T}{2} + C_r + \left(\frac{a}{b}\right).$$

This gives us:

$$q = \sqrt{\frac{C_0 D}{\frac{C_s T}{2} + C_r + \frac{a}{b}}}, \quad \text{and} \quad q = \sqrt{\frac{2C_0 D\, b}{bC_s T + 2bC_r + 2a}}.$$

This can be finally written as:

$$q = \sqrt{\frac{2C_0 D\, b}{bC_s T + 2bC_r + 2a}}.$$

Hence, optimal quantity (q^*) is given as:

$$q^* = \sqrt{\frac{2C_0 b \sum_{i=0}^{n} Q_{d_{t+i}}}{bC_s T + 2bC_r + 2a}}$$

$$q^* = \sqrt{\frac{2bC_0 \left[\sum_{i=0}^{n} g \left\{ P_0 + \frac{(h-a)}{(b-g)} \right\} \left(\frac{g}{b} \right)^{t+i-1} - \sum_{i=0}^{n} g \frac{(h-a)}{(b-g)} + \sum_{i=0}^{n} h \right]}{bC_s T + 2bC_r + 2a}}$$

$$EOQ = \sqrt{\frac{2bC_0 \left[g \left(P_0 + \frac{(h-a)}{(b-g)} \right) \sum_{i=0}^{n} \left(\frac{g}{b} \right)^{t+i-1} - \frac{gn(h-a)}{(b-g)} + hn \right]}{bC_s T + 2bC_0 + 2a}}$$

$$EOQ =$$

$$= \sqrt{\frac{2bC_0 \left[\left\{ g \left(P_0 + \frac{(h-a)}{(b-g)} \right) \left(\frac{g}{b} \right)^{t-1} \left(1 + \left(\frac{g}{b} \right) + \left(\frac{g}{b} \right)^2 + \ldots\ldots + \left(\frac{g}{b} \right)^n \right) \right\} - \frac{g\, n\, (h-a)}{(b-g)} + hn \right]}{bC_s T + 2bC_r + 2a}}$$

$$EOQ = \sqrt{\frac{2bC_0\left[g\left\{P_0 + \frac{(h-a)}{(b-g)}\right\}\left(\frac{g}{b}\right)^{t-1}\left(\frac{1-\left(\frac{g}{b}\right)^{n+1}}{1-\left(\frac{g}{b}\right)}\right) - \frac{g\,n(h-a)}{(b-g)} + hn\right]}{bC_sT + 2bC_r + 2a}};$$

$$EOQ = \sqrt{\frac{2bC_0\left[g\left\{P_0 + \frac{(h-a)}{(b-g)}\right\}\left(\frac{g}{b}\right)^{t-1}\left(\frac{b^{n+1}-g^{n+1}}{b-g}\right)\left(\frac{1}{b^n}\right) - \frac{gn(h-a)}{(b-g)} + hn\right]}{bC_sT + 2bC_r + 2a}};$$

$$EOQ = \sqrt{\frac{2bC_0[g^t\left(P_0(b-g)(b^{n+1}-g^{n+1})\right) + g^t(b^{n+1}-g^{n+1})(h-a) -}{b^{n+t-1}(b-g)^2 \times}}$$

$$\frac{-gnb^{t+n-1}(h-a)(b-g) + hnb^{n+t-1}(b-g)^2]}{\times(bC_sT + 2C_rb + 2a)}$$

$$EOQ = \sqrt{\frac{2C_0[g^t(b^{n+1}-g^{n+1})\{P_0(b-g) + (h-a)\} -}{b^{n+t-2}(b-g)^2 \times}}$$

$$\frac{-gnb^{t+n-1}(h-a)(b-g) + hnb^{n+t-1}(b-g)^2]}{\times(bC_sT + 2C_rb + 2a)}.$$

Hence, total optimal variance and fixed costs are:

$$TVC^* = \frac{C_s q^*{}_t T}{2} + \frac{C_0 D}{q_t^*} + C_r q_t^*$$

$$TFC = TFC^* = C_{stp}T + C_wT.$$

Finally, total optimal cost is given as:

$$TC^* = \frac{C_s q_t^* T}{2} + \frac{C_0 q_t^* T}{2} + \frac{C_0 D}{q_t^*} + C_r q_t^* + C_{stp}T + C_wT.$$

5. Lerner's Measure of Monopoly Power

We define the Index of Monopoly Power as:

$$IMP = \left(\frac{P_t - MC}{P_t}\right);\ 0 \le IMP \le 1.$$

IMP shows the power of monopolistic firm at time *t*. IMP is nothing but it is inverse of price elasticity. So, it is defined as below:

$$IMP = \frac{\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)} - \left(\frac{-a}{b}\right)}{\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]} =$$

$$= \frac{\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)} + \left(\frac{a}{b}\right)}{\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]}.$$

This obviously falls in between o and 1 and which is given as:

$$0 \le \frac{\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)} + \left(\frac{a}{b}\right)}{\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]} \le 1.$$

This further implies that:

$$\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)} + \left(\frac{a}{b}\right) \le$$

$$\le \left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right].$$

And hence $(a/b) \le 0$ and $a \le 0$ or $b \le 0$.

It shows that monopoly of firm exists under the above condition and in another condition when $a \ge 0$ the marginal revenue will become negative so it will be not suitable for monopoly of the firm.

6. Determination of wage rate under imperfect competition

When imperfect competition in product market and perfect competition in Labor Market are satisfied then:

$MRP = MPP \times MR$
$VMP = MPP \times$ Price of an object.

Since:

$MR <$ Price of an object.

$MRP < VMP$

Under conditions of monopoly or imperfect competition in product market, assuming perfect competition in the labor market, the labor will get wage less than the value of its marginal product, we find that:

$$C_w = MRP_L \quad or \quad C_w = MR \times MPP_L \quad and \; C_w = \left(1 - \frac{1}{e_d}\right) P_t \times MPP_L .$$

Since $P_t \times MPP_L = VMP_L$, we have:

$$C_w = \left(1 - \frac{1}{|\,e_d|}\right) VMP_L = \frac{-aVMP_L}{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]}$$

But whenever in product market, there exists an imperfect competition or monopoly then the price elasticity is less than one, so in this condition $\left(1 - \frac{1}{e_d}\right) < 1$; hence, $C_w < VMP$.

Thus,

$$TFC^* = C_{stp}T + C_wT$$

$$TFC^* = C_{stp}T + \left(\frac{-aVMP_L}{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]} \right)$$

$$TC^* = \frac{C_s q^*{}_t T}{2} + \frac{C_0 D}{q_t^*} + C_r q^*{}_t + C_{stp}T \;\; \text{or}$$

$$TC^* = \frac{C_s q_t^* T}{2} + \frac{C_0 D}{q^*{}_t} + C_r q_t^* + C_{stp}T .$$

Finally:

$$TC^* = \frac{-aVMP_L T}{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]} .$$

7. Wage determination under monopsony

Monopsony means the monopoly to hire the labor or only one individual employer or firm to hire the labor. At the equilibrium state, it is important to know that MFC is equal to both *VMP* or *MRP* and thus
$TFC = C_w T$.

8. Numerical computing and graphical representation of the model

With the help of hypothetical data and using simple computer programming in C++, we present the following numerical computations in the form of tables and corresponding graphs are also given in Anexes (vide the tables and graphs at last).

9. Conclusion

We conclude that whole paper has been focused on the various aspects of the imperfect competition under the cobweb nature of the production and its consumption.

Important results have been drawn by using optimization technique of marginal revenue and marginal cost. These include EOQ of inventory item at time instant *t*, price elasticity of demand, supply elasticity of demand, Lerner's measure of monopoly power, determination of wage rate under imperfect competition and wage determination under monopsony. These results are very useful instruments in controlling the inventory and its management.

Tables and Graphs

Table 1

Computation of TOC and EOQ

For given parameters P_0= Rs.50, n = 8, a = – 50, b = 100, g = 200, h = 100, $C_s = Rs.20$, Co= Rs.25, $C_{stp} = Rs.10$, T = 4 year, Cr = Rs.20 VMP = Rs.20;

t	P_t	e_d	EOQ	C_w	TOC (in10^8)
4	822.5	1.0006	4081.98	0.012158	3.29488
3	410.5	1.0012	2886.51	0.024361	1.64545
2	204.5	1.0025	2041.23	0.048900	8.20439
1	101.5	1.0045	1443.6	0.098522	4.07725
0.5	71.33	1.00706	1214.08	0.14019	0.286779

Table 2

Computation of TOC, EOQ and MR

For given parameters P_0 = Rs.50, t = 3, n = 8, b = 100, g = 200, h = 100, Ch = Rs.20, Co = Rs.25, Cstp = Rs.10, T = 4 year, Cr = Rs.20 VMP = Rs.20

a	P_t	e_d	EOQ	C_w	TOC(in10^8)	e_s (in10^{-5})	MR
–60	411.2	1.0015	2885.97	0.0291829	1.9772	2.48069	0.6
–80	412.6	1.0019	2884.89	0.0387785	2.64408	2.47836	0.8
–100	414	1.0024	2883.79	0.0483092	3.3154	2.47605	1
–150	417.5	1.0036	2881.03	0.0718563	5.01341	2.47036	1.5
–200	421	1.0047	2878.22	0.0950119	6.7394	2.46477	2

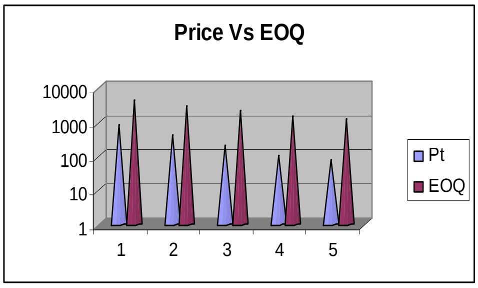
Table 3

Computation of EOQ, Cw, MR and TOC

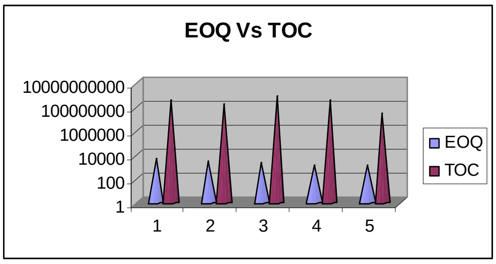
For given parameters a = – 200, t = 3, n = 8, b = 100, g = 200, h = 100, Ch = Rs.20, Co = Rs.25, $C_{stp} = Rs.10$, T = 4 year, Cr = Rs.20, VMP = Rs.20;

P_0	P_t	e_d	EOQ	C_w	e_s (in10^{-5})	MR	TOC (in10^9)
100	821	1.00244	4134.55	0.0487211	2.4187	2	1.31409
150	1221	1.00164	5089.68	0.03276	2.4878	2	1.5420
200	1621	1.00124	5891.96	0.0246761	2.4908	2	2.5943
250	2021	1.00099	6597.39	0.0197922	2.49262	2	3.2343
300	2421	1.00083	7234.35	0.016522	2.49384	2	3.8744

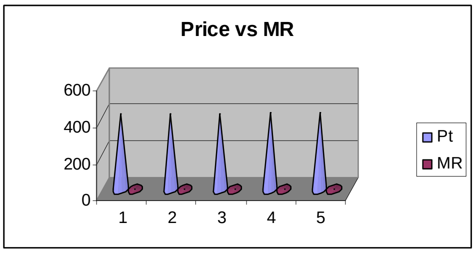
Graph 1.1



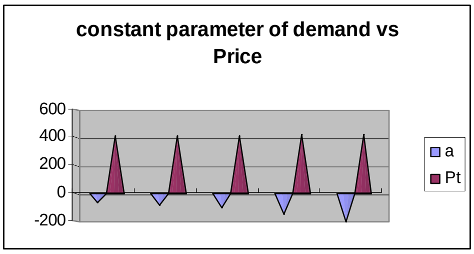
Graphs 1.2



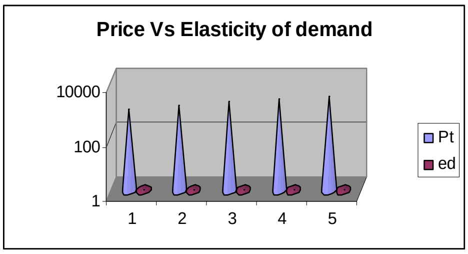
Graph 2.1



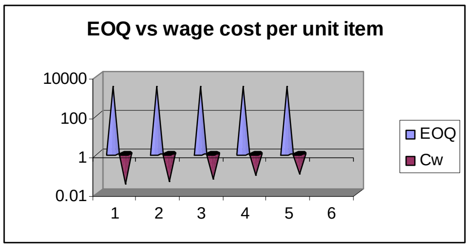
Graph 2.2



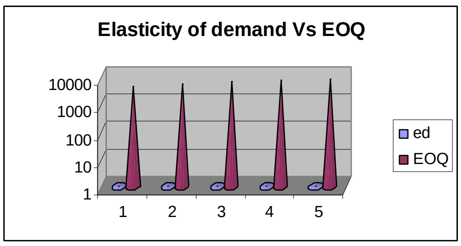
Graph 2.3



Graph 3.1



Graph 3.2



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UNDERSTANDING ECONOPHYSICS: A COMMENT ON PLATEN'S PORTFOLIO SELECTION AND ASSET PRICING UNDER A BENCH APPROACH

Carmen COSTEA*

Abstract. *The paper below is presenting an overview on estimations of risk/return correlation during the process of finding efficient portfolios after E. Platen point of view. If CAPM is provided as a good tool in measuring portfolio risk, any investor is expected to taking some of the risk. According to Markowitz a portfolio always expects both return and risk; these are related to the expected return of the securities, and even more – individual components and correlations. This approach identifies complicated feature in risk, giving a lot of trouble, when processing. When a risk factor is missing from an asset pricing model, the resulting miss-pricing is embedded within the residual covariance matrix. This phenomenon may lead to expected return estimates that are more stable and precise than estimates delivered by standard methods. But portfolio selection may be improved and the last years various methodologies have been provided the scientific literature with models that asses the prices and this is an important goal in Platen's work.*

Keywords: risk, portfolio selection, asset pricing

1. Introduction

Sharpe W.F. was a Ph. D. student at University of California when he found out "Portfolio Selection" [H. Markowitz's, 1952] that oriented him towards the "frontier of optimal investment", developed later, at Markowitz's suggestion, in his future PhD thesis investigating Portfolio Theory. From this research, [Sharpe 1964] independently developed an iconoclastic notion of investment risk finalizing with the Capital Asset Pricing Model, shortly named the CAPM. According to CAPM theory, every investment carries two different risks. The first one is the risk of being in the market – called the systematic risk or beta and it cannot be diversified away. The second one, the unsystematic risk, is specific to a company's fortunes.

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The CAPM helps measure portfolio risk and the return an investor can expect for taking that risk. Markowitz made a step further, saying that a portfolio always expects return and risk. Expected return is related to the expected return of the securities, but risk is more complicated. Risk is related to the risks of the individual components as well as the correlations. That makes risk a complicated feature giving a lot of trouble, when processing. Using a computer, we can estimate risk/return correlation and find efficient portfolios. This way, we can "get more return for a given risk and less risk for a given return", according to Markowitz.

When a risk factor is missing from an asset pricing model, the resulting miss-pricing is embedded within the residual covariance matrix. This phenomenon may lead to expected return estimates that are more stable and precise than estimates delivered by standard methods.

But portfolio selection may be improved. At an extreme, optimal portfolio weights are proportional to expected returns when no factors are observable. Such portfolios perform well in simulations and in out-of-sample comparisons. During the last years various methodologies have been provided in the scientific literature with models that asses the prices.

The CAPM has been settled as a theory of equilibrium. The CAPM comes out of two things: Markowitz, who showed how to create an efficient frontier, and James Tobin, who said that "if you hold risky securities and are able to borrow – buying stocks on margin – or lend – buying risk-free assets – and you do so at the same rate, then the efficient frontier is a single portfolio of risky securities plus borrowing and lending, and that dominates any other combination [James Tobin, 1959].

Both the CAPM and index funds come from that. The basic philosophical results carry through in the more complex settings, although the results aren't quite as simple.

The Arbitrage Pricing Theory (APT) assumes that relatively few factors generate correlation, and says the expected return on a security or an asset class ought to be a function of its exposure to those factors. The APT goes farther and makes strong assumptions about the return-generating process. It has also a limit because it doesn't tell too much about the expected return on those factors.

2. Supplementary Analysis (or Consideration)

The CAPM and its extended versions offer some notion of how people with preferences determine prices in the market. The CAPM tells you more. Whatever number of factors there may be, the expected return of a security will be related to its exposure to those factors.

Under this exposure, Portfolio Selection and Asset Pricing under a Bench Approach written by E. Platen is a paper about analysis and modeling. It is known that using this term often makes people uncomfortable. People often conjure up images of the genius wearing glasses inputting oodles of data into arcane computer programs while simultaneously sifting through reams of paper containing differential equations and advanced statistics. This image does not meet the reality of the present scientists who know that analysis does not have to be complicated or convoluted to be valuable. Despite the great steps made in the last years in terms of developing intelligent models or projects and collecting data, it can not be said the same for analysis.

The paper "Portfolio Selection and Asset pricing under a Bench Approach" develops a model for financial market relying on the description of a benchmarked portfolio, namely the growth optimal portfolio. In this paper, Platen provides a model with a certain[1] form of arbitrage in order to obtain the key features of the CAPM and APT without standard assumption required to ensure the existence of an equivalent martingale measurement.

Trying to describe the complete market case, Platen's paper (a) demonstrates that a multi- asset market composed by $d + 1$ primary security accounts including saving accounts – locally risk-less – includes also d nonnegative risky primary security account processes and (b) derives the relation existing between volatilities and risk premia.

A farther advantage of the benchmarking theory is that the derivative prices and different risk management targets – value at risk, portfolio optimization or estimation – can be performed under the real world measure. The benchmark can be extended to assets price dynamics, modeled semi-martingales incorporated both predictable and inaccessible jumps.

J. A. Yan [1998] presented Kramkov and Schachermayer's results, on optimal investment giving a review of utility-based approach to contingent claim pricing in incomplete markets. They considered a security market model in which the uncertainty and information structure are described by a stochastic basis (F, P; (Ft)) satisfying the usual conditions with F_0 being trivial (where P is the original or objective probability) that models the "real world" probability.

Starting with the concept of a growth optimal portfolio (GOP) previously developed by Kelly [1956] and later by Long [(1990], Artzner [1997]), Bajeux-Besnainou and Portait [1997], Karatzas and Shreve [1998], J. A. Yan [1998], Platen [2001, 2002] developed his own theory in

[1] Paper was presented by E. Platen at the Econophysics Colloquium, Canberra, November 2005

good papers – written alone or together with Heath [2005] – stimulating interest in reading and developing the same research.

The seven theorems presented in the above mentioned paper underline the super-martingale property of benchmarked nonnegative portfolios. For a given asset its traded price is in practice a result of a process that marches offer and demand. The difference between a benchmark traded price and any corresponding expected future benchmarked value of this traded asset is nonnegative and represents an arbitrage amount, if it's not zero. The benchmarked pricing methodology doesn't require the existence of an equivalent martingale measure. The benchmark methodology facilitate the modeling of some form of arbitrage possibly existing in the emerging and maturing markets, subjects to shocks and unexpected turbulences.

To respect the laws of conservation of value, Platen [2001] assumes that all portfolios are self-financing. Changes in the value of the portfolio are exactly matched by the corresponding gains from trade.

These days, a rich and various literature bring important data regarding no-arbitrage: Harrison and Kreps [1979], Folner and Sondermann [1986], Delbaen and Schachermayer [1994], Karatsas and Shreve [1998].

Considering benchmarked portfolios and not domestic account discounted portfolios, Platen correctly underlines that a nonnegative benchmarked portfolio process (V_δ) corresponds to a self-financing strategy (δ) which is (S) integrable and arbitrage free in the standard sense and if (V_δ) is super-martingale. Being a model for the financial markets, based on the different denominations of the growth optimal portfolio – where the risk premia for premium assets does not depend on the denomination – the arbitrage amount can be expressed as they occur after shocks or market turbulence because all benchmarked portfolio processes are arbitrage free. Therefore the APT can also be applied.

Kelly discovered the GOP in 1956; it has been the unifying object in the benchmark approach. Later, from 1959 until these days, GOP has been studied and applied it in portfolio optimization and derivative pricing, fascinating lots of scientists like Latane [1959], Breiman [1960], Markowitz [1976] and Long [1990], Hakansson and Ziemba [1995], Platen [2001, 2005]. In his papers, Platen presents, without any particular model assumption, that the MP approximates the GOP if the MP is a diversified portfolio and the continuous market has certain regularity. Thus, diversification and various forms of portfolio optimization lead to complementing results. Once the observable MP established as GOP satisfying the SDE one can now ask, a usual question is rising and concerns the natural dynamics of the GOP.

Historically, the volatility has been chosen as parameter process to model asset prices. The discounted GOP drift has been suggested as an alternative more economically based parameter process. This quantity reflects the underlying economic value that flows into the market per unit of time. By modeling an exponential function of time with constant net growth rate, the minimal market model (MMM) is obtained, as described in Platen's papers [2001, 2002]. Under the MMM the GOP volatility arises. The MMM predicts for log-returns distribution with four degrees of freedom (statisticcally confirmed by Fergusson and Platen in 2005 when studying the log-returns of a world stock index in different currencies.

3. Conclusion

The model allows explaining empirical facts stylized by its probabilistic properties and explains representative shape of the concerned volatilities of index and rate options obtained under MMM. These typical dynamics will be soon able to explain further events and phenomena, helping business analysts and decision makers to draw effective conclusions from limited data and put together information that does not yet fit at first glance. Then readers will benefit themselves and hope will become comfortable with this template.

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LINEAR FRACTIONAL MODELING OF TIME SERIES

Branko DRAGOVICH*, Dusan JOKSIMOVIC** and Jelena VELIMIROVIC***

Abstract: *In this paper we analyze possibility to use linear fractional recurrences as a new mathematical physics approach to the problem of modeling and forecasting time series. In particular, we examine their forecast ability for financial time series. To this end, we investigate how it works on different financial data, does it also have adequate forecast ability in situations when markets are not very stable, so that data vary a lot, and what error do we get in that case. The aim is also to surpass mathematical problems that may occur during the modeling. We try to find the strategy that leads to enhancement of forecast accuracy by analyzing space of parameters. The performance of forecast depends on the forecast horizon required, so we try to improve short-term forecast accuracy. Idea is to make an assessment of the suitability of the assumed mathematical model for a given data set by varying number of parameters. We also examine the behavior of the model by analyzing errors applied on the theoretically generated and real life time series, and especially questioning whether it is appropriate for financial time series. Obtained results are encouraging and further improvements are possible.*

Keywords: financial time series, modeling, forecasting, linear fractional recurrences

1. Introduction

1.1. Time series

A time series is a sequence of data with a certain chronological ordering. It can be also understood as a realisation of a random process.

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Namely, let $\{X_t, t \in R\}$ be a given random process whose realisation in certain moments is:

$$t = 1, 2, \ldots, n \qquad X_1, X_2, \ldots X_n$$

$$t = 1, 2, \ldots \qquad X_1, X_2, \ldots$$

$$t = 0, \pm 1, \pm 2, \ldots \qquad \ldots X_{-2}, X_{-1}, X_0, X_1, X_2, \ldots$$

Each of these strings is a time series. So in this sense, a time series is a string (finite or infinite) of random variables.

One of the most important tasks when studying time series is to predict their future values, both in the near and distant future. Due to a widespread and always topical need and call for the answer to such a question different methods have been developed which, depending on the nature of a time series, are all trying to produce the best possible answer to the following question: what is the value of a time series in a certain moment in the future [1]?

1.2. Linear Fractional Transformations

A linear fractional transformation (LFT) is defined as a function of the form:

$$f(z) = \frac{\alpha_0 + \alpha_1 \cdot z}{\beta_0 + \beta_1 \cdot z},$$

where z, $\alpha_0, \alpha_1, \beta_0, \beta_1$ are complex numbers satisfying $\alpha_0 \cdot \beta_1 - \alpha_1 \cdot \beta_0 \neq 0$.

This kind of mapping is also called homographic transformation or Möbius transformation. LFT defines one-to-one mapping of the extended complex plane ($C \cup \infty$) onto itself. The set of all LFT transformations forms a group under composition called the Möbius group. Various linear fractional transformations and the corresponding groups play significant role in pure and applied mathematics.

1.3. Linear Fractional Recurrences

Let us define a linear fractional recurrence relation of the first order (LFR1) as:

$$z_k = \frac{\alpha_0 + \alpha_1 \cdot z_{k-1}}{\beta_0 + \beta_1 \cdot z_{k-1}},$$

where $z_k, z_{k-1}, \alpha_0, \alpha_1, \beta_0, \beta_1$ are complex numbers satisfying $\alpha_0 \cdot \beta_1 - \alpha_1 \cdot \beta_0 \neq 0$.

Let a linear fractional recurrence relation of q-th order (LFRq) $[2]$ be:

$$z_k = \frac{\alpha_0 + \alpha_1 \cdot z_{k-q} + \alpha_2 \cdot z_{k-q+1} + \cdots + \alpha_q \cdot z_{k-1}}{\beta_0 + \beta_1 \cdot z_{k-q} + \beta_2 \cdot z_{k-q+1} + \cdots + \beta_q \cdot z_{k-1}} =$$

$$= \frac{\alpha_0 + \sum_{i=1}^{q} \alpha_i \cdot z_{k-q+i-1}}{\beta_0 + \sum_{i=1}^{q} \beta_i \cdot z_{k-q+i-1}},$$

where $z_{k-i}, \alpha_i, \beta_i,\ (i = 0, 1, \ldots, q)$ are complex numbers. In the sequel of this paper we will use rational numbers.

2. **Modeling short term forecasting time series**

2.1. Model with linear fractional recurrence relation of q-th order (LFRq)

For the given time series $x_1, x_2, \ldots, x_n$, to make the analysis simpler, we take $n = i + q$.

The predicted value x_{n+1}, i.e. x_{i+q+1}, applying the method of linear fractional recurrence relation of q-th order, is:

$$x_{i+q+1} = \frac{\alpha_0 + \alpha_1 \cdot x_{i+1} + \alpha_2 \cdot x_{i+2} + \cdots + \alpha_q \cdot x_{i+q}}{\beta_0 + \beta_1 \cdot x_{i+1} + \beta_2 \cdot x_{i+2} + \cdots + \beta_q \cdot x_{i+q}} =$$

$$= \frac{\alpha_0 + \sum_{k=1}^{q} \alpha_k \cdot x_{i+k}}{\beta_0 + \sum_{k=1}^{q} \beta_k \cdot x_{i+k}}.$$

In this model we have $2 \cdot (q+1)$ unknown parameters (α_i, β_i), where $(i = 0, 1, \ldots, q)$. Now the question is: what is the best way to determine these parameters? Here the best is in the sense of the most accurate prediction of the value x_{i+q+1}.

The natural way to find these parameters is from the condition that linear fractional recurrence relation of q-th order is correct for $2 \cdot (q+1)$ previous values of time series from the close past, i.e. for values

$x_{i-(q+1)}, x_{i-(q+1)+1}, \ldots, x_{i+q}$. In other words, that following equations are true (for $i \geq 2\cdot(q+1)$:

$$x_{i-(q+1)} = \frac{\alpha_0 + \sum_{k=1}^{q}\alpha_k \cdot x_{i-2\cdot(q+1)+k}}{\beta_0 + \sum_{k=1}^{q}\beta_k \cdot x_{i-2\cdot(q+1)+k}}$$

$$x_{i-(q+1)+1} = \frac{\alpha_0 + \sum_{k=1}^{q}\alpha_k \cdot x_{i-2\cdot(q+1)+k+1}}{\beta_0 + \sum_{k=1}^{q}\beta_k \cdot x_{i-2\cdot(q+1)+k+1}}$$

$$\vdots$$

$$x_{i-(q+1)+2\cdot q+1} = \frac{\alpha_0 + \sum_{k=1}^{q}\alpha_k \cdot x_{i-2\cdot(q+1)+k+2\cdot q+1}}{\beta_0 + \sum_{k=1}^{q}\beta_k \cdot x_{i-2\cdot(q+1)+k+2\cdot q+1}}$$

From these conditions we get homogeneous system of $2\cdot(q+1)$ linear equations with $2\cdot(q+1)$ unknown parameters (α_i, β_i), where $(i = 0, 1, \ldots, q)$:

$$\alpha_0 + \sum_{k=1}^{q}\alpha_k \cdot x_{i-2\cdot(q+1)+k} - x_{i-(q+1)} \cdot \left(\beta_0 + \sum_{k=1}^{q}\beta_k \cdot x_{i-2\cdot(q+1)+k})\right) = 0$$

$$\alpha_0 + \sum_{k=1}^{q}\alpha_k \cdot x_{i-2\cdot(q+1)+k+1} - x_{i-(q+1)+1} \cdot \left(\beta_0 + \sum_{k=1}^{q}\beta_k \cdot x_{i-2\cdot(q+1)+k+1}\right) = 0$$

$$\vdots$$

$$\alpha_0 + \sum_{k=1}^{q}\alpha_k \cdot x_{i-2\cdot(q+1)+k+2\cdot q+1} - x_{i-(q+1)+2\cdot q+1} \cdot \left(\beta_0 + \sum_{k=1}^{q}\beta_k \cdot x_{i-2\cdot(q+1)+k+2\cdot q+1}\right) = 0$$

This system has trivial solution $\alpha_i = \beta_i = 0$, which is useless. Of course, there is a possibility that this system, except trivial, sometimes has also an infinite number of solutions (α_i, β_i), where $(i = 0, 1, \ldots, q)$, but it depends on the values of time series, so we cannot expect, in advance, that this system has another solution besides trivial one. It is interesting to explore what characteristics of time series insures the solutions that are not trivial.

For all the above mentioned, we propose the modification of the model, in the sense that we in advance define value of the parameter $\alpha_0 = A \neq 0$. We call this special linear fractional recurrence relation of q-th order LFRq $(\alpha_0 = A)$.

2.2. Forecasting with a special linear fractional recurrence relation of q-th order ($LFRq$ $(\alpha_0 = A)$)

The predicted value of time series x_{n+1}, i.e. x_{i+q+1}, applying the method of special linear fractional recurrence relation of q-th order (LFRq $(\alpha_0 = A)$), is:

$$x_{i+q+1} = \frac{A + \alpha_1 \cdot x_{i+1} + \alpha_2 \cdot x_{i+2} + \cdots + \alpha_q \cdot x_{i+q}}{\beta_0 + \beta_1 \cdot x_{i+1} + \beta_2 \cdot x_{i+2} + \cdots + \beta_q \cdot x_{i+q}} =$$

$$= \frac{A + \sum_{k=1}^{q} \alpha_k \cdot x_{i+k}}{\beta_0 + \sum_{k=1}^{q} \beta_k \cdot x_{i+k}}. \qquad (2.2.1)$$

In this model we have $2 \cdot q + 1$ unknown parameters $(\beta_0, \alpha_i, \beta_i)$, where $(i = 1, \ldots, q)$, and we find them from the condition that modified linear fractional recurrence relation of q-th order is correct for $2 \cdot q + 1$ previouse values of time series from the close past, i.e. for the values $x_{i-(q+1)+1}, x_{i-(q+1)+2}, \ldots, x_{i+q}$. In other words, that following equations are true (for $i \geq 2 \cdot q + 1$):

$$x_{i-(q+1)+1} = \frac{A + \sum_{k=1}^{q} \alpha_k \cdot x_{i-2\cdot(q+1)+k+1}}{\beta_0 + \sum_{k=1}^{q} \beta_k \cdot x_{i-2\cdot(q+1)+k+1}}$$

$$x_{i-(q+1)+2} = \frac{A + \sum_{k=1}^{q} \alpha_k \cdot x_{i-2\cdot(q+1)+k+2}}{\beta_0 + \sum_{k=1}^{q} \beta_k \cdot x_{i-2\cdot(q+1)+k+2}}$$

$$\vdots$$

$$\vdots$$

$$x_{i-(q+1)+2\cdot q+1} = \frac{A + \sum_{k=1}^{q} \alpha_k \cdot x_{i-2\cdot(q+1)+k+2\cdot q+1}}{\beta_0 + \sum_{k=1}^{q} \beta_k \cdot x_{i-2\cdot(q+1)+k+2\cdot q+1}}.$$

From these conditions we get the following system of $2\cdot q+1$ linear equations with $2\cdot q+1$ unknown parameters $(\beta_0, \alpha_i, \beta_i)$, where $(i=1,\ldots,q)$:

$$\begin{aligned}
&\sum_{k=1}^{q} \alpha_k \cdot x_{i-2\cdot(q+1)+k+1} - x_{i-(q+1)+1} \cdot \left(\beta_0 + \sum_{k=1}^{q} \beta_k \cdot x_{i-2\cdot(q+1)+k+1})\right) = -A \\
&\sum_{k=1}^{q} \alpha_k \cdot x_{i-2\cdot(q+1)+k+2} - x_{i-(q+1)+2} \cdot \left(\beta_0 + \sum_{k=1}^{q} \beta_k \cdot x_{i-2\cdot(q+1)+k+2}\right) = -A \\
&\vdots \\
&\sum_{k=1}^{q} \alpha_k \cdot x_{i-2\cdot(q+1)+k+2\cdot q+1} - x_{i-(q+1)+2\cdot q+1} \cdot \left(\beta_0 + \sum_{k=1}^{q} \beta_k \cdot x_{i-2\cdot(q+1)+k+2\cdot q+1}\right) = -A
\end{aligned} \qquad (2.2.2)$$

This system has unique solution when the determinant of the system is different from zero, which has place practically always. However, if the determinant of the system is equal to zero, one can do some small modifications in values of time series, which will not affect time series but insure unique solution.

We tested this model on the data from the past (historical simulation) for $\alpha_0 = A = 1$ and $q = 1, 2, 3, 4$.

3. Test of the method and data description

We used the real time series for the forecasting with special linear fractional recurrence relation of q-th order (LFRq$(\alpha_0 = A)$), for $A=1$ and $q=1,2,3,4$. We programmed modules in the softver Matlab, which made the predictions using the data from the past, based on equations (2.2.1) and (2.2.2) for $A=1$ and $q=1, 2, 3, 4$.

We compared the forecasted values with original data from the past, calculated absolute and relative errors of deviation, and in that way analyzed the characteristics of suggested model.

All daily data are retrieved from the site www.euronext.com. For the lack of the space, in the section below we show only the results for the Belgium market, where we used stock price index BEL20, closing price over the period from 1.11.2006. till 30.11.2006.

We also tested the model for each of the 20 companies separetely from which BEL20 index is composed, using the values of their stocks (for example, Belgacom, Ackermans & van Haaren, Mobistar, Omega Pharma, etc.). Model was also applied on the other markets, like French and German market, where we used daily data of CAC40 and DAX30 indices, and values of stocks of their companies.

4. Estimation of results

4.1. Estimation of results for forecasting with LFR1 $(\alpha_0 = 1)$

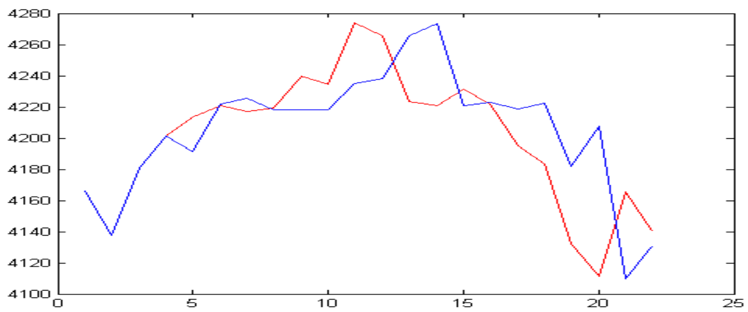
The predicted values from the past x'_{i+q+1}, we get using the formula (see (2.2.1)):

$$x'_{i+1+1} = \frac{1 + \alpha_1 \cdot x_{i+1}}{\beta_0 + \beta_1 \cdot x_{i+1}} \qquad \text{for} \qquad i = 3, 4, \dots n-2,$$

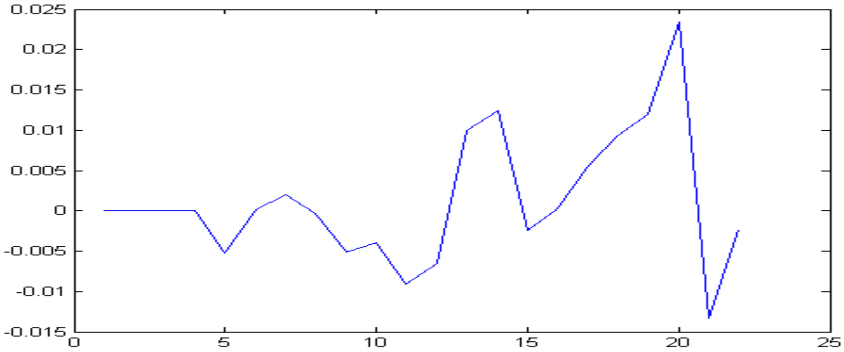
where we get $\alpha_1, \beta_0, \beta_1$ from the (2.2.2).

We then compare predicted data x'_{i+1+1} with the original ones x_{i+1+1}, calculate absolute error, $abs.err. = x'_{i+1+1} - x_{i+1+1}$, and relative error, $rel.err. = \dfrac{x'_{1+1+1} - x_{i+1+1}}{x_{i+1+1}}$, for $i = 3, 4, \dots n-2$.

Graphs of predicted and original data, and relative error, for **LFR1** $(\alpha_0 = 1)$ are shown below (Graph 1 and Graph 2):



Graph 1. November – real and prognostic process **LFR1** $(\alpha_0 = 1)$.
(red line – original data, blue line – prognostic data).



Graph 2. Relative error **LFR1** $(\alpha_0 = 1)$.

4.2. *Estimation of results for forecasting with LFR2* $(\alpha_0 = 1)$

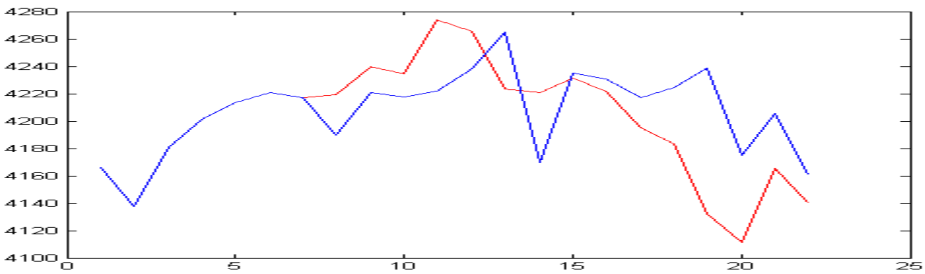
The predicted values from the past x'_{i+q+1}, we get using the formula (see (2.2.1.)):

$$x'_{i+2+1} = \frac{1 + \alpha_1 \cdot x_{i+1} + \alpha_2 \cdot x_{i+2}}{\beta_0 + \beta_1 \cdot x_{i+1} + \beta_2 \cdot x_{i+2}} \qquad \text{for} \qquad i = 5, 6, \ldots n-3,$$

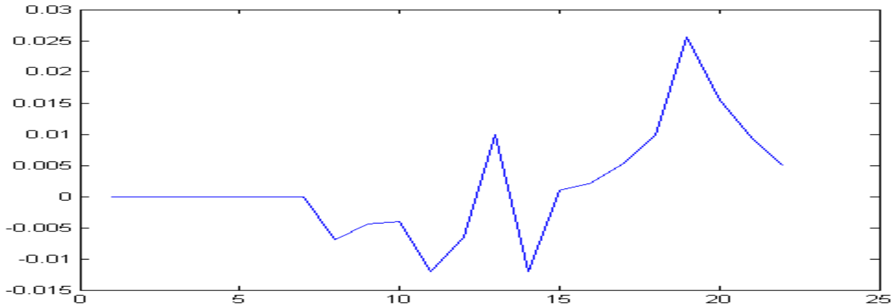
where we get $\alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2$ from the (2.2.2.).

We then compare predicted data x'_{i+2+1}, with the original data x_{i+2+1}, calculate absolute error, $abs.err. = x'_{i+2+1} - x_{i+2+1}$, and relative error, $rel.err. = \dfrac{x'_{i+2+1} - x_{i+2+1}}{x_{i+2+1}}$, for $i = 5, 6, \ldots n-3$.

Graphs of predicted and original data, and relative error, for **LFR2** $(\alpha_0 = 1)$ are shown below (Graph 3 and Graph 4):



Graph 3. November – real and prognostic process **LFR2** $(\alpha_0 = 1)$
(red line – original data, blue line – prognostic data).



Graph 4. Relative error **LFR2** ($\alpha_0 = 1$).

4.3. *Estimation of results for forecasting with LFR3* $(\alpha_0 = 1)$

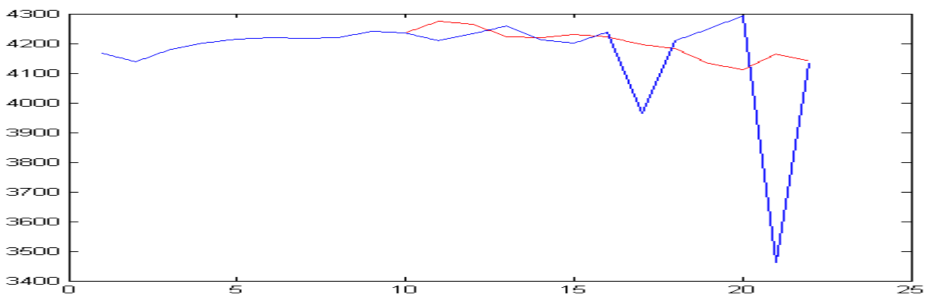
The predicted values from the past x'_{i+q+1}, we get using the formula (see (2.2.1.)):

$$x'_{i+3+1} = \frac{1 + \alpha_1 \cdot x_{i+1} + \alpha_2 \cdot x_{i+2} + \alpha_3 \cdot x_{i+3}}{\beta_0 + \beta_1 \cdot x_{i+1} + \beta_2 \cdot x_{i+2} + \beta_3 \cdot x_{i+3}} \quad \text{for} \quad i = 7, 8, \ldots n-4,$$

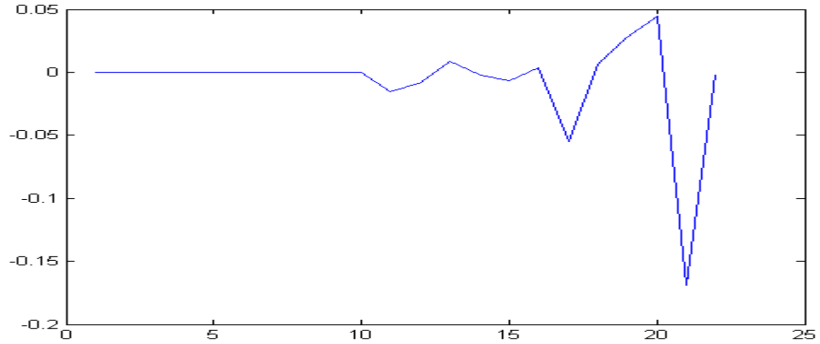
where we have got $\alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2, \beta_3$ from the (2.2.2).

Then we compare predicted data x'_{i+3+1} with the original data x_{i+3+1}, calculate absolute error, $abs.err. = (x'_{i+3+1} - x_{i+3+1})$, and relative error, $rel.err. = \frac{x'_{i+3+1} - x_{i+3+1}}{x_{i+3+1}}$, for $i = 7, 8, \ldots n-4$.

Graphs of predicted and original data, and relative error, for **LFR3** $(\alpha_0 = 1)$ are shown below (Graph 5 and Graph 6):



Graph 5. November – real and prognostic process **LFR3** ($\alpha_0 = 1$)
(red line – original data, blue line – prognostic data).



Graph 6. Relative error **LFR3** $(\alpha_0 = 1)$.

4.4. Estimation of results for forecasting with LFR4 $(\alpha_0 = 1)$

The predicted values from the past x'_{i+q+1}, we get using the formula (see (2.2.1)):

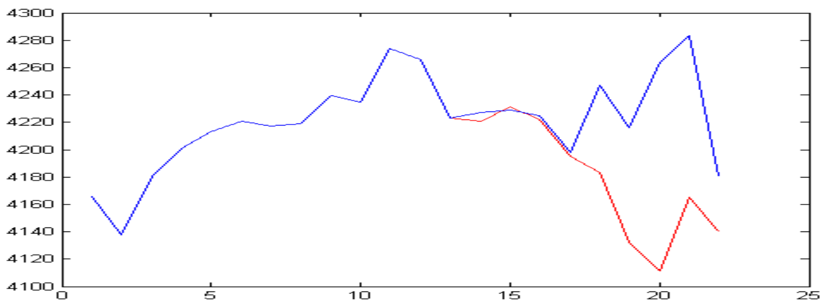
$$x'_{i+4+1} = \frac{1 + \alpha_1 \cdot x_{i+1} + \alpha_2 \cdot x_{i+2} + \alpha_3 \cdot x_{i+3} + \alpha_4 \cdot x_{i+4}}{\beta_0 + \beta_1 \cdot x_{i+1} + \beta_2 \cdot x_{i+2} + \beta_3 \cdot x_{i+3} + \beta_4 \cdot x_{i+4}}$$

$$\text{for} \qquad i = 9, 10, \ldots n-5,$$

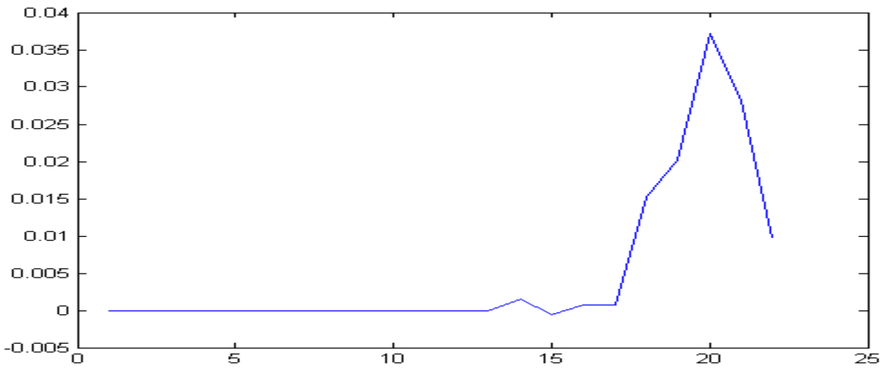
where we obtained $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ from the (2.2.2).

We also compare predicted data x'_{i+4+1} with the original ones x_{i+4+1}, calculate absolute error, $abs.err. = (x'_{i+4+1} - x_{i+4+1})$, and relative error, $rel.err. = \dfrac{x'_{i+4+1} - x_{i+4+1}}{x_{i+4+1}}$, for $i = 9, 10, \ldots n-5$.

Graphs of predicted and original data, and relative error, for **LFR4** $(\alpha_0 = 1)$ are shown below (Graph 7 and Graph 8):



Graph 7. November – real and prognostic process **LFR4** $(\alpha_0 = 1)$
(red line – original data, blue line – prognostic data).



Graph 8. Relative error **LFR4** (α_0 = 1).

5. Conclusion

Analyzing the obtained results we can make the following conclusions.

➢ For the tested time series which is very volatile, better prediction is achieved for smaller number of parameters q, what is expected, because in the very volatile time series, the value of time series in some point is more dependent on its value from the very close past. For the case when $q = 1$, we observe time series as the Markov process.

➢ Relative error is less than 5%, and that is encouraging preliminary result.

➢ Maximal value of the absolute error is comparable with standard deviation of successive changes in values of time series, and that is also encouraging preliminary result.

➢ Trend of the forecasted time series follows the trend of the original time series, and one gets impression that it is shifted in time, which shows us a direction for the further analysis.

➢ In the process of solving the system (2.2.2), sometimes the determinant of the system is very weakly conditioned, so we get some abnormal peaks as a consequence of mentioned weakly conditioned determinant and rounding off the values of coefficients at four decimals. Solution of these situations is another task which is ahead us.

These results are optimistic and show directions of further research and improvements of the proposed model for forecasting time series.

According to the above analysis we suggest the following natural directions of further investigations.

❖ Variation of the values of parameter *A*.

❖ Application of the proposed model for calculating the trend of time series when it is decomposed to its basic components. We expect that it will bring good results for large *q*.

❖ Try to modify the model in such way that some other parameter in the former model be a constant.

It is worth noting that in this model we used rational numbers. To have more insight into different aspects of the model, which parameters are rational numbers, one can treat it not only from the usual real point of view but also using *p*-adic numbers. On possible uses of *p*-adic numbers in econometrics one can see [3]. *p*-Adic and adelic analysis of some dynamical systems based on linear fractional transformations is presented in [4] and [5]. We also plan to extend the present time series model by its *p*-adic and adelic counterparts.

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TARAFDAR TYPE GENERAL EQUILIBRIUM

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Abstract. *In this paper, we prove a maximal element theorem as a conesquence of an equilibrium theorem formulated in the line of the Tarafdar type results. Such theorems give sufficient conditions for the existence of an equilibrium point (and of a maximal element) for a generalized abstract economy (respectively for a qualitative game).*
For the proof of our general equilibrium theorems, we can use some results of R. M. Dăneț (200) concerning the existence of a fixed point for a family of multimaps. Note that such results can be also applied to obtain various general equilibrium theorems, for example in the forms of Nash type, Yannelis-Prabhakar type and Lin-Yu-Ansari-Lai type problems.

Keywords: fixed-point, multimaps, general equilibrium, maximal element, generalized abstract economy, qualitative game.

1. Introduction

Many results concerning the fixed point theory can be applied in the equilibrium theory, giving, for example, the existence of a solution for the equilibrium in abstract economies or generalized games and in generalized abstract economies with preference multimaps.

In 1975, W. Shafer and H. Sonnenschein proved the existence of equilibria for abstract economies without ordered preferences.

Over the last thirty years, more general existence results appeared in the literature. To make a partial list of these results, we mention: A. Borglin and H. Keiding (1976), D. Gale and A. Mas Colell (1978), N. C. Yannelis (1987), C. Ionescu-Tulcea (1988), E. Tarafdar (1988), K. K. Tan and G. X.-Z. Yuan (1994).

All these results assume directly or indirectly the lower-semicontinuity of the multimaps representing the constraints of each agent.

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In 1999, G. X.-Z.Yuan and E. Tarafdar proved some existence theorems for equilibrium of compact or noncompact qualitative games and generalized games in which the constraint or preference multimaps have supplementary properties.

The existence of the equilibrium in an abstract economy with compact strategy sets in $\mathbb{R}^n$ was proved in a seminal paper of G. Debreu.

The theorem of Debreu extended the earlier work of J. Nash in game theory (Around 1950, J. Nash contributed several key concepts to game theory. When the 21-years old J. Nash wrote his 27-pages dissertation outlining his "Nash Equilibrium" for the strategic non-cooperative games, the impact was enormous. His existence proof is one of the first applications of Kakutani's fixed point theorem and gives him the Nobel Prize in Economics).

There exist many generalizations of Debreu's theorem, for example, by A. Borglin and H. Keiding (1976). Following their paper and, also, the paper of D. Gale and A. Mas-Colell (1978), on non-ordered preference relations, many theorems on the existence of maximal elements of preference relations, which may not be transitive or complete have been proved by T. C. Bergs (1976), M. Walker (1977), N. C. Yannelis and D. Prabhakar (1983), S. Toussaint (1984), N. C. Yannelis (1985), C. Ionescu-Tulcea (1988), G. Mehta (1990).

In this paper we give a maximal element theorem for a qualitative game, using a Tarafdar type equilibrium theorem for a generalized abstract economy.

For the proof of the existence of an equilibrium point, we can use some results (2006) of R.-M. Dăneț concerning the existence of a fixed point for a family of multimaps.

Now we recall **some definitions and notations**, reviewing the mathematical and economical concepts that we need.

For a nonempty set *X*, 2^X denotes the class of all subsets of *X*.

A *multimap* is a function $T : X \to 2^Y$; in another terminology, a multimap is also known as a *set valued function*, a *mapping*, a *map* or a *correspondence* (here *X* and *Y* are two nonempty sets).

A multimap arises naturally in many economic applications, for instance, *budget correspondence*, *excess demand correspondence* etc.).

The biggest difference between *functions* and *multimaps* has to do with the definition of an *inverse image*:

a) the *inverse image* of a set *A* under a function *f* is the set $\{x : f(x) \in A\}$;

b) for a multimap $T:X\to 2^Y$, the *inverse* of B by T ($B\subseteq Y$), is defined by:

$$T^{-1}(B)=\{x\in X:T(x)=B\}$$

But, there exist two reasonable generalizations: the *upper inverse* and *the lower inverse;*

– the *upper inverse* of B by T is the set $T^{+}(B)=\{x\in X:T(x)\subseteq B\}$;

– the *lower inverse* of B by T is the set $T^{-}(B)=\{x\in X:T(x)\cap B\neq\varnothing\}$.

Note that, in this paper we will use only the lower inverse of a set B by a multimap T, but we will write $T^{-1}(B)$ instead of $T^{-}(B)$. We will denote also $T^{-1}(y)$ instead of $T^{-1}(\{y\})$. Therefore, $x\in T^{-1}(y)$ if and only if $y\in T(x)$.

A multimap $T:X\to 2^Y$ is a *nonempty-valued* (*convex-valued,* or *compact-valued*) if the set $T(x)$ is a *nonempty* (respectively *convex,* or *compact*) for each $x\in X$.

The *fiber* of the multimap $T:X\to 2^Y$ at the point $y\in Y$, is the set $T^{-1}(y)$.

Let I be a *countable* or *uncountable* set of *agents* (or *players*). For each $i\in I$, suppose her/his *choice* or *strategy* set X_i is a nonempty subset of a topological vector space.

Let $X=\prod_{i\in I}X_i$. For each $i\in I$, let $P_i:X\to 2^{X_i}$ be a multimap. Following the notion of T. Gale and A. Mas-Colell, the collection $\Gamma=(X_i,P_i)_{i\in I}$ is called a *qualitative game*. An element $\tilde{x}\in X$ is said to be a *maximal point* of the game Γ, if $P_i(\tilde{x})=\varnothing$, for all $i\in I$.

A *generalized abstract economy* (or a *generalized game*) is a family of quadruples $\Gamma=(X_i,A_i,B_i,P_i)_{i\in I}$, where I is a (finite or infinite) set of agents (players) such that, for each $i\in I$, X_i is a nonempty subset of a topological vector space; $A_i,B_i:X=\prod_{i\in I}X_i\to 2^{X_i}$ are constraint multimaps and $P_i:X\to 2^{X_i}$ is a preference multimap. An *equilibrium* for Γ is a point $\tilde{x}=(\tilde{x}_i)_{i\in I}\in X$ such that, for each $i\in I$, $\tilde{x}_i\in B_i(\tilde{x})$ and $A_i(\tilde{x})\cap P_i(\tilde{x})=\varnothing$.

2. Preliminaries

In 2006, having as a starting point a result of Q. H. Ansari and J. C. Yao (see [AY]), R.-M. Dăneț proved a result (see Proposition 1 below) concerning the existence of a fixed point for a family of multimaps. This result was formulated (for example, in [DPV], Theorem 3.1) in the following framework.

Let I be an index set and for each $i \in I$, let E_i be a Hausdorff topological vector space.

Let $(X_i)_{i\in I}$ be a family of nonempty convex subsets with each X_i in E_i.

Let $X = \prod_{i\in I} X_i$. Let also $C \subseteq X$ be a nonempty compact subset.

In the result of Ansari and Yao (see[AY], Theorem 2) there are two families of multimaps $(S_i)_{i\in I}$ and $(T_i)_{i\in I}$, $S_i, T_i : X \to 2^{X_i}$, related by the condition $\mathrm{co} S_i(x) \subseteq T_i(x)$, for each $i \in I$ and $x \in X$. Also, let us observe that, imposing some conditions to the family $(S_i)_{i\in I}$, they obtained a common fixed point for the family $(T_i)_{i\in I}$.

The Ansari and Yao's result was generalized in a certain sense, in 2003, by L.-J. Lin, Z.-T. Yu, Q. H. Ansari and L. P. Lai (see[LYAL], Theorem 3.1).

Inspired by this last results, R.-M. Dăneț formulated another generalization of the Ansari and Yao's result, for a single family of multimaps which are nonempty-valued and convex-valued.

Proposition 1. For each $i \in I$, let $T_i : X \to 2^{X_i}$ be a nonempty-valued and convex-valued multimap (that is, for each $x \in X$, the set $T_i(x)$ is a nonempty convex subset of X_i). Suppose that the following conditions hold:

1) for each $i \in I$, X can be covered with the interiors of all fibers of T_i, i.e:

$$X = \bigcup \left\{ \mathrm{int}_X T_i^{-1}(y_i) : y_i \in X_i \right\};$$

2) if X is not compact, assume that for each $i \in I$ and for each finite subset F_i of X_i, there exists a nonempty compact convex set C_{F_i} in X_i such that $C_{F_i} \supseteq F_i$ and $X \setminus C$ can be covered with the interiors of all fibers of T_i at the points of C_{F_i}, i.e. $X \setminus C \subseteq \bigcup \left\{ \mathrm{int}_X T_i^{-1}(y_i) : y_i \in C_{F_i} \right\}$.

Then, there exists $\tilde{x} = (\tilde{x}_i)_{i\in I} \in X$, such that $\tilde{x}_i \in T_i(\tilde{x})$, for each $i \in I$ (i.e. $\tilde{x}$ is a fixed point for the family $(T_i)_{i\in I}$).

Let us remark that the proof of Proposition 1 (very technical but standard) follows the idea of the original theorem of Ansari and Yao, that is to use the partition of unity subordinated to a finite subcovering of a compact product space and to apply the Tychonoff's fixed point theorem ("If X is a compact set in a locally convex Hausdorff topological vector space and $h: X \to X$ is a continous function, then h has a fixed point"-see for example [6]).

The following collectively fixed point result (see, for example [DPV], Theorem 3.2) can be proved with understanding changes in the proof of the Proposition 1 and generalizes Theorem 1 of [AY].

Proposition 2. For each $i \in I$, let $S_i, T_i : X \to 2^{X_i}$ be two nonempty-valued multimaps, such that:

0) for each $i \in I$ and each $x \in X$, $\mathrm{co}S_i(x) \subseteq T_i(x)$;

1) for each $i \in I$, X can be covered with the interiors of all fibers of S_i, that is

$$X = \bigcup \left\{ \mathrm{int}_X S_i^{-1}(y_i) : y_i \in X_i \right\};$$

2) if X is not compact, assume that for each $i \in I$ and each finite set F_i of X_i, there exists a nonempty compact convex set C_{F_i} in X_i such that $C_{F_i} \supseteq F_i$ and $X \setminus C \subseteq \bigcup \left\{ \mathrm{int}_X S_i^{-1}(y_i) : y_i \in C_{F_i} \right\}$.

Then, there exists $\tilde{x} \in X$ such that $\tilde{x}_i \in T_i(\tilde{x})$, for each $i \in I$.

Remark. Obviously, according to the condition "0)", only S_i must be a non-empty valued multimap.

As a simple consequence of Proposition 2, we obtain the following result (see [AY], Theorem 1).

Corollary 3. For each $i \in I$, let $S_i, T_i : X \to 2^{X_i}$ be nonempty-valued multimaps, such that:

0) for each $i \in I$ and each $x \in X$, $\mathrm{co}S_i(x) \subseteq T_i(x)$;

1) for each $i \in I$, $X = \bigcup \left\{ \mathrm{int}_X S_i^{-1}(y_i) : y_i \in X_i \right\}$;

2) if X is not compact, assume that for each $i \in I$, there exists a nonempty compact convex subset C_i of X_i such that $X \setminus C \subseteq \bigcup \left\{ \mathrm{int}_X S_i^{-1}(y_i) : y_i \in C_i \right\}$.

Then, there exists $\tilde{x} \in X$ such that $\tilde{x}_i \in T_i(\tilde{x})$, for each $i \in I$.

Proof. For each $i \in I$ and each finite subset F_i of X_i, we define $C_{F_i} = \mathrm{co}(C_i \cup F_i)$. Therefore, it follows that $C_{F_i} \supseteq F_i$ and the set C_{F_i} is compact and convex. Now, we apply Proposition 2. □

3. Main Result

In this section, via an equilibrium result for a generalized abstract economy obtained as a consequence of Corollary 3, we give a *maximal element* theorem for a qualitative game.

Definition. A *generalized abstract economy* (a *generalized game*) $\Gamma=(X_i,A_i,B_i,P_i)_{i\in I}$ consists of: an index set *I* of agents (or players), a choice or strategy set X_i in a topological vector space E_i, two constraint multimaps (correspondences) $A_i,B_i:X=\prod_{i\in I}X_i\to 2^{X_i}$ and a preference multimap (correspondence) $P_i:X\to 2^{X_i}$.

Remark. The *index set I* is any set (countable or not) of agents (or players). The *choice set* (the *strategy set*) X_i is the nonempty set of actions available to the agent *i*. For each $x\in X$ and $i\in I$, $A_i(x)$ (respectively $B_i(x)$) is the state attainable for the agent *i*, at *x*, under the constraint A_i (respectively B_i) and $P_i(x)$ is the state preference by the agent *i*, at *x*.

Definition. A *qualitative game* is a family $\Gamma=(X_i,P_i)_{i\in I}$, where *I*, X_i and P_i are like as in the definition of a generalized abstract economy.

Definition. An *equilibrium point* $x=(x_i)_{i\in I}$ for a generalized abstract economy $\Gamma=(X_i,A_i,B_i,P_i)_{i\in I}$ is the one satisfying the conditions: $x_i\in B_i(x)$ and $A_i(x)\cap P_i(x)=\varnothing$, for each $i\in I$.

Definition. A *maximal point* of the qualitative game $\Gamma=(X_i,P_i)_{i\in I}$ is an element $x\in X$ such that $P_i(x)=\varnothing$, for each $i\in I$ ($P_i:X\to 2^{X_i}$).

Remark. Obviously $\varnothing$ is the minimal set in 2^{X_i} ordered by the inclusion. We call an element $x\in X$ with $P_i(x)=\varnothing$ a *"maximal" element* for P_i, and not a *"minimal" element*, because the multimap P_i is associated with a preference relation "$\succ$" defined on X (hence "$\succ$" is a strictly ordering on X) such that $P_i(x)=\{z_i\in X_i:z_i\succ x_i\}$, that is $P_i(x)$ is the upper contour set of x_i. If for each $i\in I$ does not exists $z_i\in X_i$ with $z_i\succ x_i$, that is such that $P_i(x)=\varnothing$, then *x* is a maximal element for "$\succ$".

The following result is an equilibrium theorem for a generalized abstract economy which can be proved using Corollary 3 (see [DP]). In this theorem, we consider K a nonempty compact subset of the product space $X=\prod_{i\in I} X_i$ ($K=\prod_{i\in I} K_i$, where, for each $i\in I$, K_i is a nonempty compact subset of X_i).

Theorem 4. Let $\Gamma=\left(X_i, A_i, B_i, P_i\right)_{i\in I}$ a generalized abstract economy (or a generalized game) such that:

(1) X_i is a nonempty convex subset of a Hausdorff topological vector space and K_i is a nonempty compact subset of X_i;

(2) for each $i\in I$ and $x\in X$, $\mathrm{co}A_i(x)\subseteq B_i(x)\subseteq K_i$;

(3) $\mathrm{co}K\subseteq\bigcup\left\{\mathrm{int}_X\left(A_i^{-1}(y_i)\cap\left(P_i^{-1}(y_i)\cup G_i\right)\right): y_i\in K_i\right\}$

where $G_i=\left\{x\in X: A_i(x)\cap P_i(x)=\varnothing\right\}$;

(4) for each $x=(x_i)_{i\in I}\in X$, $x_i\notin \mathrm{co}\,P_i(x)$.

Then Γ has an equilibrium point in $\mathrm{co}K$, that is there exists $\tilde{x}\in\mathrm{co}K$ such that for each $i\in I$, $\tilde{x}_i\in B_i(\tilde{x})$ and $A_i(\tilde{x})\cap P_i(\tilde{x})=\varnothing$ (hence $\tilde{x}\in G_i$).

Remark. For the proof of Theorem 4, for each $i\in I$, we consider the following sets and multimaps:

$$F_i=\left\{x\in X: A_i(x)\cap P_i(x)\neq\varnothing\right\}, \text{ and}$$

$S_i, T_i: X\to 2^{K_i}$ defined by:

$$S_i(x)=\begin{cases} A_i(x)\cap\mathrm{co}P_i(x), & x\in F_i\\ A_i(x) & , x\in G_i\end{cases};\quad T_i(x)=\begin{cases} B_i(x)\cap\mathrm{co}P_i(x), & x\in F_i\\ B_i(x) & , x\in G_i\end{cases}$$

Then, we can prove that the hypothesis of Corollary 3 are fulfilled.

Applying Corollary 3, we obtain the existence of an element $\tilde{x}\in\mathrm{co}K$ such that $\tilde{x}_i\in T_i(\tilde{x})$, for each $i\in I$, hence:

$$\tilde{x}\in F_i \text{ and } \tilde{x}_i\in B_i(\tilde{x})\cap\mathrm{co}P_i(\tilde{x})$$

or $\tilde{x}\in G_i$ and $\tilde{x}_i\in B_i(\tilde{x})$.

But from the hypothesis "(4)", $\tilde{x}_i\notin\mathrm{co}P_i(\tilde{x})$, for each $\tilde{x}=(\tilde{x}_i)_i\in X$. Then it follows that $\tilde{x}_i\in B_i(\tilde{x})$ and $\tilde{x}\in G_i$, that is $A_i(\tilde{x})\cap P_i(\tilde{x})=\varnothing$. □

The following theorem is the main result of this paper. It is an equilibrium theorem for a qualitative game and results as a consequence of our Theorem 4.

Theorem 5. Let $\Gamma = (X_i, P_i)_{i\in I}$ a qualitative game and K a nonempty compact convex set in $X = \prod_{i\in I} X_i$ such that for each $i \in I$:

1) X_i is a nonempty convex subset of a Hausdorff topological vector space and $K = \prod_{i\in I} K_i$, where K_i is a nonempty compact convex subset of X_i;

2) $K \subseteq \bigcup \left\{ \mathrm{int}_X \left(P_i^{-1}(y_i) \cup G_i \right) : y_i \in K_i \right\}$, where $G_i = \{ x \in X : P_i(x) = \varnothing \}$;

3) for each $x = (x_i)_{i\in I} \in X$, $x_i \notin \mathrm{co} P_i(x)$.

Then, Γ has a maximal element in K (an equilibrium point), that is, there exists $\tilde{x} \in K$ such that $P_i(\tilde{x}) = \varnothing$, for each $i \in I$.

Proof. For each $i \in I$, we define the constraint multimaps $A_i, B_i : X \to 2^{X_i}$ by $A_i(x) = B_i(x) = K_i$, for all $x \in X$.

Therefore, for each $i \in I$ and $y_i \in K_i$, $A_i^{-1}(y_i) = X$ (for each $x \in X$, $x \in A_i^{-1}(y_i)$ because $y_i \in A_i(x) = K_i$).

Therefore, the conditions "(2)" and "(3)" of Theorem 4 are fulfilled. (Indeed, for example, "2)" of Theorem 5 implies "(2)" of Theorem 4, because

$$K \subseteq \left\{ \mathrm{int}_X \left(P_i^{-1}(y_i) \cup G_i \right) : y_i \in K_i \right\} =$$

$$= \left\{ \mathrm{int}\left(X \cap \left(P_i^{-1}(y_i) \cup G_i \right) \right) : y_i \in K_i \right\} = \left\{ \mathrm{int}\left(A_i^{-1}(y_i) \cap \left(P_i^{-1}(y_i) \cup G_i \right) \right) : y_i \in K_i \right\}.$$

We can apply Theorem 4, finding $\tilde{x} \in K$ such that for each $i \in I$, $\tilde{x}_i \in B_i(\tilde{x}) = K_i$ (obviously, because $\tilde{x} \in K$) and $K_i \cap P_i(\tilde{x}) = A_i(\tilde{x}) \cap$ $\cap P_i(\tilde{x}) = \varnothing$.

But $P_i : K \to 2^{X_i}$. If $\tilde{x} \in K$ is such that $P_i(\tilde{x}) \not\subseteq K_i$, then $\tilde{x} \notin P_i^{-1}(y_i)$, for each $y_i \in K_i$ and according the hypothesis "2)", it follows that $P_i(\tilde{x}) = \varnothing$.

If $\tilde{x} \in K$ is such that $P_i(\tilde{x}) \subseteq K_i$, then $\varnothing = K_i \cap P_i(\tilde{x}) = P_i(\tilde{x})$.

Hence, certainly, $P_i(\tilde{x}) = \varnothing$. Therefore, $\tilde{x} \in K$ is a maximal point of the game $(X_i, P_i)_{i\in I}$. □

4. Conclusions

The main result of this paper, shows us that a qualitative game has a maximal element, in some conditions, formulated using the concepts of “convexity” and “compacity” – see for example [2]. These to concept a very natural and well motivated for a realistic game.

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NEW ECONOMY Section

THE IMAGE IN THE REVOLUTION OF EXPECTATION

Constantin POPESCU* and Alexandru TAŞNADI*

Abstract. *From the point of view of great spirits of our world, the revolution of expectation is a change of sense which bring closer the individual to the meaning of the whole living complex to whom it belongs. The living systems from this Planet and also from the microcosms where we live in are not what they seem to be. They are* ***the image*** *of their own made during the time through knowledge, understand and faith. The new economy changed thoroughly the space and time of human being businesses. These factors of national regional or local nature, have changed into world-international factors that represents the core of an* ***economy making images****. The specialized preoccupation, more and more essential, as the image of things should precede, follow and live more in mind, heart and the human's spirit, are a vital function of a new economy and human society based on the use of scientific knowledge. Around the image – as an organic function of the society where we live in, work and love – there are together the producers and the distributors of these* ***spiritual reality****, those who are always in expectation for a new way to fulfill of their life. Through resources that they rally on – beginning with financial, human, material – natural, and scientific researches, natural –, through the specific shapes that it appears – an* ***offer of an image*** *–, through the unpublished way to be consumed, changing itself through each person in accordance with its nature and the society inside itself,* ***the image*** *becomes the first sector of scientific knowledge in the new economy and the edify the sector of the economy of human being society.*

Inside ***the commodities that we name images*** *there are incorporated both knowledge of producers and of consumers, for the first time! So,* ***the image*** *has both sides, one is the face that producer wants to show it and the other is the face that the consumer made it for itself. Both* ***sides of images****, as economical goods are made through scientific knowledge under different circumstances and shapes.*

Keywords: image (as a spiritual good), new economy, responsibility.

1. Communication through emotional images

The most powerful and effective communication technique learnt and used by the world's great leaders (Churchill, Reagan, Twain etc.) is

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communication through emotional images. This is the only form of communication that addresses simultaneously the mind (IQ) and the heart (EQ) in order to transfer "understanding" and "feelings".

Understanding (the analytical capacity) comes from the left side of the brain, while emotions (feelings) come from the other side. There are numerous arguments brought by neuroscience.

Emotional images become the most important tool we can use in order to have an **effective and convincing communication (see Figure 1).**

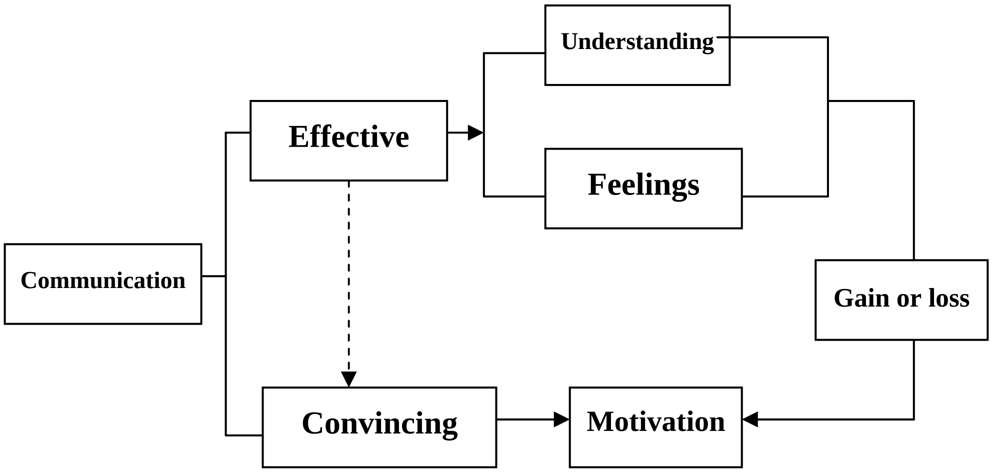


Figure 1. From effective communication to convincing communication.

As a result, **understanding** and **feeling** are the two elements needed for communication to be **effective** and two of the three ones that make communication **convincing**.

Definition: an emotional image is **a word, a statement** or **a narrative** that immediately creates an image in the listener's or reader's mind. This image:

– clarifies what you are trying to say;
– communicates a feeling you want to transfer to the listener.

The use of emotional images has a few advantages:

– They draw and direct the attention of the person we communicate with;
– They have the power to influence and change a person's thinking and life;
– They commit the words[1] to the person's memory;
– They animate communication because the person starts to visualise what he does not hear;

[1] According to statistics, words play only a 7% role in the communication process.

– They create the opportunity of more profound inter-human relationships.

In order to be convincing, communication must motivate us to take action. Usually, we resort to two of the strongest motivational factors:

a) the desire to win

b) the fear of losing.

Consequently, if we want to trigger the motivation to act we must always resort wither to someone’s **desire** to win or to his **fear** of losing.

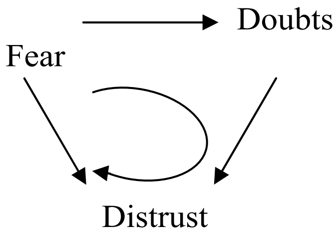
Usually, for the human being:

The fear of losing > The desire to win

The problem implies treating the “fears inside us”, the imbalance between fear (F) and desire (D) and redressing it:

$$D = F - D.$$

The progress to human self-governance gives us the possibility to break out of the **“vicious circle”** of fear:



In table 1 (Annex A) we emphasise the elements that can be considered gain or loss in a motivational act[2].

2. The image that addresses dreams

From the complex system of images (see Annex B5) we will insist on the one addressing dreams. According to Napoleon Hill: “All the achievements and riches in the world originated from an idea or a dream.”[3]

Dreams are **mental images** that basically inspire every human aspiration, from the dreams of the ancient Greeks regarding the building of democracy of the ancient Egyptians about building the pyramids to your dream of setting up a small company, a farm or of setting a better parity.

[2] Motivation is what makes you take action of make a choice. It is the one that gives you a reason. This can be an instinct, a passion, an emotion, a habit, a mood, a drive, a desire or an idea.

[3] Napoleon Hill, *From Idea to Money*, Curtea Veche Publishing House, Bucharest, 2007

Above all, dreams remind us that **the greatest achievements** always originate from **the greatest dreams**.

An extraordinary **definition** of the **dream** is the following: “A dream is a blueprint for your greatest achievements.”[4]

Consequently, when you dream, your emotions dictate to the imagination to create a **mental image** of the place you want to reach. (SF). The process of turning the dream into reality implies covering several stages (see Fig. 2).

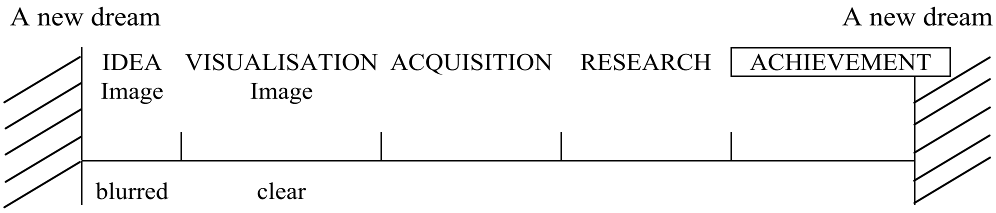
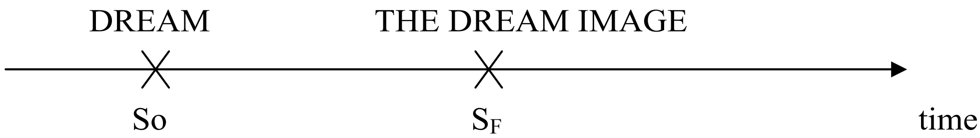


Figure 2. The stages covered by the dream.



Referring to these stages, we notice that in order to turn dreams (So) into reality (S_F) it is necessary to **visualise** them, namely to associate with them the **image** that progresses from the “blurred” to the “clear” stage:

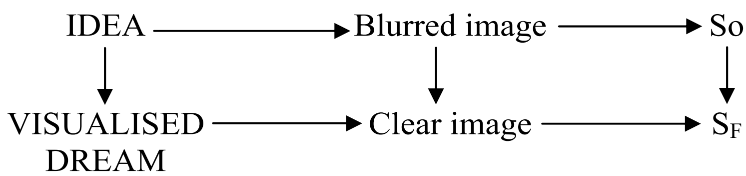


Figure 3. From blurriness to clarity.

Te process is similar to adjusting the microscope in a biology lab. At the beginning, the dream is a vague idea – with a **blurred image.** Then, as we adjust the focus, a **clear, precise image** appears in our mind. Consequently, **the image** is in your mind, not before you eyes. At this stage, you turn your dream into an **image.**

The creation of **mental images** to dreams is an art. It is the art of seeing “**things invisible**” to others. After we perceive the **mental image** of the

[4] Burke Hedges , *You Inc.*, Curtea Veche Publishing House, 2001

dream it becomes so clear, so vivid, than nobody and nothing can prevent its achievement. We can simply say you **blended** with the dream. You already see the place you want to reach and this **image** associated with the dream is nothing but a painting in an exhibition whose varnishing day will soon come. After you acquire the **image of the dream** and "taste" it with all your senses, it must be **acquired**. This means you must pay a **price**. Usually, the people who do not make their dreams come true stop at this stage[5]. They begin to avoid the dream God gave them in order to achieve their potential.

The role of visualising, of creating the image associated with our dreams, is specified in the qualitative model that describes the process of achieving them (see Fig. 4).

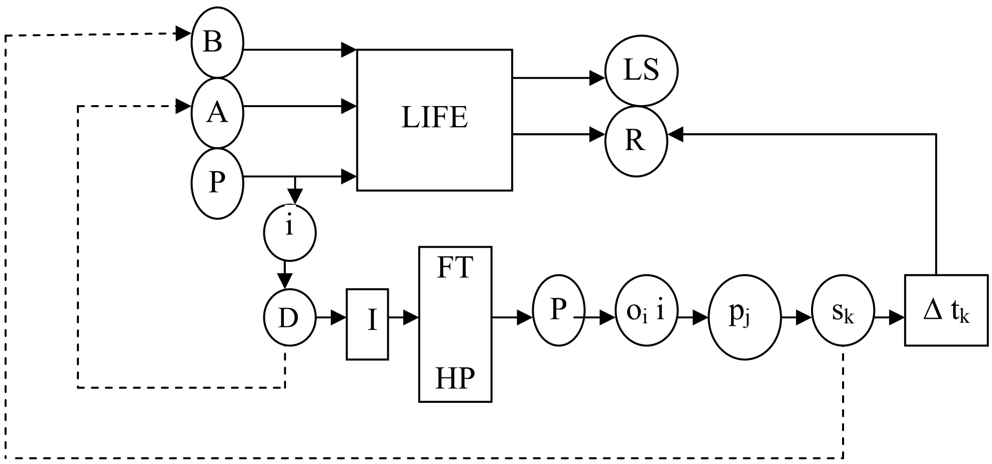


Figure 4. The dream transformation process.

Our life philosophy (P) acquired by accumulating knowledge generates new thoughts, ideas (i) which begin to "flicker" in our mind constituting the germs of our dreams (D).

These dreams are visualised, turned into clear images (I) and are fixed on two launching pillars: **faith** (FT) and **hope** (HP). **Faith** is a mood that renders the word "impossible" useless. Look around and you will notice that those who succeeded the best are those who acknowledged and used their capacity to believe. It must be cultivated through a permanent, daily use.

Hope – is the feeling that what you want is going to happen (the achievement of the dream – n.n.). It is the expectation, anticipation, optimism. It is the basis for achieving the desired thing: the dream we have dreamt of.

A person (the dreamer) reacts in a conscious manner to a thing he desires, in which he believes and which he can obtain.

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[5] John Maxwell, *Be All You Can Be*, Curtea Veche Publishing House, Bucharest, 2002.

In order to trigger the process of achieving the dream, a detailed plan (P) is elaborated in which the objectives (o_i) are set, $i = 1, n$ – which are: clear, put down on paper, measurable, and with time limits. Then, the necessary steps (p_j) in achieving the objectives are designed, $j = 1, m$ and the tasks (s_k) are set, $k = 1, l$ which must be performed every day. The daily performance of tasks gives us confidence in ourselves, strengthening our belief that the dream will come true. To each task a given time Δt_k is attached, which, through a convergent process, leads us to the result ® which depends on your life style (LS).

Let us notice that the qualitative model illustrated in Figure 4, the attitudes (A) and behaviours (B) are related to positive thinking, optimism, faith and hope.

The dreamers' aspirations influence the lives of those that work and give everyone chances to rise above a common existence. Their dreams are like a light guiding the way for other to follow. Having a clear **image** of our dream and because we are free to think, our destiny and future are in our hands.

ANNEX A

Table no. 1

MOTIVATIONAL FACTORS

GAIN	LOSS
• love	• love
• safety	• person
• acceptance	• safety
• success	• acceptance
• wealth	• success
• money	• material goods
• looks	• wealth
• health	• money
• spirituality	• looks
• closeness to God	• health
• closeness to parents	• spirituality
• faith	• closeness to God
• support	• closeness to the boss
• image	• faith
• popularity	• support
• position	• image
	• popularity
	• position

STATISTICS ANALYSIS AND MODELLING ROMANIAN FOREIGN BALANCE OF PAYMENTS, 2000-2007

Calcedonia ENACHE*

Abstract: *The international economic relations represent a wide range of trade and financial connections that are able to link together economic agents having complementary aims and coming from different national economies. The sum of the "feedback" and "feedbefore" expansions between an economy and the worldwide economic environment is to be tendentiously found in the balance equilibrium of foreign payments, due to its main trade and financial parts, indicates permanent and in relatively simple terms the method whereby each economy pays the prices for making its own way through the international trade, and the national coin evolves on the currency market.*

Keywords: balance of foreign payments, current account, foreign trade.

1. Description of the components of the Current Account balance

1.1. Preliminary consideration

Romania's balance of payments, taking as a starting point the year 2000, mirrored the worsening of the current account, due to the increase in the dynamic rate of the imports as well as to the larger imbalance under "good transports". Domestic absorption registered a constant upward evolution in the whole analyzed period, the ratio between the investment and final consumption stayed flat on an average of about 26 percent. This evolution was determined by the private sector, under the conditions in which the governmental sector limited its demand for consumption, the saving, investment and current account balance equilibrium relationship mounting progressively from 3,3 percent of GDP in 2002 to 13,9 percent of GDP in 2007.

The trade deficit, of which share in GDP increased from 3,7 percent in 2000 to 14,5 percent in 2007 has influenced heavily the current account

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deficit. The trade gap widened over EUR 1598 million due to the slowing growth rate of exports towards the developing countries and to the still fast-paced imports. The coverage of imports through exports dropped by 23 percentage points, while the openness of the Romanian economy has had an upward trend soared to 62,9 percent in 2007, amid the removal of protectionist tariffs as well as Romania's faster integration into the world trade.

Table No. 1

Romanian external sector, 2000-2007

	2000	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
	– EUR million –							
Absorption	421445	48488	51143	54239	64450	87652	108909	138853
Exports	11273	12722	14675	15614	18935	22255	25850	29380
Imports	13140	16045	17427	19569	24258	30061	37609	46966
Trade balance	–1867	–3323	–2747	–3893	–5536	–7806	–11759	–17856
Services, net	–260	–129	5	62	–213	–349	4	265
Income from abroad, net	–304	–315,0	–488	–623	–2535	–2326	–3246	–4404
Current transfers, net	937,0	1279,0	1612	1639	2972	3593	4845	4853
Current account balance	–1494	–2488,0	–1623	–2877	–5099	–6888	–10156	–16872

Source: National Bank of Romanian's Annual Reports (2000-2007), according to National Institute of Statistics, the National Trade Register Office, the Ministry of Economy and Finance, National Bank of Romania.

During the analyzed period, the annual average of exports was EUR 4624,59 million and it was increased by the partial subsidization of interest on export loans, the provision of guarantees for the export of complex products with a long production cycle, the relaxation of the industrial activities, the growth of the industrial products with a high level of processing in the total of the exported goods, the growth of the in-branch specialisation. The manufacturing industry supplied over 95 percent of Romania's exports.

The aspects concerning the seasonality presence in the evolution of the Romanian Export Trade are pointed at by the use of the Calot model, which relies on the following hypotheses: the general tendency is linear; the seasonality is at a stand-still; perturbation follows a normal distribution.

Taking the linear form: $y_t = a_0 + a_1 t + \epsilon$, as a starting point, seasonality is hereby introduced into the model by dividing the parameter a_0 into elements relating to the trend (A) and the constants s_j, every single one for each annual sub period. The variable time "t" turns into a unit for which the sub periods j (month, trimesters) and the annual periods i: $t = j + ih$

(where $j=\overline{1,h}$ sub annual periods: $i=\overline{0,(m-1)}$ years). Therefore, the regression equation becomes:

$$y_{j+ih} = A + a_1(j+ih) + s_j + \in_{j+ih},$$

the estimating items $\hat{a}_1, \hat{A}, \hat{s}_j$ come out as a result of the following calculus relations:

$$\hat{a}_1 = \frac{1}{m(m^2-1)\sum i(\overline{y_i}-\overline{y})}; \quad \hat{A} = \overline{y} - \frac{mh+1}{2}\cdot\hat{a}_1; \quad \hat{s}_j = \overline{y_j} - \overline{y} - \left(j - \frac{h+1}{2}\right)\cdot\hat{a}_1,$$

where: $\overline{y_i}=$ the sub period (trimester, month) average in the year *i*; $\overline{y}=$ the sub period average reckoned under all data[1].

The model Calot for the quarterly data has been applied regarding the export trade evolution in 2000-2007 and consequently the following equation has come out:

$$y_{j+ih} = 3664{,}126 + 58{,}2(j+ih) + s_j + \in_{i+ih}$$

Table No. 2

The evolution of the Romanian exports and the seasonality constants, 2000-2007

Year	Exports (EUR million)				Year average $\bar{y}_i$
	T I	**T II**	**T III**	**T IV**	
2000	2404,4	2505,4	2702,2	2754,3	2591,575
2001	2873	2823,3	2966,6	2722,8	2846,425
2002	2536,9	3057,9	3695,4	3882,7	3293,225
2003	4316,3	4312,9	4070,2	4026	4181,35
2004	4325,4	4692	4945,5	4930,9	4723,45
2005	5087	5423,7	5917	5785,3	5553,25
2006	6218,2	6459,8	6492,8	6678,7	6462,375
2007	7014,3	7163,7	7367,0	7835,3	
Trimester average $\bar{y}_i$	4346,937	4554,838	4769,588	4827,00	$\overline{\overline{y}}=$ **4624,591**
The seasonality constants	–190,33	–40,65	115,89	115,09	$\Sigma s_j = 0$

Source: Own calculations on the database from the Statistics Bulletins of Foreign Trade, NIS (2000-2007).

[1] Pecican Eugen, Econometrics for…economists, Economic Publishing House, Bucharest, 2005, pag. 113.

The seasonality constants point out that, on average, the amount of the exports good in the trimesters III and IV went above the long lasting tendency by EUR 115, 89 million, respectively by EUR 115, 09 million. As the dynamics of the European Union's demand for imports goods slowed down, trimester I registered the lowest seasonality level following the customs – house duties, whilst over 49 percent of the returns came out of definitive exports goods.

Regarding the import, the dynamics average rate amounted to 20,01 percent, being generated by: the international context, the structural movements into the national economy, the policy of customs measures implemented by authorities, the incentives granted to small and medium enterprises and investors (Law No. 133/1999 and Law No. 332/2001), the real appreciation of the ROL against the EUR. As far as the structure is concerned, four commodity groups have held the majority shares in total imports: machinery, equipment and transport means (29,83 percent); textiles, wearing apparel and footwear (17,88 percent); chemical and plastic products (13,43 percent); mineral products (13,38 percent).

The geographical distribution shows that over 65 percent of the trade deficit was due to Russian Federation (43,45 percent) and the European Union (23,46 percent). From trade relations with the Republic of Moldova has been achieved a surplus of 4,24 percent.

The terms of net trade recorded a positive slope from 103,5 percent in 2000 to 101,5 percent in 2003, due to higher prices of commodities exported to the European Union. In the following years, at a constant level of 104 percent in GDP was maintained. Regarding the terms of gross trade the maximum in the analysis period was recorded in 2002, when it amounted to 102,1 percent as a result of the changes appeared in the structure of exports that happened in an unfavourable external environment.

In order to analyze the elasticity of the foreign trade regarding the modification of the exchange rate and also of the way in which this has influenced the balance of the current account, arc-type values obtained in 3 variants have been used:

$$\text{a) } \frac{y_t - y_{t-1}}{y_{t-1}} : \frac{x_t - x_{t-1}}{x_{t-1}}; \qquad \text{b) } \frac{y_{t+1} - y_{t-1}}{y_{t-1}} : \frac{x_{t+1} - x_{t-1}}{x_{t-1}};$$

$$\text{c) } \frac{y_{t+2} - y_{t-1}}{y_{t-1}} : \frac{x_{t+2} - x_{t-1}}{x_{t-1}},$$

where y_t represents the 2 month mean of the export/import in t period (for the last, next and future period $t-1$, $t+1$ and $t+2$ signs have been used).

Table No. 3

The arc type elasticity values calculated for Romania's export and import

Period	Average values for:			Elasticity for:	
	Export	Import	Exchange Rate RON/EUR	Export	Import
May-June 01	864	1107	2,4821		
July-Aug 01	877	999	2,6060	a) 0,30975	a) –1,95682
Sep-Oct 01	863	1070		b) –0,01624	b) –0,67068
Nov-Dec 01	800	1151		c) –1,46754	c) 0,801916
Sep-Oct 02	1247	1512	3,2555		
Nov-Dec 02	1296	1583	3,3915	a) 0,945601	a) 1,127065
Jan-Feb 03	1379	1520		b) 2,533443	b) 0,120305
Mar-Apr 03	1474	1816		c) 4,365126	c) 4,802689
Sep-Oct 04	1716	2326	4,10735		
Nov-Dec 04	1598	2604	3,9295	a) 1,589473	a) –2,75525
Jan-Feb 05	1578	2020		b) 1,851245	b) 3,043187
Mar-Apr 05	1860	2573		c) –1,94276	c) –2,45441
Jan-Feb 06	1940	2640	3,59245		
Mar-Apr 06	2106	3117	3,49925	a) –3,29982	a) –6,96927
May-Jun 06	2294	3507		b) –7,03485	b) –12,6543
July-Aug 06	2142	3407		c) –4,02914	c) –11,2066

Source: Own calculations on the database from the Statistics Bulletins of Foreign Trade, NIS (2000-2007).

The four points of major inflexion recorded by the evolution of the exchange rate RON/EUR during 2000-2007 did not have any significant effect on the current account deficit. The condition of the MLR proposed by the economic theory according to which the improvement of the balance of payment takes place when the sum of elasticises in its absolute value, regarding both foreign trade flows is over unitary was totally verified in the last interval. Thus the elasticity taken together summed at about 3,67, respectively 5,62 for one period of being late and 7,18 for two period of being late.

The export was more receptive at the variation of the exchange course but the degree of elasticity remained low because of the limited capacity of the operators of the foreign trade to rapidly enter the more advantageous markets.

The import was less flexible at the modification of the exchange rate the rating of the national currency leading to raising the value of entering in absolute terms but generating a reduction of the rhythm of dynamics.

Since 2001, the deficit of the income balance had a devaluation accentuated from EUR 315 million to EUR 324,6 million in 2006, under the reduction of the direct investment and under the payment of interest pertaining to loans in the medium as well as long term. In 2007, the income balance reached on a EUR 440,4 million deficits, higher 35,6 percent from the previous year, due to the increasing of compensation of EUR 294 million of the employees.

The widening of the current account deficit was offset by the positive performance of the current transfer balance, which showed mirrored a surplus of EUR 485,3 million in 2007, especially on account of foreign money inflows.

2. Modelling the Romanian export's dependence on the gross national disposable income

Starting from the economic identity relation: $VND_t = E_t + TC_t$, according to Keynes theory, I suggest the next structural form:

$$\begin{cases} E_t = a + bVND_t + u_t \\ VND_t = E_t + TC_t \end{cases}$$

where:

E_t = the value of the export in the *t* year;
TC_t = the volume of the current transfers in the *t* year;
VND_t = gross national disposable income in the *t* year;
u_t = the random variable;
a, b = the parameters of the econometric model which are due to be estimated.

By applying the method of the smallest squares in the structural model, the estimators of the parameters $\hat{a}, \hat{b}$ model are obtained:

$$F(a,b) = \min \sum_t (E_t - \hat{a} - \hat{b} \cdot VND_t)^2$$

$$F'(a) = 0 \Rightarrow n\hat{a} + \hat{b}\sum VND_t = \sum E_t$$

$$F'(b) = 0 \Rightarrow \hat{a}\sum VND_t + \hat{b}\sum VND_t^2 = \sum E_t VND_t,$$

resulting: $\hat{a} = 3708,205$; $\hat{b} = 0,2196$.

The estimated model becomes:

$$\hat{E}_t = 3708{,}205 + 0{,}2196 \cdot VND_t$$

In order to verify the truth of the model the following hypothesis are formulated:

H_0: unvalued statistic model, with the alternative

H_1: valid statistic model.

The statistics used to decide which of the hypothesis is accepted is:

$$F_{calc} = \frac{s^2_{E/VND}}{s_u^2}.$$

The estimator of the systematic dispersion is calculated with the relation:

$$s^2_{E/VND} = \frac{\Delta_{E/VND}{}^2}{k} = \frac{\sum(\hat{E}_t - \overline{E})^2}{k} = 283284045{,}6.$$

The estimator of the dispersion of the residue is determined as:

$$s_u{}^2 = \frac{\Delta_u^2}{T-k-1} = \frac{\sum(E_t - \hat{E})^2}{T-k-1} = 1537773{,}64.$$

The estimator of the variable dispersion E is:

$$s_E^2 = \frac{(\sum E_t - \overline{E})^2}{T-1} = 41787212{,}5.$$

The theoretical value for a significance threshold $\alpha = 0{,}05$ and 1, respectively 6 degrees of freedom, taken from the Fisher distribution table, is $F_{\alpha,\, k,\, T-k-1} = 5{,}99$. Since $F_{calc} = 184{,}22 > F_{\alpha,\, k,\, T-k-1}$, H_0 is rejected, inanely it consider that model is valid.

The interference for the parameters of the regression model is made by estimating these on the intervals of trust and testing the statistic hypothesis.

For the parameter a, the tested hypotheses are: $\begin{cases} H_0{}^a : a = 0 \\ H_1{}^a : a \neq 0. \end{cases}$

The t statistics is: $t_c^a = \dfrac{\hat{a}}{s_{\hat{a}}} = 3{,}11$,

where: $s_{\hat{a}} = \sqrt{s_{\hat{u}_1^2}\left(\dfrac{1}{n} + \dfrac{\overline{VND}^2}{\sum(VND_t - \overline{VND})^2}\right)} = 1193{,}13.$

For a significant threshold of 5 percent, the theoretical value of the *t* test is $t_{\alpha/2;\,6} = 2{,}447$. Because $t_c > t_{\alpha/2;\,6}$, it is established that the free term is significantly statistic.

For the parameter *a*, the trust interval is given by:

$$\hat{a} - t_{\alpha/2,\;T-2} \cdot s_{\hat{a}} \le a \le \hat{a} + t_{\alpha/2,\,n-2} \cdot s_{\hat{a}}$$

$$0{,}021 \le a \le 788{,}71.$$

For the *b* parameter, the tested hypotheses are: $\begin{cases} H_0^b : b = 0 \\ H_1^b : b \ne 0. \end{cases}$

The *t* statistics is: $t_c^b = \dfrac{\hat{b}}{s_{\hat{b}}} = 13{,}57$, where the square medium of the variable residue is calculated with the relation: $s_{\hat{u}_1}^2 = \sqrt{\dfrac{1}{n-k-1}\sum u_{1t}^2} = $ $= 0{,}219$ and the square medium deviation of the estimator: $s_{\hat{b}} = \sqrt{\dfrac{s_{\hat{u}_1}}{\sum (VND_t - \overline{VND})^2}} = 0{,}016.$

For a significant threshold of 5 percent, the theoretical value of the *t* test is $t_{\alpha/2;\,6} = 2{,}447$. Because $t_c > t_{\alpha/2;\,6}$ it is considered that *b* parameter is significantly different to zero. The trust interval for the *b* parameter is given by:

$$\hat{b} - t_{\alpha/2,T-2} \cdot s_{\hat{b}} \le b \le \hat{b} + t_{\alpha/2,\,n-2} \cdot s_{\hat{b}}$$

$$0{,}18 \le \mathrm{b} \le 0{,}259.$$

In order to measure the intensity of the relationship between the variables, the correlation report is used and also the testing of the significant is made with the help of the Fisher-Snedecor test:

$$R = \sqrt{1 - \frac{\sum u_{1t}^2}{\sum (E_t - \overline{E})^2}} = 0{,}9841$$

$$F_{calc} = (T-k-1)\frac{R^2}{1-R^2} = 184{,}21.$$

The theoretical value for a significance threshold $\alpha = 0{,}05$ and 1, respectively 6 degrees of freedom, taken from the Fisher distribution table, is $F_{\alpha;\,k;\,T-k-1} = 5{,}99$. Since $F_{calc} = 184{,}22 > F_{\alpha;\,k;\,T-k-1}$, H_0 is rejected, inanely it consider that *R* it is statistically significant.

After an analysis of the acquired results it has been ascertained that the model is fit for service, which explains for 96,84 percent of the whole rise and fall of the endogenous variable. Applying the *t* test in order to control the significance of the parameters, makes it obvious that these ones are statistically significant; the same statement is upheld by the limiting values of the credibility periods which have the same sign. The marginal rate of the exports shows that in 2000-2007, under a rise of EUR 1 million of the available national income, the Romanian exports went up by EUR 0,2196 million.

Throughout the analyzed period, the current account deficit was subsidized over 80 percent by the net entering of direct investments, as the most significant capital inflows were due to the privatization of the National Company RomTelecom, the Romanian Development Bank, the "Automobile Dacia Pitesti", the Romanian Commercial Bank and SNP Petrom.

Among the most important investors there have been: Ispat Sidex, Raiffesein Bank, Daewoo, Colgate Palmolive, whilst, according the social capital criterion of mentioned above, the main investing countries have been: Netherlands, Austria, France, Germany and Italy.

3. Conclusions

The Romanian economic and financial environment has lately recorded a noticeable progress, yet the gaps in comparison with some Central European countries are still considerable in some aspects.

The Romanian foreign balance of payments showed a progressive worsening of the current account from 3,3 percent in GDP in 2002 to 13,9 percent in GDP in 2007. Despite the fact that the economics theory suggests keeping the deficit up at most 6 percent in GDP, the experience of some new states joining the European Union, has emphasized that, as the outside credibility grew and the economic performances improved, the amount of the capital investments which is the main source of support for the import goods raised, which lead to the going down the current account.

Although now over 90 percent of the financial account is being covered up by direct and portfolio investments, still the relatively high level of the current account deficit needs not being given additional impulses that are correlated with the growth of the public and private consumption.

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THE STATISTICAL ANALYSIS OF SEVERAL SOCIO-ECONOMIC AND ENVIRONMENT INDICATORS BY USING THE SINGULAR VALUES

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Abstract: *We propose a methodology to establish an hierarchy between given environment and socio-economic indicators. The classification difficulty arises from the presence of many attributes which should be evaluated together. In this context we decompose the frequencies matrix of the indicators into singular values to define new aggregate coefficients. This method allows a suggestive unidimensional and bidimensional graphical representations of the clusters. The decomposition procedure was applied for classifying 14 known indicators from socio-human field.*

Keywords: statistical analysis, singular values method, socio-economic indexes, aggregate coefficients.

1. The problem formulation

We intend to establish an hierarchy for 14 socio-economic and environment indicators by using the subjective marks received from the individuals of a specified population.

We'll partially depict some aspects of a research started in 2003 and concerning the diagnosis of the quality of life (Institute for the Quality of Life – Romanian Academy). The data were collected by using a questionnaire which imposed a national survey with more than 1000 subjects. The research analyzed the behavior of about 200 social and economic coefficients.

In the subsequent we'll present some methodological aspects by selecting only 14 environment and socio-economic variables. The significance of the coefficients A-N is given in Table 1.

A comparative statistical study will emphasize the quantitative but also the qualitative differences which exist in Romania between the urban and rural environment.

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A statistical analysis concerning the volatility of the indicators A-N will later be detailed by a similar study on various particular subgroups of the initial population. We mention here the men and the women subsets, different ethnic groups (Romanian and Magyar people), the behavior of the age categories, the influence of the school performances or the current professional activity of the respondents.

Table 1

Indicators A-N (questionnaire CV2003)

A.	Individual health
B.	Family relations
C.	Individual household
D.	The quality of the environment
E.	Work conditions
F.	Relations with your neighbors
G.	Family income
H.	Access to drink water in your community
I.	Health services received in your community
J.	The police activity in your community
K.	The quality of the education in your community
L.	The information received through mass –media (press, radio, television)
M.	The quality of public transport in your community
N.	The current possibility to spend your free time (recreation facilities)

2. The sample characteristics

Our statistic study is based on the results given by a national representative sample **E** which has 1018 individuals. The sample was designed in the spring of 2003 at the Institute for the Quality of Life – Romanian Academy (ICCV).

At the questions A-N the questionnaire mentioned the following possible answers:

R1. "Very bad" – code 1; **R2.** "Bad" – code 2; **R3. "**Satisfactory" – code 3; **R4.** "Good" – code 4; **R5.** "Very good" – code 5; **NR.** NonRespondent; **NC.** Not the Case.

Table 2 presents the frequencies for the answers **R1-R5** taking into consideration all the 1018 persons from the whole sample **E**.

In this paper we will emphasize the differences of perception for the people which lives in the rural and the urban localities too. It is pursuing the subjective importance given by the community to the questions A-N. For every query it was computed the percents values at the answers **R1-R5**. All these details are summarized in Table 3.

Table 2

The frequence response for the indicators A-N (the whole sample)

Total	R1	R2	R3	R4	R5	NR	NC
A	84	205	306	343	80	0	0
B	7	16	101	590	262	1	41
C	23	57	251	585	100	2	0
D	29	128	256	508	87	10	0
E	23	87	163	208	34	61	442
F	10	19	112	688	179	10	0
G	162	343	342	153	10	8	0
H	75	187	199	480	66	11	0
I	37	122	300	483	59	17	0
J	31	83	311	508	51	34	0
K	11	77	207	566	73	84	0
L	10	66	248	588	77	29	0
M	74	199	247	403	38	57	0
N	101	245	304	294	27	47	0

Table 3

The percents of the answers R1-R5 at the questions A-N considering the **ur**ban and the **ru**ral localities (the **NR** and **NC** responses are neglected)

Ind.	R1		R2		R3		R4		R5	
	ur	**ru**	**ur**	**ru**	**ur**	**ru**	**ur**	**ru**	**ur**	**ru**
A	0.052	0.118	0.169	0.238	0.297	0.305	0.400	0.265	0.083	0.074
B	0.006	0.009	0.006	0.029	0.096	0.112	0.581	0.632	0.311	0.217
C	0.018	0.027	0.052	0.061	0.229	0.268	0.596	0.553	0.105	0.091
D	0.041	0.015	0.184	0.062	0.323	0.175	0.416	0.606	0.037	0.143
E	0.041	0.050	0.132	0.219	0.294	0.347	0.459	0.329	0.074	0.055
F	0.011	0.009	0.013	0.026	0.106	0.118	0.680	0.686	0.191	0.162
G	0.120	0.207	0.312	0.371	0.368	0.305	0.187	0.111	0.013	0.006
H	0.084	0.064	0.177	0.195	0.198	0.197	0.465	0.490	0.076	0.053
I	0.045	0.028	0.118	0.126	0.309	0.289	0.464	0.503	0.064	0.054
J	0.046	0.015	0.108	0.058	0.323	0.308	0.475	0.563	0.048	0.056
K	0.016	0.007	0.074	0.093	0.225	0.217	0.612	0.599	0.074	0.084
L	0.007	0.013	0.061	0.073	0.206	0.304	0.616	0.569	0.109	0.040
M	0.060	0.096	0.179	0.239	0.274	0.239	0.444	0.392	0.044	0.035
N	0.085	0.127	0.207	0.307	0.307	0.320	0.354	0.241	0.047	0.005

We remark here that the sample **E** has 475 persons from a rural environment (that is 46.7 %) and 543 individuals living in the urban localities (53.3 %).

Analyzing the percents from Table 3 we observe significant differences regarding the individual opinion in the urban and in the rural communities. Taking into consideration the whole held information (the possibility of 5 types of answers) it is very hard to evaluate quantitatively the distinct perception manner of the problems A-N from a subpopulation which live in a rural or in an urban environment.

3. Methodological aspects

To obtain a classification concerning the importance allotted by the people to the problems A-N, it is absolutely necessary to measure the similarity and dissimilarity levels between the "objects" A-N which belong to a space **S**. In fact the set **S** includes all possible distributions which could be obtained from the questionnaire answers. In this context it were used diverse "distance" and "similarity" coefficients adjusted to the domain **S**.

We remark that in the specialized literature are known a lot of similarity measures and also many types of "distances" (see for example [4]).

For this research, in the initial phase, it was used only the Euclidean distance ([4]). The concrete structure of the data and the specific combination of the old groups to produce the new ones impose different other types of "distances". Often it imposed that the selected similarity measures must remain "nearly invariant" when we build some particular classes.

If we consider the different variants of the answers R1-R5, every indicator A-N could be regarded as a point in the space $\underline{\mathbf{S}} = \mathbf{R}^5$ (see also [1]). For this approach we neglected In Table 3 the answer variants NR and NC.

Since it is very difficult to have a graphical image of the points A-N which belong to the space $\underline{\mathbf{S}} = \mathbf{R}^5$, it is necessary to find a "projection function" from the 5-dimensional space **S** inside an unidimensional **R** space. This projection application must preserve, as much as possible, the "similarity rapports" between all the objects A-N (more details in [3], [4], [10]).

Essentially, the unidimensional space **R** is characterized by an appropriate combination of the significant attributes of the objects A-N. The selection of the essential "factors" which characterize the distribution of the indexes A-N is, in our case, based on the singular values approach (see also [8], [9]).

The projection functions used to reduce the dimension of **<u>S</u>** do not preserve the old origin of the Cartesian coordinate system and in the same time do not retain neither the "positive sense" on the new coordinate axis ([2], [6], [7], [9]).

Therefore, after the approximation of the space **<u>S</u>** with an uni-dimensional metric space it is absolutely indispensable to establish "the positive direction" in this new representation. We mention here that "the positive sense" has a subjective meaning which must be in concordance with the concrete interpretations of the data set ([8], [9]). In practice the term "positive" is relatively, being defined by the multiple correlations between the implied entities.

Taking into consideration the structure of the A-N group of coefficients we decided to choose the point O(100, 300, 400, 300, 100) as the origin of the 5-dimensional space $\mathbf{R}^5$. The selected point O, being characterized by a symmetric distribution, represents a relative equilibrium state.

Though we must remark that the proposed selection of the point O as origin in the multidimentional space **<u>S</u>** = $\mathbf{R}^5$ could be controversial for the studied population.

Indeed, the distribution (100, 300, 400, 300, 100) which characterizes the coordinates of the origin O is not interpreted as a *"standard variant"* in all situations. More precisely, the distribution of O represents a stable equilibrium state only if we accept as *"extremely positive"* the variant when 0.33 % from the individuals of the analyzed population have "bad" or "very bad" life conditions and another 0.33 percents of persons have "good" and eventually "very good" ambient environment factors (to interpret the imposed distribution of the point O selected as a reference origin of the data).

Concluding, the graphics 1 and 2 which use the point O as an origin for the system of coordinates depict *"more optimistic"* the reality perceived by the population.

Since, sometimes we have a lot of NR and NC cases which vary depending on the questions A-N, we preferred to express the distributions of the indicators A-N by using the percents and not the absolute frequencies.

In our case the unidimensional reduction of the initial space **<u>S</u>** to **R** is extremely accurate. So, we got a precision coefficient equal to 0.955 (see the computation formulas from [8] and [9]).

Concluding, if is used an unidimensional depiction of the indices A-N, the proportion of the lost initial information concerning the relations between the objects A-N from $\mathbf{R}^5$ is very small, that is about 0.045.

4. The importance of A-N indicators

Applying the singular values procedure (details in [8] and [9]) we obtain the unidimensional coordinates of the A-N indices. These coordinates are listed in Table 4.

Table 4

The unidimensional coordinates of the indicators A-N
(we considered the origin O(1,3,4,3,1)).

	A	B	C	D	E	F	G
Total	0.04712	0.16749	0.19038	0.14766	0.09829	0.22359	–0.06553
Rural	–0.00072	0.18742	0.17887	0.18971	0.05840	0.22639	–0.10184
Urban	0.09042	0.15412	0.20227	0.11034	0.12949	0.22388	–0.03357
	H	**I**	**J**	**K**	**L**	**M**	**N**
Total	0.11381	0.14485	0.17037	0.20686	0.20616	0.09255	0.02935
Rural	0.12540	0.15652	0.19792	0.20100	0.20291	0.07013	–0.00728
Urban	0.10498	0.13589	0.14634	0.21349	0.21113	0.11454	0.06129

The data from Table 4 permit us a comparative study, on the urban and rural localities, regarding the indicators A-N (see Figure 1).

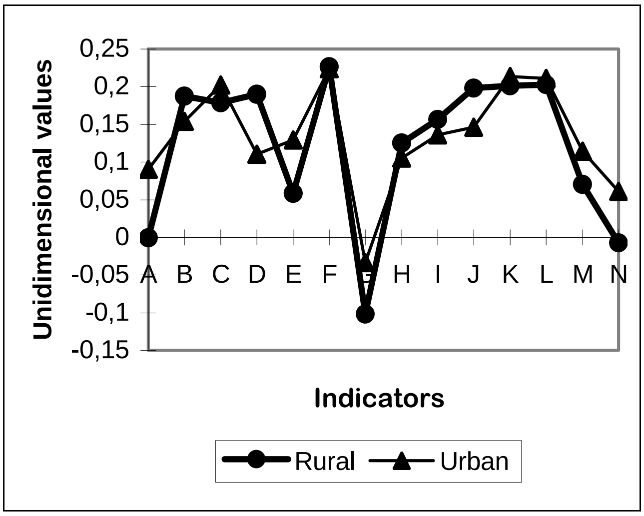

Figure 1. The perception of the indicators A-N
(rural and urban localities).

The figure 1 has the major inconvenience of an "individual representation" for the indices A-N, without having in mind the whole imagine of the all reciprocal relations between the analyzed variables.

Taking into consideration the previous mentioned aspect we suggested the Figure 2 representation. This imagine have a significant improvement for permitting us to establish an hierarchy of all the implied variables.

Interpreting in Figure 2 the positions of the points A-N we could assert with certitude the existence of some major differences between the rural and urban environment. To pursue, for example, the behavior of the variables G, N, A or E. All these mentioned differences are not so pertinent in Figure 1.

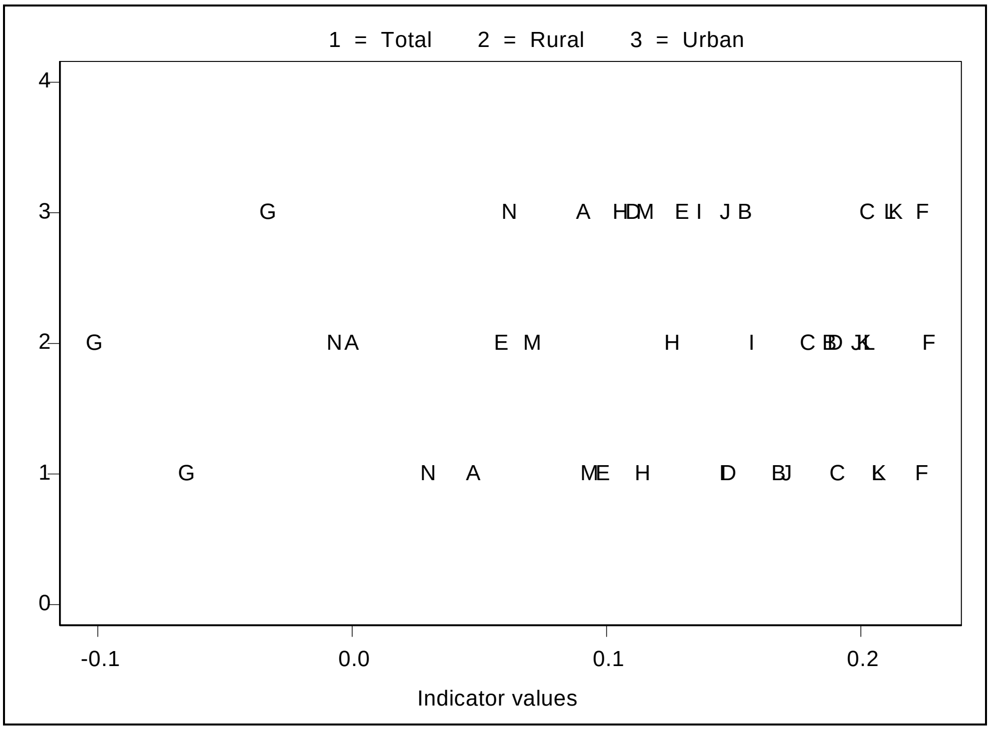

Figure 2. The classification of the indicators A-N.

5. Conclusions

– The present statistical approach is validated indirectly since the Figures 1 and 2 confirm many intuitive and practical aspects.

– The proposed mathematical procedure, based on the use of the singular values decomposition, gives us additional quantitative and qualitative information concerning the effective values of the differences between the 14 studied indicators. More, we remark here that the point representation in Figure 2 is very accurate and approximates extremely well the "distances" between the real distributions of the indicators A-N.

– The advantage for obtaining a good graphical representation is incontestable since in this case we have a general image of the whole multiple interdependent relations between the variables A-N.

– Figure 2 suggests us new statistical hypothesis which will be validated for any other subgroups of the initial population (for example to get new hierarchical classifications depending on the age or on the academic qualification of the individuals, to compare the ethnic opinions).

– The present study will be extended by considering a structural approach. A lot of new other socio-economic and environment indicators will be taken into account. More, all these indices will be grouped into several classes as: **m**aterial **a**spects (MA), **p**ublic **s**ervices (PS), **l**ife **c**onditions (LC), **e**nvironment **f**actors (EF), **i**nter-human **r**elations (IR). The new classes could have a similar approach as A-N indicators.

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INTERNET, A KEY FACTOR OF THE NEW ECONOMY WORLDWIDE

Gheorghe POPESCU [*]

Abstract. *Internet represents, according to experts, the most stunning technological of the history. Along with the unparalleled development of the Internet and with a remarkable focus on information and fast information access, traditional companies have faced difficult questions, such as:* "How may Internet controlled speed and amplitude? How may Internet generate the increase of the business efficiency? May Internet help to increase and develop the activity of a company? How may be approached the issue regarding the companies' competitiveness given the conditions of the New Economy?" *The present work aims at put knowledge in terms of basic resources of the New Economy, able to trigger an extremely fast evolution of the Internet, to define and clarify notions regarding signs, data and information, so frequently used in regular language, to show the information classification and the significance of the implicit information compare with explicit information, to present the main features of the digital economy and to highlight, in the end, the Internet's role on global level in terms of increase of competitiveness upon the level of the New Economy. It is deemed very useful delimitation and profound knowledge of the notions the New Economy operates with and, thus, the significance regarding the presentation of the Internet as component of the digital economy and "engine of the New Economy" is considered a key and reference factor given the present international environment.*

Keywords: internet, new economy, business efficiency, knowledge, economic growth, digital economy.

1. Knowledge as continuous and vital process within the New Economy

Knowledge is seen as a continuous process able to differentiate individuals, by means of motivation and knowledge capacity. The motivetion we have to know more or less is tightly connected to the effort we have to make to increase the knowledge capacity, which effort comprises intellectual, financial and time resources. A highly significant notion is the

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knowledge cost which depends on the social, economic and politic context where an individual is at a certain point and on the society's interest for knowledge.

On individual level, there is a paradox of knowledge. It addresses our attempt to represent the *infinity* of the universe where we live in a *finite* knowledge system. The biological and psychological limitation of the human brain hinders it to identically and entirely reflect on our universe and, thus, knowledge becomes a relative cognitive dynamic process. Metaphorically, Andrei Pleşu has enclosed this knowledge paradox in an elegant wording:" *But we think intermittently and differently: we think with a limited device the infinity of each thought*." (Pleşu, 2003, p. 82). People think based on patterns, which are formed in time by means of education, family, school and universities, society or religious cults. Such individual patterns are shaped and depend on social, cultural, politic and economic patterns existing in a certain geographical area and in a given historic age.

Some authors even speak about the organizations' cognitive systems, which are not mere collections or linear compilations of the individuals' knowledge. Even if there is a certain dynamics of the employees' number in a company, by means of their employment or departure, the company has the capacity to preserve its database, behavior, norms and values (Baumard, 2001).

2. Signs, data and information – concept clarification and specific aspects

In regular, the concepts of data, information and knowledge are used most times equivalently and inter-changeably. With respect to knowledge management, they are not equivalent in terms of semantics. The difference between them is relative and it triggers the difficulty to define them.

Signs are unitary graphic representations, invented by humans to communicate. For instance, the mathematical figures and symbols are signs we use frequently in our daily life. Separately, these signs don't tell anything. They gain meaning when they are complied together and immersed in a semantic context.

Data are compilations of signs, which represent different characteristics of events or processes. Therefore, data are processed signs. For instance, 2 and 0 are two signs with no signification. If we compile under the form of 20 and put them in a semantic context, we have obtained data about the event whose context we have considered. For instance, we could consider a table with the evolution of temperature in a city during a certain month of the year. In this context, 20 means the value of the temperature of a certain day. All the values listed under the respective table represent data. Them, in themselves, do not carry meaning, but lend the meaning of the context considered. For instance, in a table with the ages of youth, 20 means the age of a person and the value in that table will represent characteristic data to evaluate the ages of the respective youth.

Information is data with meaning. For instance, 20°C represents information regarding the temperature in a certain day and place and 20 years old represents the age of a certain person. Thus, the data have been put in a certain semantic context and have become information. In other words, information is processed data. Moreover, *to inform* means *to inform*, respectively to put *in a form*, to structure a certain meaning by processing the data, which are compiled signs.

Information is a concept having various semantic valences and construal, which captured the attention of various researchers, mainly after the publication of the book written by Claude E. Shannon, *A mathematical theory of communication* in 1948 (Guiaşu, 1968). This explains by means of the attempt of the American author to develop a *metric* to measure the information quantity conveyed by means of a communication channel. According to CE Shannon, information must be a measurable and abstract quantity. Its value must not depend on the information type as well as length and temperature is physical condition measures with values which do not depend on the object subjected to measurement.

3. Knowledge and its classification in implicit and explicit knowledge

Knowledge is information processed in order to understand the events taking place in our environment. Our entire biological and social existence depends, mainly, on the knowledge about natural, social, politic, economic, scientific and technologic environment we live in. The knowledge's processing can be performed on different levels of complexity and abstraction, from scientific theories to applicative knowledge necessary for daily existence. In an organization, the processing of data into information and of information into knowledge is made both on the individual level of employees and on the level of work teams. Organizational skills occur by implementing knowledge as intangible resources and their processing capabilities. Properly developed, these skills may lead to the achievement of the competitive advantage (Probst, Raub, Romhardt, 2000).

To define knowledge is yet another goal of science due to its complexity. Knowledge is a peculiar combination between conscious and subconscious, rational and irrational, direct life experience and experience gained by learning. The child touching with his/her own hand a hot object got burnt and learned by direct experience the danger related to hot objects. He/she has not yet an explanation for this fact, but learned to avoid touching hot objects in the future. Later, when he learned about thermodynamics and heat transfer, he would have understood rationally the danger related to hot objects and would avoid touching them.

Knowledge is classified as *implicit knowledge* and *explicit knowledge* taking express ability as classification criterion.

Nonaka and Takeuchi underline the significance of *the implicit dimension in knowledge generation*, especially in the oriental culture, by considering the existential unit between body and (Nonaka, Takeuchi, 1995). If the western philosophy focuses on the delimitation of the physiological processes in the human body from the cognitive ones, the oriental philosophy focuses on the unity and integrality of such processes, namely on the significance of the sensory knowledge process for the development of the cognitive processes. From this, the attention paid by Japanese companies to the development and capitalization of the knowledge's implicit dimension of their employees. The wisdom of each individual is, in this context, an outcome of all the existential processes specific to his/her personality.

The explicit dimension of knowledge addresses its capacity to be communicated by means of written and oral language. It is the dimension which accompanies the individual and organization communication process, a process so familiar to us that we most times live under the impression that it is the only possible process for knowledge communication. However, we underline that, in terms of quality and quantity, knowledge in the explicit field is only a small part of the entire knowledge a person may acquire, respectively may generate during their entire life.

Explicit knowledge is learned in school and society; it allows the formation of the culture of a people and its capitalization by means of communicating knowledge and wisdom to each generation. Our universities are organizations specialized in communicating knowledge, but they have developed teaching technologies which process explicit knowledge. It is the knowledge allowing the development of science and technology, namely the society's progress. Computer technology operates with explicit knowledge and, may be, that's why western companies have focused in its processing and use for accomplishing the competitive advantage. (Fuller, 2002).

The knowledge management, especially the one developed by Japanese companies, centers the human capital and thus both implicit and explicit knowledge is considered. Although it may seem curious, it means to consider both the rational and irrational, the conscious and unconscious, the explicit and emotional intelligence. For Japanese, creating new knowledge exceeds the explicit stage and that's why the convergence between the implicit and explicit dimensions is very important.

4. Strategic knowledge significance

Recent research on strategic management and performance management underline *the knowledge's significance in achieving the competitive advantage*. Knowledge is the most valuable intangible resources an organization may have and has already become the most important production factor, after entire ages were dominated by land, labor and capital as production factors. A survey performed by the renowned consultancy company „McKinsey" has clearly proved that the success of large companies is based on performing knowledge management. If a success metric is considered based on market capitalization, it may be highlighted that, for large companies, the most important contributions belongs to intangible resources and not to the tangible ones.

5. Digital economy

Digital economy is based on Internet and has as actors – communities of manufacturers, merchants, consumers or electronically transacted services.

5.1. Digital economy and new economy – definitions and characteristics

The outcome of the implementation of this new economy addresses the creation of new markets and the expansion of the existing ones, the occurrence of new behavior patterns for manufacturers and consumers, the transformation of the patterns within the traditional economy.

Digital economy implies new laws, new currencies new mentalities and behaviors for manufacturers, sellers and buyers, new distribution networks, etc. The digital economy concept has several synonyms: e-economy, Internet economy, web-economy, virtual economy etc.

The "new economy" concept must not be associated directly and unilaterally with the digital economy. **The new economy** is the concept which defines the role, weight and dynamics of the traditional and digital economies given the increase and powerful influence of the latter.

As "real" economy, the digital one is based on market, which is transforming from a standard one into an electronic one, based on information technology, having the following functions (similar with the regular ones): meeting between buyers and sellers; offer contents (features of the products offered by the seller, agreement on various products); search for business partners (information about product and price, auction organization, offer adjustment by the sellers according to buyers' preferences); price discovery (price assessment and comparison); transaction easement; logistics for supply of information, goods and services to buyers); payment (payment transfer to seller); trust: credit system (reputation, online agents for consumer protect-tion, assessment agents); organizational infrastructure; legal frame (comercial code, contract law, intellectual property protection, dispute settlement); regulations (regulations and rules; monitoring, enactments).

The main particularity of the business fulfillment in a digital world is that the distribution processes are based on *information gathering, selection, processing and distribution*. The chain of a digital business starts with supply and demand and concludes with prices and competition. **The main components of the digital economy are**: digital products, consumers, sellers, company infrastructure, agents, maintenance and support services, web designers.

5.2. Components of digital economy

In order to classify digital economy, the main components can be classified (mentioned above) and there can be other classification means based on the complexity and inter-dependence of the companies acting in the digital economy. Such system developed by CISCO Systems and Texas University, in their report from June, 2000 [www.internetindicators.com], identifies four components in a hierarchic structure of the digital economy: Internet infrastructure; applications of the digital economy infrastructure; agents; online transactions.

The identification of the digital economy components are based on this structuring manner and on elements generating maximum incomes.

Other researchers [www.idc.com] structures structure digital economy on three components, joining the last two components in a single class: Internet infrastructure (which represents the support for transaction fulfillment); commercial infrastructure (which facilitates trading on Internet);

online trade (which represents the trade itself). In time, the components' structure has changed and each stage of the Internet evolution has coincided with the dominance of one of the levels mentioned. Thus, in the first years of the Internet, investments were focused on technologies and services developing the infrastructure, attracting users for online trade. At present, the dominance of the expenses not related to technology are increasing, sales and marketing and web design gaining a significant role for the improvement of the Internet environment and increase of the online trade. Several large companies, such as Microsoft and IBM are playing various roles on different levels. This layering of the digital economy allows an analysis of the company in terms of its level to enter the e-economy and the possibilities to expand its activities to other architecture levels.

5.2.1. Internet Infrastructure – general presentation

This level comprises companies whose products and services help the creation and development of the network infrastructure based on TCP/IP protocols. This class comprises telecommunication companies, Internet providers, Internet infrastructure support companies, Internet access companies and manufacturers if network equipment, computers, security products and services, etc.

The companies comprised in this level have become more powerful than those in other layers of the digital economy. The Internet infrastructure has become a huge industry on its own, larger than the pharmaceutical industry, baking system and aircraft industry. Ensuring the spine of the digital economy, infrastructure eases the increase of the companies from the other three levels. Several of the most powerful of the companies in the Internet infrastructure operate in the other levels of the digital economy, generating significant incomes in the electronic trade or ensuring products and services for the application level. Within this level, there operate companies such as IBM, Dell, HP, Cisco, GlobalNet, GSM, RDS, EasyNet etc.

5.2.2. Diversity of the applications of the digital economy infrastructure – a challenge of the New Economy

This level comprises companies whose products and services allow the optimal use of the infrastructure in order to fulfill electronic businesses. On this level, there are manufactured the software products necessary for direct web transactions. Additionally to the manufactures of software necessary for web transactions, the application level includes the consultancy and services companies which design, build and maintain all the web sites, from portals to complete sites for electronic trade. The achievements of the companies within this level are products and services which allow the optimal use of the infrastructure for the fulfillment of online businesses: Internet trade applications; multimedia applications; web developed software; search engine software; online training, web databases. As digital economy and network infrastructure develop, it is expected for the development of applications (audio and *video* technologies and other web applications) to expand significantly. On this level, there activates companies such as Adobe, Macromedia, Borland, Genesys Software etc.

5.3. Features and particularities of digital economy

The particularities of the digital economy are provided in a research performed by Texas University, the Center for Research in Electronic Trade. The research has considered the structure on the four levels of the digital economy with examples of the American achievements for 1998-2000. The research's findings have led to the delineation of the following definitive characteristics of the digital economy:

a) The level of the electronic transactions, out of which a significant preponderance is held by the electronic trade, has noticed a spectacular development, unforeseen. Digital economy is developing rapidly, an example being the American digital economy which developed with a rate of approximately 174.5% from 1995 to 1998, compared with the average global rate (which includes the American digital economy as well) of 3.8% in that period.

b) The dimension of the digital economy has become compatible with the traditional one only in a few years from the implementation of WWW, already contending in size with traditional sectors such as energy, vehicles and telecommunications. The average per employee in the digital economy is over US 250,000, 65% higher than in the traditional economy.

c) The digital economy has become significant worldwide [http://www.idc.com], one of the first 20 economies in the world being the American digital economy. Although the overall incomes in the digital economy are not strictly comparable with the IGP if the American one was (in terms of overall income) "recouped" as nation, it would have the 18th position worldwide, after Swiss and before Argentina.

d) The digital economy has significant impact regarding new jobs and related responsibilities, remodeling labor market. Most worker classes (say, web designer, Internet advisor) did not exist before 1994/1995 and most organizations have changed the work places' structure to adjust to new challenges and opportunities within digital economy. Over six million Americans work in what we call high technology field and, as electronic businesses expand and traditional businesses are depending more and more on the Internet, new jobs will be created and the existing ones will be reformed.

e) The staff working in the infrastructure and application levels generates more than half of the digital economy's income. The incomes of these levels are higher than those of agents and online transactions, which shows the crystallization condition of the digital economy and suggest the need to increase rapidly the levels remained behind. Given that the overall economic activity is heading toward the Internet – or TCP/IP networks, the levels of agents and business will become more powerful. In parallel, there are developed the infrastructure and application levels due to the Internet's expansion worldwide and as organizations will agree to built Intranet and Extranet networks.

f) Organizations within the infrastructure level have the highest income per employee. The Internet infrastructure is the most lucrative of all the levels with the digital economy, many organizations within the infrastructure being powerful and enjoying competence and significant savings. Although they generate a high added value, organizations within the application level decrease their savings because they don't sell just software, but they spend on Internet assistance and web projects to model their products according to each client. This issue is significantly acute for organizations within the online transaction level. Moreover, organizations within the levels of agents and traders have started to operate in the electronic environment recently and traditional organizations which enter into online activities are not able, most times, to operate as accurately as the ones "born" on the Internet.

g) Large companies supply digital economy. Although thousands of new organizations are established within the digital economy, large companies continue to play a significant role. For instance, the first 15 IT companies own a third of the jobs within this field.

h) Even if it is agreeable the image of an electronic world without agents, studies highlight that they play a significant role regarding the establishment of digital economy. For instance, only fees (which are lower than 10% of the value of a transaction) have represented important incomes for certain classes of agents, such as tourist agencies and auction houses.

Understanding the characteristics of digital economy depends on knowing the relations products-costs-quality, analyzed according to the following economic aspects:

■ The curve of the costs for *tangible* (traditional) products and services has a “U” shape because, up to a certain point, as quantity increases, the cost decreases and then the two variables increase simultaneously. For *intangible* (digital) products, the variable cost per unit is very law, almost fix, compared to quantity. The cost per unit decreases as the quantity increases. The classification of products and services in tangible and intangible has taken into account the possibility of their direct examination (which situation suits traditional economy) or the impossibility of this process (situation characteristic to digital economy).

■ The products/services connection becomes stronger in digital economy. Unlike the physical products sold, plus technical services such as installation, repair, etc, the intangible products are subject to update and development services (update, upgrade) which endorse daily update and upgrade of initial parameters.

■ The link between purchase and rental is over-unitary in traditional economy while in digital economy it tends towards a unitary or over-unitary value. This is due to the fact that tangible products could hardly be partitioned, while intangible ones, for instance, software, are easily partitioned among several users.

■ Given that digital economy costs are high, it becomes efficient when a “critic number” of buyers and seller is “achieved”. In traditional economy, products and services may be efficient, regardless of the number of buyers and sellers. In digital economy, the transaction attendees will increase when television and personal computers will be equally spread worldwide.

■ Quality incertitude and the need to increase it are more considerable in digital economy. It represents the consequence, on one hand, of the high prices which must be paid for intangible products and, on the other hand, of the impossibility to “touch” and test the product directly, “at work”. In digital economy, quality is tightly related to the brand, name of the manufacturer, thus new digital companies hardly enter the market.

■ The prices’ nature is a lot more flexible in digital economy. Obviously, prices are important in any economy because they determine the volume of the sales, market partition and profit. In digital economy, products may be offered for discriminatory prices for different clients, according to demographic aspects, preferences and past behaviors etc.

Online agents may allow actors to set the desired product and price offered and then to correlated them in real time. Moreover, in digital economy, the prices' negotiation becomes real for expensive products as well;

- There are two types of prices in digital economy: *online* and *offline.* While certain organizations offer low prices for online products (for instance, *Pacific Brokerage Service -www.tradepbs.com,* offers a 50% discount for online products), others (such as banks) maintain the same price or charge a higher price (certain banks charge a higher fee for online payments than for offline ones; another example in this respect is grocery shop *www.peapod.com*, which offers online products for a price significantly higher than in supermarket). This price diversity changes the microstructure of the consumer's market, distribution channels and the force of the business concluded between parties;
- Within the **new economy**, potential winners and insolvents are easily differentiated and, in general, there will survive organizations which move fast towards the electronic world. Traditional manufacturers must automate and computerize manufacture, to include more electronic components in their products, to appeal to cyber-marketing, to expand their market, using Internet facilities, etc. In their turn, native organizations for digital economy must "humanize" their products, to lower their prices, etc. It is envisaged that potential winners will be the suppliers of Internet services, portal owners, electronic trade software companies, owners of private networks, average manufacturers, interconnection equipment suppliers, advertising and target marketing agencies, large traders, suppliers of payment systems, special and security infrastructures, online business companies, conventional retailers who use frequently online markets, market creators, etc. The potential insolvents will be small whole sellers, brokers, regular sellers, manufacturers without novelty elements etc.

6. Conclusions

The knowledge dynamics becomes crucial for any precise understanding of the way organizations function. Therefore, the knowledge management may determine the competitiveness of an economic agent. As any individual, the organization has a certain memory, a certain capacity to learn and a specific knowledge generation and processing. The Internet infrastructure has become a huge industry on its own and that is why the article stresses the most important aspects regarding the new forms of technology nowadays.

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IMPACT ANALYSIS FOR RISKS IN INFORMATICS SYSTEMS

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Abstract: *In this paper are presented methods of impact analysis on informatics system security accidents, qualitative and quantitative methods, starting with risk and informational system security definitions. It is presented the relationship between the risks of exploiting vulnerabilities of security system, security level of these informatics systems, probability of exploiting the weak points subject to financial losses of a company, respectively impact of a security accident on the company.*
Herewith are presented some examples concerning losses caused by excesses within informational systems and depicted from the study carried out by CSI.

1. Introduction. Problem statement

The International Standard ISO/CEI 17799 [1] *Informatics technology – Security techniques – Code of practice for Informatics security management* defines **risk** as the combination between the probability of occurrence of certain event and its consequences. From the point of view of informatics systems' security, the risk represents the combination between the probability of occurrence of a break in the security system of informatics systems and the impact on the capacity of those systems for carrying out the designed security functions.

The risk can be considered like as a threat that could exploit the possible vulnerabilities of the system, with a certain probability. It is an undesired and unpleasant event, which waits to occur, but that due certain reasons may not appear or through certain methods may be avoided. In order to prevent the occurrence of an undesired event generating considerable impact on the security of informatics systems, security measures should be taken. These security measures are called simply **measures** or **controls**. In this article we are not talking about the security measures that can be adopted in order to reduce the risk level concerning the security.

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Risk level is an arbitrary indicator, denoted L, which allows grouping certain risks into equivalency classes. These classes include risks which are placed between two limit levels – acceptable and unacceptable – conventionally established. It is determined through risk evaluations, on the basis of an adequate combination between the occurrence probability of a security event and the maxim consequences (impact) that the event may have upon the respective system.

The acceptable risk level is the risk level conventionally admitted by the organization management, regarded as not causing undesirable impacts on its activity. This is determined by methodic and specific evaluations.

The residual risk is considered to be the reminder after the risk treatment. As a general rule, the residual risk may be regarded as risk on acceptable level.

Assuming the risk definition set forth upwards, this is a positive real number R which may be represented by the area of a quadrangle surface, having as one side the **P**robability of occurrence of certain event, noted with P, and as the other side the consequences of that event occurrence, respectively the **I**mpact of the undesirable event, noted with I, upon the security of the studied organization. Mathematically speaking, the same area may be obtained through various combinations between P and I, of which the preferred one is quite the product between probability and impact. There are a lot of **P**robability – **I**mpact couples generating the same **R**isk R, defining quadrangles of same area as illustrated in figure 1.

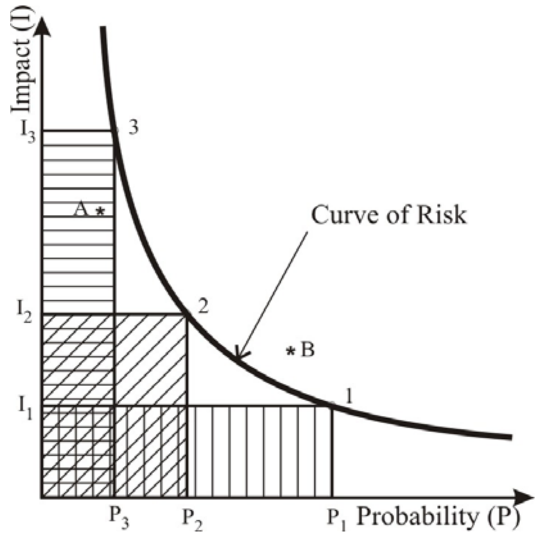

Figure 1. Graph representation for equivalency of Risks defined by different Probability – Impact couples.

If the vertexes of such quadrangles, which are not on axes, are linked through a continuous line it results a **hyperbolic curve** C, named the **Curve of Risk** [2]. This curve allows the differentiation between the **acceptable risk** (Tolerable – T) and the **unacceptable** one (Non-Tolerable – NT).

Thus, the risk of occurrence of a certain event A, with high impact, with serious consequences but low probability of occurrence, defined by coordinates placed below the represented acceptability curve is considered **acceptable**, while the risk of event B, with less serious consequences but high probability of occurrence, of which coordinates are placed upwards the curve, is considered **unacceptable**.

Hyperbolic curve of risk based on couples (**P**robability, **I**mpact) is illustrated in figure 1.

Security function is a function of a system (equipment) by help of which the risk is eliminated or reduced, or is only drawn the attention of a risk occurrence.

Security level is an indicator showing in general the security status of a system. It is determined indirectly, by determining the risk level and it is in inverse proportion to it.

The following relation describes the connection between the risk level and the security level:

$$Security = f(Risk) = \frac{1}{Risk} \tag{1}$$

and is presented in the table 1.

Concerning risk levels, in the present paper, due to practical reasons, for significations present in column 2 we will use the abbreviations present in column 3 of table 1.

Table 1

Relation between the risk level and the security level

No.	Risk Level	**Risk signification**	**Notation**	Security Level	Security signification
0	1	2	3	4	5
1	R_1	Minimum, Non risk	N	S_7	Excellent
2	R_2	Very low	VL	S_6	Very good
3	R_3	Low	L	S_5	Good
4	R_4	Medium	M	S_4	Acceptable
5	R_5	High	H	S_3	Low
6	R_6	Very high	VH	S_2	Very low
7	R_7	Critical	C	S_1	Insignificant

Risk criteria are reference terms against which the meaning of the risk (level) is determined.

There are several risk criteria categories, such as for example:

– specific consequences;
– costs or associate benefits;
– socio-economical aspects;
– the perception of interested parties;
– the occurrence frequency of security incidents;
– the cumulated effects of some incidents occurrence;
– uncertainty rate of determined risk level and of accepted trust level;
– the residual risk level admitted by each organization.

2. Economic improvement of the Security Level

All risk *t*, 20% of the costs are reflected in accomplishments of desired benefits in a percentage of 80% – in our case minimization of the security costs. A maximum security S_7 (respectively an increase of only 20%) can be obtained with addition of extremely high expenses, practically with 80%. In figure 2 is shown the graphic illustration of this law concerning relation costs-benefits in order to achieve security of informatics systems.

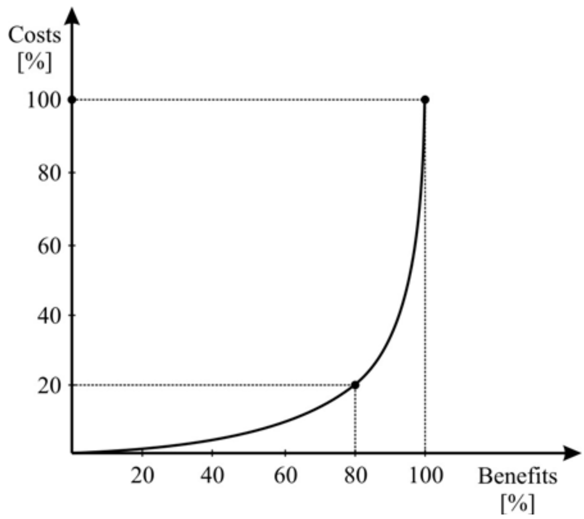

Figure 2. Relationship costs – benefits.

The manager of the organization is the person deciding how much is he willing to pay for the security, decision that is taken on the basis of a technical-financial analysis.

Security is very difficult to quantify. One can never state that within the organization we have a certain level of security. It only can be estimated as an excellent, very good, good, acceptable, low or insignificant level of security. Even though, an evaluation (at least financial) can be made concerning the security level.

Always implementation or tests and improvement the security generate costs concerning equipments, personnel and time spent for implementing security, including checks or simulations Those risk assessment are carried out mainly in the big companies and eventually in the medium-size ones. In small organization there is no enough personnel trained to do that kind of analysis and also there is no money to pay a specific company to do it. Taking all that into account, the least they could do is to spend some money for a minimum set of security measures. It is well known that managers do not invest in things that are not profitable at that moment and that, when they do spend different amounts of money, those are under the imposed limit for that kind of costs. In such case, there must be assured a security and its costs has to be under the line of a certain amount of money.

In order to measure the security level within a company with concern to means utilized for automatic data processing there can be utilized a ***financial Security Indicator***, noted as SI_f, which is defined by the following formula:

$$SI_f = \frac{Ce + \sum_i^n P_i \cdot Cc_i}{Ce} - 1 \tag{2}$$

where: Ce – the cost of calculation technique equipment and software used;

Pi – the weight of the measure agreed in the security system;

Cc – the cost of the controls for the equipment or software adopted.

In the case in which there are no investments in equipment assuring security the value of this indicator is zero.

When $0 < SI_f \leq 1$ – there is a minimum security level, but not inexistent.

When $SI_f > 1$ then we can say that the way in which it is assured the security is more expensive than the equipment itself. In this case, the reasons are several, such as:

– the risks were not well evaluated and/or security measures were exaggerated;

– the equipment (computer) is not of quality and needs additional equipments;

– the up-dated equipment value is low as compared to the cost of security measures.

In the first case, not to lose information, there has been made an exaggeration about the security measures – like financial investment or like number. In other words, the prices for those investments are too expensive or it is invested more than it is needed.

The second case occurs when there are bought poorly-qualitative computers and later equipment that may compensate this situation are required. This is the case of computers ”no names” or even if they have, they are old and no longer present safety under operation. It is quite often encountered the situation when new bought computers get blocked in case of electric voltage fluctuations.

The third case is encountered when the cost of equipment is up-dated to the new value. If the computers as well as the control equipments were bought at close data and the cost of the equipment as well as that of control measures are simultaneously up-dated, then the SI_f does not change so much.

When is desired an optimization formula for the software price, then we have to consider that the license to be bought can be utilized for each personal computer (PC) to be installed on and that only certain operating soft beneficiate of installation licenses only on the server serving these PC.

There will be taken into account the costs for all software installed on the respective computer, up-date cost for each of it, possibility of a breakdown for the hard disk and need to reinstall the program on that computer, cost of antivirus to be installed and its permanent updating. All these special soft have various prices. For example a Windows XP 2003 license, type OEM (can be installed only one time) costs about 150€ and a license that can be reinstalled, for the same software product, exceeds 350€. There must be carried out an analysis that should consider the probability of PC breakdown, due to several reasons, requiring reinstallation about 2 (two) times within 3 (three) years – period of time agreed as moral wear of this software, after which a re-installable license is purchased.

The price for an AutoCAD license exceeds 3500 € and it can be reinstalled on any computer, subject that the program is not running on several computers simultaneously. The acquisition of “subscription” option for the respective AutoCAD costs approximate 220 € per year and allows an automatic up-date to the versions to be created.

There is also the opportunity to “rent” AutoCAD license, for a certain period (for example 3 years), for 40-60 € per year for each PC, an offer that is starting to be successful.

All these economical analyses are part of the organization development strategy, representing an independent and difficult actions.

3. Economically analysis of the impact reduction

There are several methods for estimating the possible losses and the calculation of recovery costs.

An already classical method for economical analysis of the threat impact mentioned in ISO/IEC 13335-3 *Informatics technology- Guidelines for the management of IT security – Part 3: Techniques for the management of IT security* is called **A**nalyzed **L**oss **E**xpectancy – ALE. It is a simple quantitative method, which allows estimation of the possible losses based *on already existent company records, kept by an expert, in the organization.*

ALE depends on the value of the asset, on the possibility to loose its value, vulnerability to the respective threat and threat occurrence frequency. This method is below briefly presented:

The following abbreviations are proposed:

AV = **A**sset **V**alue – the costs needed for replacement and those caused by losses of any of the organization assets (tangible and intangible);

PVL = **P**otential **V**alue **L**oss – measures the impact on assets or quantifies the loss when the event occurs, being expressed in percentage (%) against the asset value;

ARO = **A**nalyzed **R**ate of **O**ccurrence – frequency the event is expected to occurred. It is a statistical estimation usually provided by some specialty institutions based on some data that it already has.

ALE is calculated according to the formula:

$$ALE = AV \times PVL \times ARO \tag{3}$$

There can be included another measure – recalculated ALE – representing the maximum value of the security investments that we agree to spend for protection against threats.

Calculation example:

ARO (*frequency*) – *has value* $\frac{1}{10}$ (*the event can occur once every 10 years*)

PVL (*potential asset value loss*) – 50%;

AV (asset value) is considered at 50000 €;

ALE = AV × PVL × ARO = 50000 × 0.50 × 0.1 = 2500 €

In other words, there is the possibility that due to not taking into account the risk associated to this asset loss, the organization may loose every year 2500 €.

Sometimes another indicator is utilized – SOL (**S**ingle **O**ccurrence **L**oss) – when the company is interested on the value of the damage caused by a single occurrence of a threat in case of extremely important assets and the second occurrence does no longer count. It is similar to ALE, but the result of this analysis forms the response to the question "**It is possible to assure or not**" to continue the business in case of disaster.

Calculation example:
AV = 5 millions €;
PVL = 10%;
$SOL – AV \times PVL = 5 \times 10^6 \times 10^{-1} = 500000$ *€.*

Both methods may serve as basis for impact assessment, when all assets, weak points, threats and their occurrence frequencies have been identified. The impacts can be determined and consequently grouped into logical structured groups.

There are several such structure modalities, the basic example having only with 2 impact levels: "tolerable" (T) or "Non-Tolerable" (N).

The method recommended by ISO IEC TR 13335-3 is to complete a simple matrix, while considering the possible loss value and the threat occurrence frequency. For both variables there are decided 5 classes, the highest one being 4 as can be noticed in table 2.

Table 2

Impact assessment considering loss value and the threat occurrence frequency

Possible losses	0	1	2	3	4
Frequency					
4	0	NT	NT	NT	NT
3	T	NT	NT	NT	NT
2	T	T	NT	NT	NT
1	T	T	T	NT	NT
0	T	T	T	T	0

In this case the impact can be associated with the risk.

4. A case study about the evolution of the impact

Every year, Computer Security Institute (CSI) carries out different studies with the participation of the San Francisco Federal Bureau of Investigation's Computer Intrusion Squad.

These studies are used by every company to concentrate their efforts to counteract the most frequent attacks. The survey results are based on the opinion and the responses of different computer security practitioners in U.S. corporations, government agencies, financial institutions, medical institutions and universities.

The CSI survey it is said to have been conducted anonymously as a way of enabling respondents to speak freely about potentially serious and costly events that have occurred within their networks over the past year. Organizations covered by the survey include many areas from both the private and public sectors. The sectors with the largest number of responses came from the financial sector (20 percent), followed by consulting and education (11 percent), informatics technology (10 percent), and manufacturing (8 percent). The diversity of organizations responding was also reflected in the 9 percent designated as “Other” [6].

Taking into account the information given by CSI, we made comparisons between the last two years, 2006 and 2007 for distinguishing the losses suffered by the companies. The average annual loss reported in 2007 survey shot up to $350424 from $168000 the previous year. The result are presented in table 3.

Table 3

The evolution of the losses between 2006 and 2007

Type of abuse	Value 2006 USD	Value 2007 USD	%
Financial Fraud	2556900.00	21124750.00	7.26
Virus	15691460.00	8391800.00	–0.47
System penetration by outsider	758000.00	6875000.00	8.07
Theft of confidential data	not mentioned	5685000.00	
Laptop or mobile hardware theft	6642660.00	3881150.00	–0.42
Insider abuse of Net access or email	1849810.00	2889700.00	0.56
Denial of services	2922010.00	2888600.00	–0.01
Fishing	647510.00	2752000.00	3.25
Bots	923700.00	2869600.00	2.11
Theft of proprietary info from mobile device theft	6034000.00	2345000.00	–0.61
Theft of confidential data from mobile device theft	not mentioned	2203000.00	
Sabotage of data or network	260000.00	1056000.00	3.06
Unauthorized access to informatics	10617000.00	1042700.00	–0.90
Web site defacement	162500.00	725000.00	3.46
Telecom fraud	1262410.00	651000.00	–0.48

Type of abuse	Value 2006 USD	Value 2007 USD	%
Misuse of wireless network	269500.00	542850.00	1.01
Misuse of public application	not mentioned	251000.00	
Instant messaging abuse	291510.00	200700.00	–0.31
Password sniffing	161210.00	168100.00	0.04
Blackmail	not mentioned	160000.00	
Exploit of yours organization DNS server	90100.00	104500.00	0.16
Abuse of wireless network	469010.00	not mentioned	
Others	885000.00	123500.00	–0.86
Total	**52494290.00**	**66930950.00**	**0.28**

At this survey participated in the year 2006, 616 computer security practitioners and in 2007 – 494. The results of this survey indicate that cyber crime is a critical concern. Every organization is vulnerable to numerous types of attack from many sources and the result of an intrusion can be devastating in terms of lost assets and good will.

Insider abuse of network access or email edged out virus incidents as the most prevalent security problem.

It is easily seen that the most important losses are because of the system penetration by the outsiders and the financial fraud. The losses caused by those factors are very import – the sums were raised with 8.07% and with 7.26%.

It is seen that the biggest loss in 2007 is due to financial fraud and not by viruses like in 2006. I cannot say that these results can be expanded to all the companies because of the small number of respondents but I can consider them important because the survey is sent to roughly the same group each year.

About the costs of computer crime – there are some informatics about those cost and the percentage of IT Budget Spent on Security. 61% said that their organizations allocated 5% or less of their overall IT budget to informatics security like it is seen in the figure 3.

A quick comparison of the bars at 3 to 5% level shows a significant uptick in 2007 and we must notice that in 2006, 47 % said their organization allocated less than 3% of the total IT Budget, whereas this year only 35% fell into that range.

The general picture is that security programs budgets are slightly up. Expressing the budget as a percentage of the IT budgets means that the actual number of dollars spent depends on whether the IT budget is growing or decreasing.

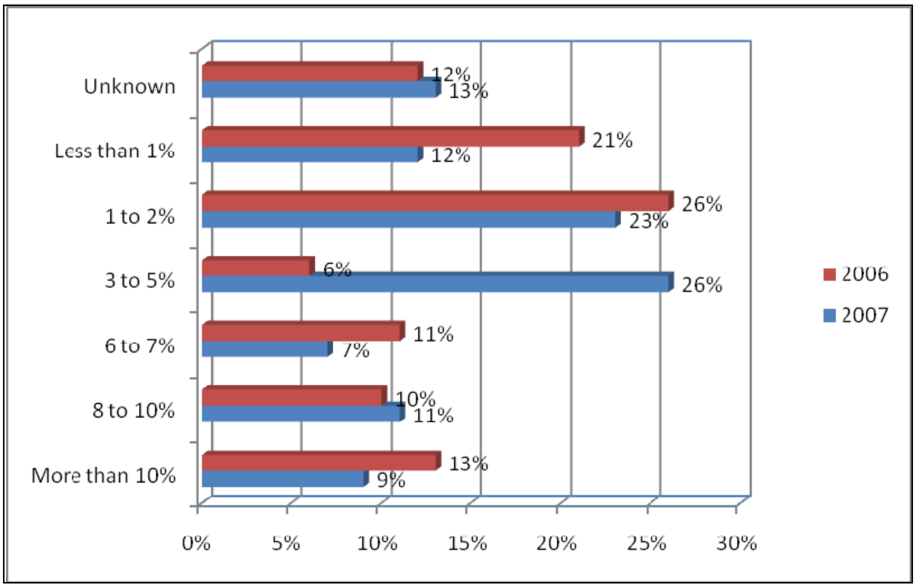

Figure 3. The percentage of IT Budget Spent on Security.

5. Conclusions

Can say that security is improving within companies, but, the losses to cybercrime are bigger this year than the last one.

The country's economy relies on networked computer informatics systems for communications, energy distribution, commerce, transportation and in other domains. It is known that cybercrime and the attendant threat of identity theft reduce user and consumer confidence, reducing the acceptance of e-commerce.

And so, computer security has moved to a position of prominence in most organization being a critical activity that helps protecting the systems. But, we can say that if we want to have a certain security level we must first know the threat we are bewaring of. And after that, when testing the organization's security, we must act like a real hacker because in that way we can discover all the lacks in our system and then, periodically we must develop new vulnerability tests.

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PARALLEL NEURAL NETWORKS FOR EDGE LOCALIZATION AND CONTOUR INTEGRATION

Florin ȚENEA* and Tudor NICULESCU**

Abstract: *The paper describes a new type of artificial neural network – ANN for edge-based pattern extraction problems. In the model, each neuron represents an edge with continuous state variables describing its location and orientation. Each neuron adjusts its state to increase its membrane potential, which results in highly adaptive dynamics of the synaptic weight distribution. The network allocates multiple neurons with different orientation modes for each edge. The strategy allows accurate modeling of multi-modal distributions at key-points such as corners and junctions. As a result, the network delineates edges at sub-pixel accuracy while preserving key-points. It is also capable of processing a sequence of images and following moving objects.*

1. Introduction

The human vision system can rapidly extract patterns from a set of features such as edges with extreme reliability. Although it uses an extensive amount of high-level knowledge to separate patterns from noise, we believe that the underlying signal processing mechanism with local operations alone still outperforms most artificial counterparts. It is evidenced by the fact that we are capable of extracting arbitrary patterns from noisy observation as long as the patterns form some well-organized shapes.

One of traits existing in most biological systems but lacking in common artificial systems is a high degree of adaptability at many levels: biochemical, anatomical, morphological, and architectural.

Such high adaptability in biological systems has been claimed responsible for their superior performance, flexibility, and fault tolerance. More recent studies have reported that morphological changes can take place in the order of seconds to enhance the perceptual capability of the system.

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Traditionally, edges are first detected by a simple gradient operator and salient sets of edges are selected to form patterns. Typically, an edge is represented by its location and orientation angle. The pixel coordinate is used for an estimate of the location, and the direction computed from the gradient field is used for an estimate of the orientation angle. Thus, the performance of the system is heavily dependent on the initial estimates.

In this paper, adaptive behavior is introduced to the system by allowing each edge representation to change so that the edge fits better with its neighbor edges. Each edge is represented with continuous adjustable attributes:

- orientation: $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2},\right]$;
- spatial location of the receptive field center: $(x, y) \in \Re^2$.

The system uses multiple neurons to represent different representations of the same edge, and provides a competition mechanism with which only one neuron survives to represent the edge.

We first design a network for edge localization task and later, we extend it for contour integration.

2. Edge localization

The system has multiple sheets of orientation sensitive neurons, which are divided into two layers, as shown in figure 1:

- input and
- relaxation layers.

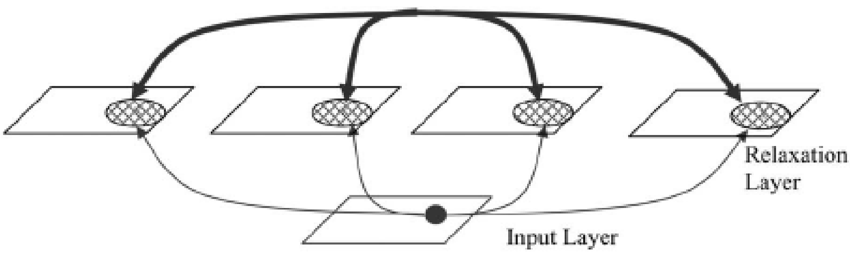

Figure 1. The neural network of the edge localization system.

Neurons in the input layer are sensitive to illumination changes in their receptive fields, and act as gradient operators. Each input neuron computes the strength of the gradient and the direction perpendicular to the gradient from a sub-image inside its receptive fields. The activities of the input neurons are conveyed to the next K sheets of neurons called relaxation neurons, where K is the number of orientation modes available to the system. Each relaxation neuron receives feed-forward inputs from input neurons in its retinotopic neighborhood, excitatory inputs from relaxation neurons in its topological neighborhood, and inhibitory inputs from relaxation neurons at the same receptive field but in different orientation modes.

In the relaxation layer, each neuron adjusts its orientation and location so that it fits better with the neighbors. Criteria for the fit are problem-dependent. In general, each neuron obeys the following discrete dynamic rules:

$$s_{im} = \sum_{j \in n_i} g_{ijm} p_{jm} + \alpha \sum_{j \in \hat{N}_i} \hat{g}_{ijm} \hat{p}_j,$$

$$p_{im} = \frac{p_{im} s_{im}}{\sum_n p_{in} s_{in}}.$$

where $N_i and\_\widetilde{N}_i$ are sets of relaxation neurons and input neurons that are neighbors to e_{im}; respectively, g_{ijm} and $\widetilde{g}_{ijm}$ define the compatibility of e_{im}; and α is a constant.

Second eq. provides competition among neurons across the orientation modes.

i, j, k – location indices;
l, m, n – orientation indices;
e_{im} – a relaxation neuron at the site i with the orientation m;
$\widetilde{e}_j$ – an input neuron at the site j.

Each relaxation neuron is represented by three attributes, x, y, θ. We denote the attributes of e_{im} as x_{im}, $y_{im.}$ and θ_{im}. We also use s_{im} and p_{im} to represent the membrane potential and activation level of e_{im}, respectively.

Similarly, we use $\widetilde{x}_j, \widetilde{y}_j, \widetilde{\theta}_j, \widetilde{s}_j$ and $\widetilde{p}_j$ for the state variables in $\widetilde{e}_j$: $\widetilde{\theta}_j \in \left[-\frac{\pi}{2}, \frac{\pi}{2},\right]$.

For edge extraction, we favor straight edge alignment and we are using the following formulas:

$$g_{ijm} = g_r(e_{im})\xi(\theta_{im} - \theta_{jm})$$

$$\hat{g}_{ijm} = G_e(e_{im})\xi(\theta_{im} - \hat{\theta}_{jm})$$

where:

$$G_r(e_{im}) = \exp(-((x_{im} - x_{j0})\sin\theta_{im} - (y_{im} - y_{j0}) \times \cos\theta_{im})^2 / 2\delta_r^2)$$

$$G_e(e_{im}) = \exp(-((x_{im} - x_{j0})\sin\theta_{im} - (y_{im} - y_{j0}) \times \cos\theta_{im})^2 / 2\delta_e^2)$$

represent synaptic weights from e_{jm} to e_{im} and from $\widetilde{e}_j$ to e_{im} and

$$\xi(0) = \begin{cases} \cos(2\theta), & \cos 2\theta \geq \cos(\pi/k), \\ 0, & \text{otherwise} \end{cases}.$$

Each neuron updates its attributes, (x, y, θ) to increase the membrane potential. The spatial attributes, (x, y) are updated with simple gradient ascent.

$$\frac{\mathrm{d}x_{im}}{\mathrm{d}t} \propto \frac{\partial s_{im}}{\partial x_{im}} = \sum_{j \in N_i} \frac{\partial g_{ijm}}{\partial x_{im}} p_{jm} + \sum_{j \in \hat{N}_i} \frac{\partial \hat{g}_{ijm}}{\partial x_{im}} \hat{p}_j,$$

$$\frac{\mathrm{d}y_{im}}{\mathrm{d}t} \propto \frac{\partial s_{im}}{\partial y_{im}} = \sum_{j \in N_i} \frac{\partial g_{ijm}}{\partial y_{im}} p_{jm} + \sum_{j \in \hat{N}_i} \frac{\partial \hat{g}_{ijm}}{\partial y_{im}} \hat{p}_j.$$

An update rule for the orientation is given by the following formula:

$$\frac{\mathrm{d}\theta_{im}}{\mathrm{d}t} \propto \frac{\angle F_{im}}{2} - \theta_{im}$$

When no significant stimulus is present within the receptive field (so that there is not enough amount of stimuli from the input layer), the neuron does not obey the dynamics described above but returns to its default state.

The input is considered significant if the gradient strength at the receptive field exceeds some threshold. The default spatial attributes for a site i is the location of the corresponding fixed lattice location (x_{i0}, y_{i0}); the default orientation for an orientation mode m is $m\pi/k - \pi/2$ and the default activation level is $1/k$.

The update rules of the internal attributes induce adaptive changes in the synaptic weight distribution. When a strong gradient is present in the receptive field of a neuron and the orientation of the gradient is close to the neuron's default orientation, the neuron adjusts its internal attributes so that the synaptic distribution matches more closely with the gradient strength field in the receptive field. The interactions among relaxation neurons alter the attributes so that they approach a collinear formation. As the pattern moves, the neuron keeps modifying the attributes so that the distribution profiles of G and $\widetilde{G}$ follow the moving pattern. When the gradient orientation deviates too far from the neuron's default orientation or the pattern leaves the neuron's receptive field, the neuron adjusts its internal attributes back to the default state.

3. Contour integration and edge linking

The contour integration process is to group edge fragments and produce a contour that agrees with our visual perception.

Figure 2 shows our contour integration network. An additional layer is added on top of the relaxation layer and is called saliency layer.

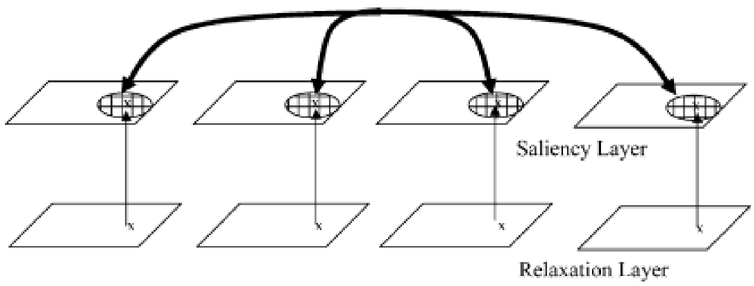

Figure 2. ANN with saliency layer.

The new network is responsible for computing saliency measures of relaxation neurons, linking those with significant saliency measures to form contours, and detecting key-points.

The main problem is to select a subset of edges and the order of links that best describes the underling contour of the object. Edges are selected based on some saliency measure computed locally and iteratively.

For solve that problem we propose the following interaction model:

$$\tilde{s}_{il} = \max_{jkmn} r_{ijklmn} \sqrt{\tilde{p}_{jm} \tilde{p}_{kn}} + s_{il} p_{il}$$

$$\tilde{p}_{il} = \tilde{s}_{il}$$

where $\tilde{s}_{il}$ and $\tilde{p}_{il}$ denote the membrane potential and the activation level of a saliency neuron. *s* and *p* (without ~) denote the membrane potential and the activation level of a relaxation neuron.

Empirically, we use the following formula:

$$r_{ijklmn} = \cos^2(\theta_{il} - \theta_{jm}) \cos^2(\theta_{il} - \theta_{kn}) \cos(\alpha_{jikl/2}))$$

where α_{jikl} is an angle formed by line segments connecting from $(x_{j0,} y_{j0}) -$ $-(x_{ie,}\ y_{ie})$ and from $(x_{ie,}\ y_{ie}) - (x_{k0}, y_{k0})$.

After the convergence of the network, p gives a saliency map. A neuron's activity is suppressed if it is not on a ridge of the saliency map along the edge direction of the neuron. A neuron's response is also suppressed if its saliency is not the maximum across the orientation modes.

A remaining active neuron searches its neighborhood for a pair of active neurons that provides a positive saliency measure.

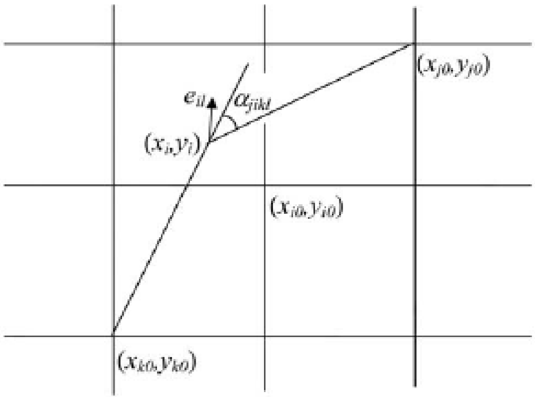

Figure 3. Segment and angle representation.

The search starts with a neighborhood of the checker-distance.

1. If no grouping within the neighborhood with a positive saliency measure is found, then the neighborhood area is increased to the checker-distance;

2. The search continues until a set is found or the maximum neighborhood size is reached. If the search is successful, a contour segment is formed using the spatial attributes of the three neurons. Otherwise, the neuron's activity is suppressed. In our experiments, we stop the search when no grouping is found for the checker-distance;

3. Even with such small neighborhood size, most edges are grouped properly in our experiments.

4. Conclusions

The paper proposes a new type of ANN with sub-pixel edge localization and contour integration for all kind of patterns.

The novelty of the network is that it implements controlled synaptic adaptation in terms of neuron's internal attributes.

Another important characteristic of the neural network is that it provides competition among different candidate representations at each retinotopic site of patterns.

That ANN is considered applicable to other vision tasks including texture segmentation, color perception and patterns classifications.

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