ECONOPHYSICS Section
GROTHENDIECK TOPOS, ZETA COMPLEXITY, AND ARROW OF TIME: NEW CONCEPTS FOR A MODERN PROJECT MANAGEMENT

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Abstract. A critique of mathematical paradigms that implicitly underlie project management methodologies leads the authors to consider a new class of project management based on Grothendieck's Topos. The design of the management based on these Topoi must be associated to uncertainties and arbitrariness that always govern the partition of projects into linear elementary tasks (disjoint or not) and then render their organization highly determined (either through determinism or random features). Zeta properties associated to the matching between overall and local constraints are emphasized leading the design of a new class of hyperbolic management named zeta management. The fundamental role of scaling between the different tasks and different tempi is analyzed in detail. We show why the role of the project manager must be rethought as well as the role of the field actor whose psychological profile must be enlarged to account for responding to the new constraints of the globalization. Despite the formalism, this note is not intended for any mathematical purpose but it aims a managerial opening onto a geometrical point of view in place of the usual analytical vision.

1. Introduction

Economic growth is a major factor in the stability of modern societies. The idea that generally prevails is not that of social equilibrium but that of dynamic stability. In this context, the mastery of evolutions, which may not be confused with the notion of progress, is a requirement imposed to the managers. It enforces itself from top to bottom of the hierarchies which structure ours societies (Government, Countries,
Enterprises and now Individuals). It appears all the more necessary because the acceleration of change generates frustrations and gives rise to resentments. These resentments feed the nationalisms. It is easy to understand the roots of this ideology: the need to rebuild a lost causality or a dream for linear historic tale. The most paradoxical avatar of this need for narrative is the role of substitution played, if required, by chance. The notion of project that historically intends to avoid the pitfalls noted above (determinism and chance) is not new. If we leave aside the idea of a “project of marriage” mocked by Moliere, one must, in economics, go back to the beginnings of the industrial era, in the 17-18\textsuperscript{th} century, for discovering a primitive formalization of rational projects. The idea of projection that underlies the idea of project gives a humanistic status and a meaning at historical evolutions. In other words, any action takes its source somewhere in a universal exteriority reified by the being. The notion of rational management of the action becomes explicit and leaves aside the being during the 19\textsuperscript{th} century. It is expanding since that period. The concept subsumes military expectations that take into account only the uncertainties for zero-sum games, and then goes beyond the hypothesis of additive balances. The economic project thinks the production of new wealth, the profitability, the added-value, the reinvestment. It opens onto non-additive thermodynamic issues. However, the absence of additivity is generally considered as external with respect to the project (The idea is an independent input; the market is treated as a thermostat etc.). The aim of project management is to consider the process as a sum of steps able to be added arithmetically \[1, 2, 3\]. In this respect, its financial optimization is fundamental for the project manager because the overall cost must be identified with a thermodynamic extensive variable (energy, utility etc.). Evolution and changes are considered manageable if the organization is able to segment the overall action within perfectly independent set of tasks \[4, 5, 6\]. However, the context of globalization doubly undermines this constraint because (i) the project must take into account intermediate externalities that link the items between them (for example, the price of inputs depends on local needs but also of the overall requirements of the project and of international context), and (ii) because it interacts with medium and long-term strategy of the organization, in an economical context characterized by large fluctuations \[7\]. This context, which recalls the military context in the times of crisis, must be compared to the shift from a locally Euclidean environment without major retroaction, – namely having at disposal “entropic bins” (colonies, oceans, space of conquests.
etc.) – to a hyperbolic universe; a world of Pokémon type. In a Pokémon world, discrete and cumulative multiscale local interactions construct a closed geometry around a fuzzy narrative in the context of an algebra (depicted by pre-defined grammar) applied to the items and successive steps, where entrance and exit are in equivalence. The request for (i) a dynamic causality and (ii) a recursive earning, becomes a dream difficult to manage by using another method than the mere intuition. This situation can be highly destabilizing for all the people that, in the organization, does not play dice with the future. We should be able to assert that the rise of populisms and the powerlessness of the rulers are, in our democratic societies, the direct consequence of our inability to think rationally about hyperbolic context. The world became the finite checkerboard of a game characterized by fuzzy scale rules. It becomes more and more difficult to anticipate the consequences of even well-thought actions. One must consider the evolution of certain societies towards “elected dictatorship” as attempts to reconstruct a form of Euclidean determinism and of vanished simplicity. Similarly, the use of referendum in the framework of binary choices and the war as well, are also the expression of a need for clear causality, including under its degraded expression of risk and chance.

In this context, the responsibility of the intellectual elites is large because their duty is to light up the minds and to offer cognitive alternatives for depicting the world from models different of those which structure our “Euclidean Education” [8, 9]. Today, thought and pictured in the Euclidean framework (based on linear causality), many rational of actions fail and thus cause a loss of cognitive confidence. The failure of these types of representation generates social frustrations, social dissipative drags and even a dangerous increase of death instinct. In the international political context that is now ours under the yoke of new technology, the need for rethinking a modern management of action could prove to be vital at short term [10, 11, 12]. But the new ways of thinking can no longer put aside the scaling process from local to global. In an arithmetic framework this scaling amounts to emphasis the arithmetic role of lattices bounded by the greatest common multiple (that projects the organization towards the future), and by the smallest common divisor (which requires to take in consideration the individuals requirements, like part of overall history). The overcoming of the linear usual intuitions could probably be written in a more rational way in geometrical terms; this is the main hypothesis of this note dedicated to econophysics.
In this new cognitive landscape, a constructive criticism of economic tools is necessarily based on a criticism of the usual concepts of physics, too elementary to be relevant in complex environment. Obviously, all metastable and multi-phasic states, too complex to be formalized, calculated and even more transposed, are forgotten or more seriously artificially justified for chancy reasons. These reasons can cross out Euclidian determinism but this last is only reduced, in this shift, to the sole deterministic hyperbolicity associated to the chance. Because of its purely operational scope (the current state of knowledge in maths) the substitution propagates this paradigm in the field of economics and finance [13, 14, 15] with numerous damages. On the theoretical level, this choice is due to the fact that besides to the Euclidean determinism, it is the only scientific representation perfectly well assured on its formal foundations: the theory of measurement and its children, the theory of probabilities and that of ergodicity.

The risk is the only theoretical field able of being brought back quite simply to a determinism that does not assert of its name. Psychologically, chance is often chosen by people disoriented by history because, within Euclidean way of thinking, a total uncertainty is paradoxically more intelligible for the doxa than a partial uncertainty. It is well known that the man who loses his keys in a shade tends to look for them in the only light of the lamppost preserved lit by instituted powers. To give an image that is unfortunately not a caricature, the only alternative that mathematical physics offers to our refusal to be a grain of sand on a drumhead excited by chance, is that men and women who work for creating wealth devote half of their efficient energy to access to their working place. We see herein the danger of being prisoner of inconsistent physics proposal for economy. The absurdity of this image is obvious but it is nevertheless inscribed in the filigree of physic doctrine of almost every statistical representations of the economic world.

Since the question posed herein is the question of the link between physical models and economics, let us observe that the most advanced works in physics are now based on a “geometrization” of our relation to the world [16, 17]. The prevailing idea is that a physical process follows a geodesic on an underlying geometry and that this geometry is thus the relevant representation of our environment. The symplectic geometry is for instance the geometry able to represent the mechanics of the continuous of space-time surfaces. It introduces the general relativity. But geometry is also a group of authorized transformations upon this surface. After von
Neumann's analysis, we are today indebted to Alain Connes for focusing the thought on the role of operator algebras and of the groups for explaining the quantum mechanics and the standard model relevant for the physics of particles [18]. The associated geometry is called non-commutative geometry to take into account the irrevocable uncertainty of our relationship to the world. The link between all these ideas is due to the relation which associates the notion of transformation group, hence of reversible or irreversible evolution, and the concepts of the geometry (Symmetries, Ambiguity, Tiling, Topology, Homology, Morphisms [19]). The well-assured formalization of ambiguity in the framework of the algebraic geometry explains the current interest for non-commutative geometry [18]. The use of this geometry for building modern project management upon geometric concepts, might contribute to introduce naturally the irreversibility and the fact that project always aims to cause a marginal change of the state of the world. We shall analyze herein the advantage that might be reached by designing a new “geometrization” of the economy. By assimilating the monetary exchange to the dual of the Riemann zeta function seen as a mere Kan's extension of functors [11] in the framework of category theory, this research has already been undertaken. The present note reports the continuation of this preliminary analysis by considering the project management in the framework of Topos. The aim of the work is to build an atlas of our modalities of handling economical complexity analyzed by the means of an optimization of the efficiency of action.

The results of previous work on the Riemann zeta function, both as a universal function for approximating multi-valued analytic functions and as a trace of our ability to think some categories of arithmetic complexity [20, 21], have led to consider the rebuilding principles of hierarchisation and of division of a finalized project into a set of elementary tasks. It is by means of the $\alpha$-exponentielle concept that we suggest to reformulate some of the most elementary modalities of project management (Euclidian environment, additive framework, external added value etc.). Certainly, the suggested extension does not shut the issues involved by the notion of progress in hyperbolic environment, – progress that must be analyzed in a non-linear and non-additive framework –, but, due to concepts introduced through the Topos, the analysis boosts the heuristic potential of a geometrization of the economy, at a level unknown to this day. We shall back our reflections to the notion of Topos, because this concept gives an almost geographical intelligibility to complex relationships. In doing so we
are convinced that then, clear scientific representation of the complexity of our human relationships is again logically thinkable. The Topos will help us to overcome of the elementary paradigms that coerce unfortunately today any relationship with a world that escapes to minds if it is too much Euclidean and Cartesian. The current social confidence being largely compromised by obsolete cognitive reflexes, Topos should help us to rebuild a confidence in an understandable future. The limiting cognitive paradigms, leading to the political regressions to which the world is now dangerously confronted, should then be overcome. Moreover we have to be able to attribute the current democratic regressions to a cognitive dissonance of the elites. These dissonances perfectly beheld since the static borders of a society in rapid evolution. In order to deal with them rationally, we will back our reasoning by starting from Alexandre Grothendieck’s scheme theory herein based on the Yves André lectures [19] and those of Alain Connes [18] in College de France. The theory will be pictured within the scope of our physical representation of the zeta-complexity, theory currently in progress, here applied to project management.

2. Category. Sheaves. Topos

The tools usually required for managing the creativity are mostly ineffective. Indeed, creativity is associated with open-systems in the dual meaning, namely (i) mathematically open (systems without borders, only limited by infinite values or characterized by internal singularities) and (ii) physically open (crossed by external flows). As our relationships with the world are governed from global quantitative representations (and influence) through local qualitative measures (fields, densities) we must take into account the links between these two categories for getting a practical relevance of our actions. To fulfill this requirement we can use the mathematical Grothendieck’s works. They allow us to understand that mapping based on scales and called Topos, may usefully be considered. The Topos is the overall set of analogies, relationships and links (applications, transformations, morphisms, functors), between objects within different categories of our experimental universe. The Topos projects our local state onto a global representation of this universe. The Topos appears as a kind of GPS able to guide our will, hic et nunc, within this overall representation (after and anywhere). Thus, Grothendieck gives a mathematical formulation to the Schopenhauer's thought: a world
as a will (local sheaves, bundles of relations) and as representation (global).

2.1. Topos and sheaves

The study of the set of “points” \( X \) and their spatial relations is usually called topology. This name theorizes the notion of vicinity and of continuity. B. Riemann led the foundations of this concept in the mid of 19\textsuperscript{th} century. It was later metrically upgraded by Felix Hausdorff, a German mathematician who perished in Nazi camps. This theory is based on the notion of "open set", namely a part of a space without boundary whose algebra is based on the notion of gluing (sum) and Intersection (difference). A vicinity \( v(x) \) of a point \( x \in X \) is an open part of \( X \) which includes \( x \). An open-set is a subset of \( X \) which is close to each of its points. The vicinity of any point \( x \), \( O(x) \) defines what is called a mathematical filter for \( x \) (any part of \( X \)) that contains a vicinity \( V_{o}(x) \) of \( x \), likewise, all intersections except the empty set; indeed empty set cannot be an element of any vicinity since \( x \) belongs to all its adjacent sets. A set is naturally open if and only if it is its own interior (the interior \( A \) of a part of \( X \) is the set of points of \( A \) for which \( A \) is a vicinal set). Interior is written \( \mathring{A} \subset A \). A boundary is neither in the interior of \( A \) nor in the interior of its complementary \( X \setminus A \). The closed complement of an open set is qualified as "closed". We can observe that this topological definition requires the definition of a "flag" to characterize the set. A Topos must be inhabited and ordered. The notion of inclusion introduces a partial order and union \( \cup \) and intersection \( \cap \) rise associative composition law which grants the set of the mathematical properties \( O \) a trellis whose maximal value is associated to \( \cup \) and with a minimal value is associated to \( \cap \). In practice under the reserve of weak restrictions it is possible to construct \( X \) only starting from the trellis of the open sets \( O \); namely, we identify the point \( x \), with the filter of its vicinal open sets \( O(x) \). Thus it is easy to understand that if is a continuous application such as \( f : X \to Y \), the inverse image of an open set of \( Y \) is an open set in \( X \). It must be observed that all above definitions did not call for the concept of distance. It is a degree of freedom. This freedom associated to the notion of open set, justifies the thermodynamic notion of “state” and naturally introduces many features which may give rise to self-similarity if overall properties and local properties attune together.
Figure 1. Simplified diagram representing the main concepts of sheaf theory. To illustrate the purpose of this note, we have to distinguish properties concerned by the attribution of a metric over the basis from the properties involved by the fibration, functor dual of the projection of the manifold upon the basis. Both build an adjunction one of the main concepts of the category theory.

The notion of state can be merely associated with the zero of a certain continuous functions. However, as we know with some algebraic functions defined using complex variables, a variable does not necessarily refer a value (or a figure), but to several of them, giving rise to what Galois named mathematical ambiguities. These ambiguities are mainly based on symmetries. To overcome these ambiguities Riemann suggested defining the functions no further in the complex plane but using a multi-layered covering of the complex plane (Riemann surface). These sheets may touch themselves at certain points giving birth to different branches of the function, and therefore to some instabilities (catastrophe theory). If we shift from the continuous algebraic function to the open set of its definition, the function being considered as a field, one has to apply the Grothendieck’s substitution to the open sets $X$ in the complex plane by
their coverages. Obviously the generalization of the Riemann surface to Algebraic manifolds of arbitrary dimensions (space defined by polynomial equations with several variables) is then a categorical generalization from the notion of point into the notion of site.

A site is a covering category, namely a set of objects $U$ and a family of arrows (morphisms having $U$ for goal $U \to U_i$ where $U_i$ is a collection of open sets located inside $U$ and whose union $\cup$ is stable under the shift of the basis and under composition. Any topological space provides a site that is equivalent to the lattice of its open spaces seen as categories. The morphisms are then given by the relation of order of inclusion $\subseteq$. All sites giving rise to a manifold without any branching will be qualified as “etale” sites. The covering layers do not intersect themselves. In addition a Topos has a physical meaning only if it is “inhabited”, for instance a topological space can be inhabited by continuous functions, by tangent vector fields $T_X$ if the manifold is smooth etc. One calls pre-sheaf $F$ over $U$ a set $F(U)$ of local data, namely these objects that, associated with a natural restriction such as $V \subseteq U$, leads $F(V) \subseteq F(U)$ keeping the validity of algebra based on $(\cup, \cap)$. We call pre-sheaf morphism $F \to G$ any functorization $F(U) \to G(U)$ compatible with the algebra. The elements $F(U)$ are called sections of $F$ above $U$. By taking only into account the inclusion $\subseteq$ one forgets major other issues. Indeed it is also necessary to imagine the way in which the local sections can be glued together while keeping the vicinity of similar points. For instance, if $s_i \in F(U_i)$ and $s_j \in F(U_j)$ with $U_i$ open set of $X$ one has to match $s_i$ and $s_j$ in $F(U_i \cap U_j)$ with $s \in F(\cup U_i)$. We call sheaves the pre-sheaves which mesh together as indicated above. The obstruction to glue themselves correctly, namely to shift from a pre-sheaf to a sheaf, is a source of knowledge because this obstruction can be view as a mathematical object (a singularity possessing its own symmetries). The object of the theory of obstruction gives birth to the “cohomological issues”, and provides tools to account for various geometrical operations such as gluing, deformations, surgical cutting etc. For example, the obstruction to deforming a manifold $X$, must be seen as an element of a cohomologic group $H^1(X, T_X)$ of the tangent sheaf. For example, let us consider a sheaf of commutative groups on a manifold $F$ (abelian sheaf) and a sheaf of subgroup $G$. A quotient can then be defined. It is a pre-sheaf
Generally this is not a sheaf and the obstruction for being a sheaf must be seen as a certain cohomology group $H^1(G)$. Canceling the cohomology group removes the obstruction. Beyond this, the obstructions that live on the varieties themselves pose problems of gluing, which give birth of higher order cohomology groups $H^k(U,F)$ with $k > 1$. Fractal geometries introduce implicitly these Cohomologic groups when $\alpha$-dynamics (presheaves or sheaves depending on $\alpha$) are considered in such geometries. $\alpha$-dynamics requires the consideration of what we have called “sismic cohomological group”.

### 2.2. Grothendieck’s topos

Practically the concept of pre-sheaf and sheaf can be transposed from open set to the site $S$. A pre-sheaf on a site is a contravariant functor $F$ on $S$ in the category of sets and it is a sheaf for all covering family $U_i \rightarrow U$. An overall section $s \in F(U)$ must be identified with a local section compatible with $s_i \in F(U_i)$. If $S$ is identical with the lattice of the open sets $O$ of a topological space $X$ one find again the notion of sheaf over $X$. Grothendieck called Topos any category $\chi$ equivalent to the category of the sheaves defined over the site. Examples: (i) the punctual Topos, the category of sheaves for a space reduced to a point, is the sets category; (ii) if $G$ is a group the category of set on which $G$ proceed is a Topos; (iii) the “etale” Topos for an algebraic varieties $X$, is the category of sheaves over the “etale” site $X$. Each Topos gives rise to a theory of obstruction (cohomology). Different sites can give birth to equivalent Topos but if the topological space is reasonable, the Topos of the sheaves is able to characterize the overall space. In fact, step after step, we replace the topological space $(X,O)$ by the mathematical trellis authorized by the algebra and then by the sites end up sheaf $\chi$ (Topos) putting at disposal all the familiar operations in the category of sets. By focusing on gluing constraints, it has been possible to extract the intrinsic localization properties of $X$, by forgetting the points. It is a “metamorphosis of the notion of space” since the point can be any type of categories. We will give below an example of this extraordinary property. This example was born by facing the practical engineering problems in matter of energy storage. It is a model of Topos over arithmetic sites and fractal cohomologic effects appear experimentally [20,21,23].
3. Dynamic Topos on Fractal Interface

We owe Laurent Nottale, the designer of “scaling relativity”, the initial idea aiming to relate scaling properties of space and time complex [23], namely the introduction to a physical dynamics in fractal space-time. At the beginning of the eighties, this theory aroused an important noise in media that, out of the academic main stream and competing for a conceptual leadership for scientific displaying about physics in fractal environment, led to the ostracism of its author and his theory. L. Nottale’s personal experience led, some other scientists, to care for aiming the same end, – the dynamics features in fractal medium –, by dedicating the works only to the engineering and to practical applications without unveiled scientific aims. These were: (i) Oustaloup team researching on automatics techniques and focused on the CRONE control system [24] and (ii) these of one of the present authors, Le Méhauté’s teams at the research laboratories of CGE (later Alcatel-Alsthom) in Marcoussis. The latter team conceived and experimented with the TEISI model (Transfer of Energy on Interface Auto Similar) whose initial object was the optimization of lithium batteries [25, 26]. One of the indirect consequences of these studies was the design and pre-development of the reprographic process 3D-ALM (Addive Layer Manufacturing) and test of waves interferences with fractal geometries. Based on the self-similar geometrical structures of the interface used for exchange of energy, both models (CRONE and TEISI) gave a geometric content to the fractional integro-differentiation operators by attaching them to the fractal geometry [27-30]. This geometry possesses extraordinary properties. They not only actualize the infinity but also locate it at finite distance. These properties must be taken into account in physics by distinguishing both aspects (distance and scale). Therefore, fractality has not to be considered only as a mere Dirichlet’s like boundary condition. This radical option distinguishes the TEISI and CRONE models from the models based on the usual differential operators (Laplacian, d’Alembertian, Heat and Diffusive, Polya Hilbert, Dirac etc.) forced by fractal constraints. Putting aside the concept of velocity, the fundamental question to which these two models give rise is that of the links between space and time when both are ruled by fractal geometry. The TEISI model is especially interesting because it carries in it all the ingredients of the
theory of Topos while adding thereto elements of hyperbolic metric and category theory. Let us analyze briefly these aspects from figure 2. The TEISI model is based on the idea that the electrochemical exchange interface is a “covering family” associated to an experimental interface that can be assimilated to a discrete topological space. The covering is determined by both the scaling (distance) and the Fourier dynamics components (time) which defined the approximation of the interface of exchange. It is also determined by the basic fractal pattern which defines, at a differential level, therefore locally, the non-integer metric. The object of this model is, hence bases a “metric site” in the meaning given above. The model links to the site a functor, $F_E(B)$ which ensures, along a vertical fiber, the link between the base $B$ featuring the spatial motive $M$ and the dynamic interface, category determined over all scales. At this step, the interface is only a particular layered space characterized by a simple non-integer metric. The specificity of the TEISI model is, on the one hand, to consider that the “metric site” is itself a functor that binds the pattern $M$, seen as a differential object to a minimum scale (monad on lattice) so that $\frac{1}{d}F_M(M) : B_1\rightarrow M$ and on the other hand to assign to the origin and to the differential functor respectively, a category called “Space” and a dual category that we can denominate “Time”. It is here that on the one hand the differential character and on the other hand the functorization gives to the concept of space-time its status of a priori synthetic category. More precisely, physical analysis leads to the construction of the following functors $U^{\frac{1}{d}}F_M(M) : \eta\rightarrow u(i\omega\tau)^{\frac{1}{d}}$ with $\omega\tau = n$ is the number of pieces of the a certain layer of the covering family associated with the scale $\eta$ in the category $B$ [25-30]. In the same way that the notion of velocity linearly reduces Space to a Time, the proposed morphism assimilates space to a non-integer differential operator in Fourier space; the complex factor $i^2 = -1$ allows the transformation of the functor into a continuous function of complex argument. It leads to a non-integer dynamic model (namely the canonical Cole and Cole impedance $Z_0(\omega)$).

As can be seen in Figure 2, the canonical dynamics is reducible to an exponential when $d = 1$. This observation leads to the conception of an extension of exponential function [31] that the authors have latter called a $\alpha^{-\text{exponentielle}}$ (with a) with $\alpha = 1/d$. If $d = 2$ the covering will be called a Peano's coverage. $\alpha^{-\text{exponentielle}}$ dynamic category is more general than the exponential but condensed many properties of this last.
Figure 2. Diagram synthesizing the foundations of the TEISI model in which the notion of covering of the base is unveiled by a dynamics involving a non-linear space-time relationship and therefore geometrically spreads as a motive, by the application of the functor carrying the metrical analogies from local scales to global scales. This diagram gives a meaning to many experimental data related to the electrodynamic, electrochemical and mechanical impedances which led to the conception of the notion of $\alpha$-exponentielle. These impedances express the existence of forces associated with the coincidence between global symmetries and local symmetries.

Practically the impedance $Z_{\alpha}(\omega)$ is a model of this extension. It is associated with all possible internal epimorphisms resulting from the decomposition on the basis of prime numbers $p \in \wp$ of the number $n$, seen as an hyperbolic distance on the diagram, and such as $n = \{\omega \times \tau_n\} = \prod_{k} p^r_k = \{\prod_{i} p_i^{r_i} \times \prod_{j} p_j^{r_j}\}$; in other words $n \rightarrow \{\tau_n\}$, if $\omega$ is a degree of freedom. The combinatorial operations give rise to a functor (Fibration) from the category of impedances into the category of the Riemann Zeta functions $Z_{\alpha} \rightarrow \zeta(s) \{\tau_n\} \rightarrow \zeta(1-s)$ [20, 21, 32, 33]. The function $\alpha$-exponentielle $Z_{\alpha}^{\tau_n}$ bases the fibration, expressed henceforth as a disjoint sum of two arcs $Z_{\alpha}^{\tau_n} = Z_{\alpha} \cup \{\tau_n\}$. In the context of research on the resolution of Riemann's conjecture [35], arcs disjunction open both the way to a physical approach to the meaning of this conjecture [20, 21, 32, 35, 36] and a non-thermodynamic [37, 38] understanding of the notion of arrow of time [20, 21, 32]. Practically, if the complex plan is oriented, it
can be shown that this sum (set union) is analogous – according to the fibration modalities explained elsewhere leading to reconstruct a time parameter in the field of complex numbers \( s \in \mathbb{C}^- \), to the functional relations attached to the zeta functions by means of the set operator \( \zeta(s) \cup \zeta(1-s^*) \) and \( \zeta(s^*) \cup \zeta(1-s) \). Three consequences result of these observations (i) zeta Riemann function is a self-similar function, (ii) via fibration, the internal epimorphism give to a sign to the time parameter, therefore an arrow of time emerges which is not due to a physical process but to the metric of the underlying fractal geometry; in other words the notion of time can be immersed in the field of complex number to report about this new orthogonal property; (iii) The epimorphism is reduced to an isomorphism when the condition conjectured by Riemann is fulfilled, namely \( s = (1/2) + it \) result which leads symmetrically to a disappearance of the orientation of the complex plane, namely disappearance of the arrow of time, \( s = -s \). This situation is the sole able to arose the Riemann no trivial zeros \( \zeta(s) = 0 \), which justifies and physically validates the hypothesis expressed by Riemann (1859). If we also observe that the constraint \( d = 2 \) validating the Riemann hypothesis, makes it possible to depict all the properties of the stochastic processes, these properties are therefore naturally associated with Interface of the Peano, namely interfaces which, in 2D varieties, have no metric exteriority. Then analyzed by using zeta functions, these systems must be self-referenced, and their arithmetic partition can only be associated with a set of states within an arithmetic morphism with prime numbers (Montgomery-Odlysko conjecture [33]). The introduction of the quantum of state is thence analogous to the introduction of the digit One, the only way for the system to respond to a mechanics (and therefore to evacuate the arrow of time) is then to consider the dual-fractal systems, by playing with the only even symmetries and prime number at the same time, namely by playing with the digit Two (binary choice, excluded middle principle, Markov chains etc.). Thus, beyond the concept of \( \alpha - \text{exponentielle} \ (Z_{\alpha \tau}^n) \) with \( \alpha \) rational or real in the interval \([1, 2]\), \( \alpha - \text{exponential} \) is not only a very natural extension of the notion of exponential (valid for non-additive systems) but also, under the rules of the Topos theory, it offers a heuristic potential that we will now exploit further by seeing the consequences of our analysis in the frame of project management.

4. Reversibility. Commutativity. Arrow of time

Every project is by construction, attached to an irreversible action. However, considering the equivalent models transposed from physics [39]
we have to discern at least two notions of irreversibility attached to it: (i) a notion reduced to a simple dissipation due to the limited yield of any individual action (Entropy as Heat production). For example, the employee loses some amount of time in transportation to reach his/her place of work; (ii) a radically different notion related to a modification of the social organization or hierarchy (Negentropy, Productive Work, Humanitarian assistance). These last actions involve the putting in correlations of local scales and global scales, as well as a recursive links coupling different moments of the action-process (Before, After). These correlations can, not only be ensured by a project manager, but also in a democratic way, from all branches of the social scaling, by every actors of the project. The “deliverable” is then more than a mechanical-object, but a consequence that irreversibly changes the realm of the work itself (economic, social, intellectual, artistic). Let us examine this distinction, which, from a theoretical point of view, relates to issues of algebraic order.

**Figure 3.** Diagram of the traditional representation of the division of a project into different tasks linking the notion of topos and the sequence of tasks at all scales. The cost (entropy) is standardized over the horizontal axis. If the tasks are separable, the functor associated with the projection on the basis can mathematically be associated with the Fourier transform of an exponential function whose time constant is associated with the cost of the quantum of action. Taking into account that the clock or the film of the action can be unwound in the opposite direction leads to consider the loops, namely in category theory the adjunction or equivalently reverse morphisms.
4.1. Non commutativity and irreversibility

There is an arithmetic issue that, to this day, seems without useful consequences for the engineer or physicist. This question is the following: the set of natural numbers $N$ and of rational $Q$, gives rise to well-ordered suites. How can this order be transferred to the set of real numbers $R$. Indeed, these numbers are defined by a cut in the set of rational numbers. Why, does this cut not erase the order (the cut corresponds in fact to two convergent sequences of inverse order)? If this remark is relevant, the generic order for natural numbers (the continuity being represented by the real line) can only be extrinsic. Fortunately, as we have noticed above, the transition from the continuity of the real line to the binary quantification may always be associated, to the concept of inversion of this line with respect to any external point. Henceforth, we use the duality 1D versus 2D to implement this transformation. This is implicitly expressed with the notion of impedance in physics (representation within the complex plane). The addition of a point outside of the line, and therefore the embedding of the line (1D) into a larger space (2D), can hence provide a representation of the quantification, as well as, conversely, the extension sought. But what is the theoretical artefact hidden behind this inversion which seems very elemental operation? Let us start, by reasoning from with the idea emitted by Connes that continuity can be expressed by the conjunction of a discontinuous and a non-commutative group ruling the algebra (namely irreversibility for the physicist) of a discrete set of “points”. In metric terms and through analogy, the “minimum distance” from the point to the right discretized on $N$, explicitly measures the dissipation of the physical process (entropy production) hence representing its irreversibility. This distance is geometrically fixed by the scaling normalization, or likewise the location of the point 1 on the line. It is therefore not only by transferring a sequence of order but by defining the differential sequence of this order, in other words the unity, that the irreversibility and hence the order over the physical process is expressed. The fact that inversion shall be involved, is thus herein associated to the notion of entropy by the simple mathematical definition of the transcendent logarithmic function (integral over $f(x)=1/x)$). This observation makes it possible to assert that this irreversibility can be associated with the absence of commutative properties of the discrete group, having a single-parameter $\tau$, which define the geometry whose physical process follows a geodesic. The inversion which is the basis of the chosen example allows associating the irreversibility of the process with the curvature of the geometry which determines the global
form of the geodesic locally reduced to a straight line but involved in 2D or complex plane environment. The usual notion of physical time seen as an instant between a past and a future is here only the expression of \((\mathbb{N}, \times)\) the monoid ruling the embedding and the nesting of successive partitions, considered through sole even prime numbers, namely based on two (2, 4, 8 etc.), number associated to the sequence of semi-circles in the covering family of the overall dynamics without crossing.

Hence the transfer of order on the natural numbers to the set of real numbers can be based on the fact that two straight lines exist. The first line supports the quantification through an initial monoid \((\mathbb{N}, +)\). The none commutative properties is ensured through the inversion of the first line, via a second monoid \((\mathbb{N}, \times)\) which uses a partition to express an overall irreversible order (entropy production through the partition into ideals intrinsic structure). As it is known from category theory, the matching of both monoids gives rise to a non-commutative group whose properties are determined according to a scaling law, which, in physics, account for the irreversible consequences of the moving along the line. The ordered set being associated with the inversion of the initial straight line, and therefore also associated to a circle which may be discretized or not, is thus naturally associated with the major transcendental invariants (e, \(\pi\), \(2^{1/2}\)). These are therefore associated to the continuity even if a discretization is implemented.

If we add that this point of view is compatible with the work of Tomita-Connes – concerning the unitary nature of the uni-parameter group resulting of the matching of the monoids underscoring explicitly the symmetry of the sign of this parameter (i.e. absence of relevance of the sign of rotation of the clocks at this stage) –, we can infer from this, that the symmetry induced by the coincidence of the two monoids is global. The order of the continuum along the parameter \(\tau\) can therefore be thought as Connes asserts it, by conjunction, including in physics (Figure 4), of the application of a non-commutative group on a discrete set. Space (segregation of points) and the time of physical clocks (scaling associated to the propagation in scales over the arc: arithmetic sites) are herein two distinct categories whose functorial bond constructs the continuity involving the transcendent binary number \(2^{1/2}\) by matching two lips of cut (line and arc) at infinity. As is usual in physics, this analysis attributes irreversibility (therefore of mathematical non-commutativity) only to the physical process. There is then no objection for the process to be closed upon itself (loop) and thus that a cyclic temporality does exist (Aïon), but this conclusion precludes the existence of an arrow of time, without any relevance if time is a real number. This is the conclusion of Alain Connes’
works, who put off the existence of an intrinsic arrow of time and replaces it by a Thermal-Time.

This scale of time considers – as J. P. Badiali reports it in his works [33] –, the extrinsic existence of a thermostat determining both the distribution of energy on quantum states and an absolute value of scaling for the breaking symmetries between quantum regime and thermodynamic regime. The reaching the coincidence between local symmetry and global symmetry in the scheme of thermal-time explicitly introduces a non-standard cut-off, fore infinity in the order of the scales. As it is underscored in Connes’ works this aspect involved the use of Dixmier’s trace, a cut-off is then allowed (thanks to the fitting of the dimensionality ($d = 2$; integer operators)) between the metric of the thermostat and the metric associated with the geometry of the non-commutative group.
(Fig. 5). But, even perfectly well adapted to the physic of particles, this is still a narrow vision of the general physical problem that requires a non-integer field of definition for $d$. What about the physical problems we are really faced with, when we are led to consider geometries and groups such that the dimension is non-integer, situation preventing any dimensional fit with standard differential operators and involving the emergence of cohomologic groups? That is exactly this kind of problems that engineers and managers are today led to consider.

4.2. Arrow of time

Our assumption is the following: the symmetries evoked through non-commutativity are global symmetries whose principles extend locally by association of the renormalization groups with traditional differential operators (heat equation for instance). The a priori hypothesis of a Noetherian energy in the framework of usual physics links linearly (or quadratic) time and space through the concept of velocity locking any heuristic breakthrough. The situation changes radically if the dimension is not integer because the non-integer differential equations are then linked to local groups which are no longer associable to a change of Galoisian basis, thus naturally reducible of global groups (Figure 4). As it is shown through the $\alpha^{-exponentielle}$, and based on zeta Riemann functional relations, overall symmetries can be rebuilt but these symmetries are here submitted to constraints (distortion spinning and local spreading) resulting of local contextual epimorphisms in $Z_\alpha^3$. These symmetric extensions operate from the bursting of singularities affecting the space-time relation, while shifting, jointly the tricky question of the energy as a Noetherian constant. It is also the discontinuity in the transition from local symmetry to global symmetry-pointed out for instance by the Voronin’s theorem-which is involved through (i) the extension of the dynamic time variable in the field of complex numbers, (ii) the role of the scaled covering of the process $s \rightarrow s + it$ and (iii) the role of universal functions whose archetype is Riemann's zeta function.
Figure 5. Schematic representation of the development of the above paragraph, pointing out the role of inversion on the establishment of an order on the set of real numbers. In the right part, the conceptual linchpin of non-commutative geometry (Connes) are represented in the case where $a = 1/2$. In this physical model, it can be seen how the inversion of the order of application of the operators $A$ and $B$ modifies the referential explicating, in the complex plane, the non-commutativity by means of a phase angle. The value of this very specific phase angle is such as the Noetherian properties of energy are kept and as well the meaning of usual differential operators.

Moreover the authors have proved the links between the burst of local singularities and the operators of non-integer order, therefore the validity of the non-linear relationship between space and time. But the extension of the local to the global cannot be useful without the intervention of the functor required for linking together the complementary categories able to give rise to an involution: herein $Z_a \rightarrow \{\tau_n\}$ and then $\zeta(s) \rightarrow \zeta(1-s^*)$ namely the functional relation Physically, this functor accounts for the existence of a force, the constraint of adjustment of the global onto the local. Given the work already done, we call this force “Arrow of Time” because it manifests itself, at the margin of the irreversible process which gives birth to this force. The strictly internal fitting properties confer an entropic status (more precisely herein a negentropy status) to the sign that
characterizes the additional parameter $\tau$ used for the renormalisation. This sign (namely, the 1D orientation of this force) is the main characteristic resulting of the internal automorphisms of the overall integrated system which must be required in order to give the system the status of a being open to an environment strictly external to it. This arrow of time, – which is more precisely a local gauge of contextual adjustment of the whole field of possible non-integer dynamics –, is kept and transferred through the scales, by the means of a phase. The irreversibility, visible on the representation $Z_a^\tau$ is experimentally underscored through negentropic effects that must be distinguished from the local irreversibility of the dynamic process. These effects are associated with the self-similar internal overall correlations ($D < 2$) of the dynamics when compared with the local metric of the variety ($2D$). They justify the use of the concept of energy. Thus the arrow of time results from the properties of the cohomologic group (obstruction of gluing), which comes from the issues for fitting both local and global properties. This presentation might be a modern echo of Prigogine's works when analyzed with 50 years of stepping back (40). To summarize the state of the art known at this stage:

- If there is an arrow of time, it is always associated with an action, thus with energetic dissipation, but it is partly independent of its value. Arrow of time emerges from a cascade of energy characterized by a pair of distinct orders (i) in space (total order) and (ii) in scales (partial order due to the shift of energy from high frequencies to low frequencies $HF \rightarrow BF$).
- This arrow is at first a reactive force which, written within energy units, is none other than the inverse of a length therefore also a time since exchange of energy imposes the notion of velocity at boundary.
- Its theoretical vocation is to establish the non-trivial coupling of the global symmetries expressible by means of energy and local symmetries whose energy is not necessarily a Noetherian invariant.
- This coupling can be expressed by means of dualities over natural and rational numbers ($\mathbb{N}/\mathbb{Q}$) and the additive and multiplicative monoids ($+/\times$). These dualities use also the quadratic self-similarity of the natural numbers $\mathbb{N} \times \mathbb{N} = \mathbb{N}$.
- The emergence of this force under external constraint is due to a difference that makes it possible to distinguish the space-time relation at local scale (non linear) from the space-time relation at overall scale (Linear). In the absence of distinction the force disappears.
• This specific relation is expressed through the Riemann zeta functions on the one hand by the sequences which constitute them \((\mathbb{N}, +)\) and \((\mathbb{N}, \times)\) and on the other hand by the functional relations which connect the Riemann dual functions.

• The above complexity can be expressed by the coincidence of both categories (i) Site category covering of the dynamics (local group with a real parameter) and (ii) a category related to the variety matching the overall covering highlighted via the zeta functions (as bundle of functors using a complex parameter).

• Allowed by the existence of an “étale” sheaves, the common fibration parameter is then called time. Its sequence does not give birth to any arrow of time. This arrow emerges only from the singularity appearing at the boundary of sequence and by the means of a phase. Complex time appears at this step. The phase leads us to distinguish, the succession when considered in the positive sense in the complex plane, of the succession considered in the negative sense. We call arrow of time this asymmetry which possess overall geometric status linking the phase with both kinds of parametrizations: on line for fiber and within site-scaling for the tangent bundle on the dynamics.

• The time arrow (the phase) being independent of the absolute values of the space measurements, the time unit parametrization is a contravariant nonlinear function (mainly a power law) of the action. The power factor, if it exists, is related to the Archimedean metric of the hierarchies involved in the cascades of energy shaping the steady state of the open systems.

• These properties explain the renormalization laws often observed including in macroscopic physics (WLF, Peukert, Zipf, Aggregation laws etc. [26, 27]).

4.3. Self-organization of the open systems

As expressed through the Voronin’s theorem and its Topos interpretation, the disappearance of the commutativity is due to the gap characterizing the pair of covering functors (i) in line \((\mathbb{N},+)\) and (ii) on the arc \((\mathbb{N},\times)\). It is a particular expression of the sensitivity to initial conditions. Except for \(\alpha=1/2\), \(+s\) and \(-s\) cannot be matched one to another; this characteristic results from a break of symmetry, and more precisely from the breaking associated to \(\alpha=1/2\). The fibration associated with the
covering of any dynamics is then oriented. It is herein distinguished from temporality along the dynamic of the physical process which by itself does not exhibit intrinsic asymmetry. Thus appears through the phase a strictly internal factor of irreversibility (due to automorphisms), having herein of pure geometrical origin. The associated force is required for putting the overall system in coherence. This arrow of time is expressed fairly simply for self-similar systems by relocation of usually real unit of time into the field of complex numbers. The categorical constraints over \((\mathbb{N},+)\) and \((\mathbb{N},\times)\) are then separated. The internal correlations over \(\tau\) arise an internal metric of dissipation (phase and amplitude over the cascade of energy) and they give birth to a duality: dissipation \textit{versus} self-similar hierarchy, for all open systems. The fractional factor binding space and time explains that such open systems (e.g. living, economic or social systems) crossed by flows of energy and therefore subjects of local dissipations, organize themselves according to certain systemic geometry which becomes a monad by embedding systems of systems up to give birth to an overall system. Since this geometry is self-similar, one can match the functorial limits (limit and co-limit) in the aim of stabilizing, all over the scales, the algebra associated to the geometrical composition. The limit/co-limit functorial correspondence, then render dynamically coherent all the subsystems with respect to the overall system, while these systems stay open in topological and physical meaning. This is exactly what Prigogine disclosed 50 years ago by working about self-organizing reactions and by attempting to design thermodynamics of irreversible processes. It is also upon such kind of fractal geometry that the self-fulfilling forecasts systems are built. In the absence of adequate mathematical tools, and in particular in absence, in fractal geometries, and of a relevant expression of the cohomological groups to account for distinguishing both space and time scaling, the use of traditional integer differential operators fails on the issue of non-additivity characterizing the dynamic complex-sets ...whence, at his epoch, the weak endorsement, and the academic doubts expressed about Prigogine’s works [39]).
Figure 6. Extension using fibration of the base of the dynamics: the \( \alpha \)-exponential, leads to the Riemann zeta functions in the framework of a constructivist approach. This fibration facilitates the understanding of the impact of difference between local and global metric. This difference involves cohomological obstruction for matching independently space and time scaling. This impact is herein equated to the definition of a particular time unit leading the emergence of an arrow of time if \( \alpha \) is in the set \([1,1/2]\). This diagram also makes it possible to understand why \( Z_{\alpha} \rightarrow \{\tau_n\} \) hence \( \zeta(s) \rightarrow \zeta(1-s^*) \) appear as Kan categorical extensions that gives birth to a dual involutions. These involutions are based on a relationship between overall and local constraints. They generalize the notion of Integration versus Derivative through the epimorphism that plays a role quite similar to a Fourier Transform.

His works was at that time mainly focused (i) on the research of the theoretical foundations of the thermodynamic equilibrium [40] and (ii) on the existence of hypothetical cyclic states considered as the alpha and the omega of physics. The notion of dynamic fix-point as a substitution to the notion of equilibrium will appear later. The role of the Riemann zeta function and of the related functional relations to represent the loss of additivity expressed by means of the extension \( Z_{\alpha} \rightarrow \{\tau_n\} \). Therefore \( \zeta(s) \rightarrow \zeta(1-s^*) \) now appears fundamental because, in spite of the singularity induced in the \( \alpha \)-exponential extension, these functions ensures the link between the system that is self-similar and the energy that does become relevant as Noetherian invariant, only through his zeta expansion. Due to long range correlations the exchange of energy across fractal interface gives rise to an internal counter reaction. This one then carries out an
internal space-time reorganization of the dissipative system, with the emergence of an arrow of time as the main expression of the breaking of space-time symmetry with respect to quadratic or linear (Integer) symmetries. The closure is achieved only after the creation of a bridge (carried by the total flow of energy) between local properties and global properties affecting both space and less naturally, the time. This bridge has exactly the meaning that must be given to the phase factor. This one, induced by the fractal geometry removes by mere shifting, the reversibility the functors along the fibration \( Z_a \to \zeta(s) \{ \tau_n \} \to \zeta(1-s^*) \) as expression of the arrow of time (Fig. 6).

The consequence of above analysis is the requirement for considering two classes of project management. A first additive class asserts that the means implemented, therefore the sum of costs, determine the efficiency. A second class, none extensive, is characterized by an efficiency which benefits of a multiplicative factor attached to an internal multiscale organization for actions and actors. The multiplicative factor has amplitude strictly related to the internal multiscale automorphism outdoing the sequences of the tasks, the usual planning and the sharing of means. The first category of management is well known and well taught but, except for Gantt Methodology, the academic recognizing does not still exist for the second class. This exception is due to the triviality of the time parameter which authorizes in the Gantt diagram a superposition of the tasks without intrinsic modification of the unit of time. Our proposal which involves a nonlinear coupling between task and time is much more sophisticated at theoretical level because base on topos and at practical level because it requires robust social long ranges organization (in space and time) based upon non-additive characteristics of the vicinal human being relationships. The aim in this case is not only the access to steady state production line (one entrance and exit) but the reaching of a dynamic stability based upon the fix-point theorem, in the framework of a multi entrance system in contact with an irreducible “exteriority” always variable. The deliverable stays the main purpose of the project but it takes place among infinity of constraints; among them the long term survival of the organization is one of the more important aims for the system.

5. Outlook about the mathematical foundations for a modern project management

By linking the overall vision of the project with local point of view, the project manager reifies in micro-economy the meaning of the sheaf
theory and as well the main aspects of category theory (adjunction and self-similar emerging structure). The project manager personifies within the mathematical framework of categories a bundle of practical actions. It can, however, epitomize it in various ways. We will distinguish two main embodiment of project management: (i) traditional or deterministic project management, management which is not mathematically distinct from the stochastic management and strongly mimics the concepts of the Mechanics; (ii) Conversely the “zeta project management” – so named for highlighting the role of the zeta function –, will be characterized by the fact that beyond his own task, the actor of an elementary task is strongly related with the overall project through every level of the cascade of actions. In this case, the hierarchical tree is different of the one for the hierarchical relationships within the first class of management. It is depicted by a lattice (trellis) of links which is then a Topos over the site covering the hierarchy of tasks from top to bottom. Before analyzing both options let us first come back to a generic issue which concerns the passage from the continuous set to the discrete set.

The need to share the overall project under a set of different tasks involves the shift from the continuity depicted by the means of the (dissipative) finite segment (dissipative via compactification) & its associated arc and an infinite straight line which may be discretized. In the parametrization of arcs some critical points are may be distinguished (zero, unity and the infinity). They are enough for characterizing the whole dynamics. As it has been shown, an outstanding tool is available in mathematics for explaining these properties: the inversion of line with respect to a point outside this line. Virtually a geometric equivalence exists between the half circle that binds the two critical points at edges via a hyperbolic geodesic (semi-circle) of the dynamics and the length of segment that links straightly (along Euclidean orthogonal line) both critical points. For analogical reasons based on the physical meaning of impedance, energy may be taken as reference along the orthogonal line. The length of this segment (which is equivalent in physics to the density of the entropy production and in mathematics to a mere distance) is directly related to a temporal unit involved by the normalization if implemented jointly with the inversion (Fig. 3 and 5). The symmetries allow the renormalization within scales using a single parameter: \( \tau \) the time constant defined above. The set of relaxations gives birth to an involution thus insuring whatever might be the singularities along the geodesic, the outside significance of the concept of energy (utility, cost). These properties give a physical meaning at the model of passage from discrete to continuous
representation (compactification) as a thermodynamic expression (via negentropy) of the arrow of time. This last then appears as a particularly well chosen expression if compared to the force able to curve an arc. In this way that, far from chance or thermal effects evoked above, the arrow of time may appear as the key of the project, seen herein as teleonomic aim, geodesic action, and hierarchy of tasks, aiming techno-social irreversible actions.

5.1. Traditional project management considered as 1-topos over open set: Poincaré’s tiling

The central paradigm which usually does not suffer from questioning is that of the project manager’s ability to decompose the overall project into singular tasks that can be performed in series or in parallel. In general, for example in Gantt methodology, we admit that a cost (time, energy, utility) can be assigned to each task and that every task can be quantified by using this unit. This constraint also suggests that for each task there is a velocity of the process that binds space (through dissipation) and time (partition of the task itself and covering of the task as an arithmetic site), which allows us to use the execution time as an efficiency parameter. Figure 3 and 7 gives an idea of the constraints involved by these assumptions. The serial / parallel partition of the tasks appears as the covering of a base which is none other than the total action which as to be performed. The encompassing semi-circle represents the final deliverable. The under layered semi circles, represent intermediate deliverables. The replacement of the straight line arrows by semi circles (Fig. 3) to represent the task expresses the fact that formal initialization of an arrow at some point cannot be defined very precisely and as well that the arrow never reaches its end, even if practically the result of action is a “local deliverable”. The sums of “almost nothing” inter-correlated virtually determine in this case the overall efficiency of the project. A task is therefore an open set in topological terms as well as the sum of subsets that constitute the set of the common action. Like the motion of bike, the action is therefore only a sum of converging approximations. The division into distinct tasks constitutes multi-coverage of the action (Grothendieck’s site) which thus corresponds a Topos through partitions using the pre-sheaves. The action is therefore a category characterized by a covering family and a disjointed sum of tasks remains a task. The organizational presheaf then
emerges very naturally as functors of a category named Base (B) onto the covering family (FC) which represents the variety of overall action. According to the above analysis we can represent a task by means of a semi-circle parameterized by the inverse of a characteristic time that is to say by a critical frequency. These data provide the state of progress of the task in hyperbolic units, the infinity pointing to an end of the task, the “unit for normalization” pointing to the half “life time” of the task while zero focusses onto the starting as a teleological will. This representation immediately emphasizes the following features: due to the Kan extension involvement (each task being not necessarily separated from the others) and in spite of the requirement for finite energy threshold (entropy, utility or cost), the delay required for doing every task is always infinite (let us analyze for instance the mass and the boundary of Koch’s fractal flake to understand the idea). This property is due to the singular nature of the boundaries of the tasks, needing for power (time) and energy (space), and thus that mimics fractal limit behavior (actualization and localization of infinity in scale at finite distance). Except if acceptance of time delay (approximation for the end of tasks) and since the partition of tasks imposes that the geodesic of action (arc) remain open, the methodical and tidy worker may have a feeling of having in charge an unbearable task that is never finished. This feeling can contribute to create social tensions especially if time delay are arbitrarily (thermally) determined. Moreover, if the tasks are strictly separated, the actor cannot have the feeling of belonging to a collective action. Besides this, the approach of determinist management being strictly analytical, the only competences able to be recognized from the top of the hierarchy are the skills of the limbic left brain which limits the exploitable human potential within any creative project [41].

5.2. Project management considered as a $\alpha$-topos over Grothendieck’s sites

The dynamic treatment of self-similar systems naturally leads to power laws. Among these, the exponential function is a generic form linked, like the logarithmic function, to the hyperbolic function $1/x$ and such as the Fourier transform is connected to the semi-circle $Z_1(\omega)$ parametrized via the application $\exp(-t/\tau)\rightarrow 1/(1+i\omega\tau)$. 

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Figure 7. Poincaré’s tiling of the complex half-plane as a representation of the Grothendieck's thoughts in project management. It is schemed herein in its traditional mechanical form (determinist management). All the main analytical, geometric and mechanical properties are present but they are precisely the major impediment to innovate because they do not correspond to the constraints encountered by the field actor, the project manager and more generally the company. This kind of project management is certainly a useful guide but requires of being revisited by the project manager or their advisors avoiding an application to the letter of its recommendations. All managers know that human and environmental factors, most often unpredictable build the success or failure of a project, otherwise perfectly managed in a quantitative way. Project manager need other tools to think the management that why we suggest to consider through the zeta management (Figure 8) which could be compared to the CRONE command efficiency [24] or to SPADD systems damping [42].

This law which also bases indirectly the Gauss distribution makes it possible to construct probability measure (via for instance Dirac operator). It will be observed as it is done above that the semi-circle is reducible to the inversion of a semi-infinite line for all infinite energy at disposal. It will be observed that the root functions are also derived form of such
transformations via the Fourier transform \((t/\tau)^{\alpha-1} \rightarrow 1/(i\omega\tau)^{\alpha}\) provided that the radius of the semi-circle \(Z_{1/2}(\omega)\) tends to the infinite. Analytical generalizations can be deduced quite easily if \(\alpha=1/2\) (see Laplace table of transformation). However, although the mathematical application \((t/\tau)^{\alpha-1} \rightarrow 1/(i\omega\tau)^{\alpha}\) is also at disposal the constraint on the field of definition of \(\alpha\) does not allow the Fourier (Laplace) transforms to exist because the relation is valid if and only if \(\alpha > 1\), which is prohibited because \(\alpha = 1/d\) with \(d > 1\). In spite of this constraint, the mathematical transformation remains interesting in heuristics terms because it states that there is a usual link between the power function with its Fourier transformation by means of a mathematical integration. This property involves a mapping of an overall property onto a local property. It is nothing else than an involution using scaling which can be identified to a new type of differential operator [43]) for any value of the fractal dimension \(d\). Except in a specific case where \(d\) is integer, herein \(d \in \{1,2\}\) the Fourier transformation is not valid.

We can, however, conceive what would be the equivalent to this transformation by using the notion of tangent bundles and sheaves, well known for differentiable smooth manifolds. For example, if the sheaf of the vector fields on \(X\) is given, i.e. a data, for every point \(x \in X\) of a tangent vector \(v(x)\) varying continuously with \(x\), the bundle allows the construction of what the mathematicians call a connection, one of the terms associated with the Leibniz’s analytic rule when a vector field on the variety varies with each point of application; this connection is also associated with what is named the parallel vector transportation giving birth to a phase angle if the displacement implements motion upon a variety characterized by a curvature [19,44]. If \(X\) is a smooth topological space, the differentiable continuous functions form a sheaf on \(X\) which is also a differentiable smooth manifold. What about these properties when we pose the problem with varieties smooth only almost everywhere when based on the \(\alpha\)-exponential transfer function \(Z_{\alpha}^N=1/(1+(i\omega\tau)^\alpha\) with \(n \in \mathbb{N}\). They are not secured, – even in the general case where \(n \in \mathbb{R}\), by a Fourier Transform? It will be recalled that, in the light of the foregoing, \(Z_{\alpha}^N\) is none other than the expression of the sheaf of covering families (FC) associated to a fractal coverage when the scaling of the approximation of the hyperbolic distances given by \(\eta = u/v\) (Fig. 2). By including the motive given from the hyperbolic metric through its dynamical analytic form \(\eta \times (i\omega\tau)^\alpha \sim 1\), the system must locally respond to a non-linear space-time relationship. The discretization of the dynamics on the set of prime numbers in accordance with \(n \in \mathbb{N}\) leads the epimorphism \(n = \omega\tau \rightarrow \{\tau_n\}\). In other words, for any expression of a temporal norm associated with \(\omega\) it is possible to associate
a set of discrete values \( \{\tau_n\} \). Any morphism associated with a pair epimorphism for \( n \) et \( n+1 \) hence gives birth to the equivalent of the tangent bundle \( T_X \) over the dynamics. By using of zeta function, it was shown that \( \{\{\tau_n\}\}_{n \in N} \to Z_{1-\alpha}^N \) which leads to consider \( \forall n \in N, Z_{1-\alpha}^N \) as the expression of the tangent sheaf of the dynamic for the above covering family. It will be observed that the joint implementation of the factors \( \alpha \) and \( 1-\alpha \), as expressed through Riemann's zeta functional equation, leads one to think that the approach followed is a generalization of the notion of integro-differentiation, in the case where the metric is not integral as usual. If we add as it was noticed that \( N \mathbb{Z}_\alpha \cup N \mathbb{Z}_{1-\alpha} \) gives birth to a quasi-exponential in the Fourier space as a disjoint sum of arc of circle and that this one carries in it all the information necessary to the establishment of the functional relation of the zeta functions one must assume that the approach followed is a generalization of the Fourier transformation for the framework under consideration. We shall also be able to conclude that this union carries within it the involution which can confer at the \( \alpha \)-exponential a real prospective power in physics of non-extensive systems. This quasi-exponential is thus available for formalizing some classes of peculiar complex systems. We named these systems: Zeta Complex Systems (ZCS). ZCS could be used to formalized any living systems (immunitary systems for instance) economic systems (post Keynesian developments and monetary systems out of equilibrium), web systems (machine learning and explainable artificial intelligence) etc. This heuristic power can be implemented by the means of an extension of a Poincaré tiling which was considered above for featuring the traditional project management based on deterministic and or stochastic aspects. Figure 8 provides the extension of the Poincaré pavement when the semi-circle, – herein the fundamental geodesic –, is stricken by obstructions and epimorphisms when matching the global and the local behaviors, Cohomologic damages (represented by what we call Seismic Cohomological Group in Figure 8), associated to the \( \alpha \)-singularities of the group (therefore the geometry), being solved by the means of the \( \alpha \)-exponential properties. At this step a pair of project management cases must be considered.
Figure 8. Generalization of Gantt approach by tilling of complex plan according to $\alpha$-exponential geodesics. We observe that the complexity of the organization seems to increase with respect to the figure 7, but this situation fit practically the real situation where each task is bound with all the others (like in Gantt methodology but within hyperbolic context). Conversely, the new qualification of this reality emphasizes the perverse effects that must be associated with the traditional “Euclidean” and arithmetic management. This disqualification was justified before the development of new model, but the zeta-management clearly open the way for managerial improvements taking into account the absence of additivity of hyperbolically correlated tasks. The generalization of Mandelbrot’s set within non-quadratic form and the associated Julia’s set gives an example of the cohomology group resulting from fractionalization of the automorphism. This group is called "seismic group" because it highlights sliding faults at all scales. This group is balanced by means of the extension of Kan in the frame of $\alpha$-exponential representation.

Though generally considered as universal, the case associated with $\alpha=1/2$ must be regarded as a degenerate modality of the general case and Figure 8 becomes Figure 7 with a compensation of the seismic group. In the case mentioned applied to project management, each elementary action is considered as a task (cyclic state) or a sum of independent tasks (cyclic overall states) by the project manager who leaves the choice of the local strategy at the discretion of the local actors. Observed from the board office, these local choices can appear, without conceptual damage, as random choices, the overall behavior appearing as being ruled by a kind of
Adam Smith’s “no visible hand”. This property distinguishes this degenerated form from the deterministic project management, according to which the task is fixed within a Taylorism vision. In the stochastic case $\alpha = 1/2$ solely the local unit of time-scale used by the field actor is fixed by the CEO of the enterprise. It appears like a “thermal” unit of time: lower limit of an acceptable irreversibility. Conversely, the multiple local boundary conditions optimize the path followed to accomplish the elementary tasks. This freedom lowers the dissipation of energy and the cost is *minimum*. In the deterministic case, the unit of time used to schedules the action is set by the project manager by means of a global clock able rotate in both directions (time reversibility is implicitly supposed). The actor has not any degree of freedom over a delay written using a “thermal” unit of time. The outside determination of every elementary process defines the irreversibility of the common action without any influence of the local actor. While Taylorism is associated with the modern epoch, the stochastic management is an expression of a post-modern period based on the individualism and Uberization of the social contract. It is characterized by the existence of a common extrinsic quantum of time (Thermal time) which serves as the basis for optimizing every quadratic human action (Energy, Utility, and Currency). In the context of our analysis of the degenerate forms of relationships, this optimization appears to be mechanically optimal because the agents (components) are socially neutral and without possible individuation (it is also the meaning of the stochastic degenerated model). However, as it has been shown, it can be dynamically oriented only if it is thermalized, therefore associated with an infinite amount of primary resources (primary materials, massive currency provisions, thermostat etc.) all conditions that question the resiliency and the social relevance of the model. Unlike the Taylorism model, the stochastic model, tuned on the standard statistical physics, cannot be sustainable for a long term, including for theoretical and logical reasons.

The general case is infinitely more interesting from the sociological point of view. The theory developed above shows that the succession and the coordination of the actions are in a functorial relationship of exchanges (Fig. 8) and of reciprocal normalization. The recursivity is no more stochastic. The steps of individual action, but also the links shared, must be considered as categories building a functorial adjunction (i) in a realm of action by sharing the real time and (ii) in the realm of involvement by sharing the abstract space of the hierarchic tree, (iii) in the realm of edges as forced via the cohomologic group for matching local and global. The
The local actor then appears no more as a point-agent but as a Grothendieck site covering locally all actions of the project; the sheaf attached to it is none other than a variety that connects implicitly the actors to each other. The resulting connections then give to the project a capability of "forcing" and of intangible overcoming of any local or general difficulty. In strictly operational terms such organization gives birth to a feed-back factor which will distinguish it very favorably from the Taylorism and stochastic organization because the project will become creative both within the framework of its aims but also outside of this framework. The cognitive issue with which this type of organization is nevertheless confronted is due to the fact that it is not built upon cyclic Eigen states. Therefore the system must be thought of as an overall set of open subsystems whose edges consist only of connections (just as living beings whose consciousness should emerge from the sum of subsystems and from the appearance of an arrow of time). For cultural reasons related to standard causality, the trend of the society is nevertheless very strong to go back to a Taylorism or Uberism organizations. In spite of cognitive issues and understanding the zeta management offers a flexible alternative to these binary choices. Let us analyze this opportunity.

It is known that two mechanical systems which have the same spectrum of eigenstates are not necessarily identical. There is a major reason for this characteristic: the spectra may not be tuned with each other, for scaling reasons. For instance, a chair is perfectly stable for standard scale but the multiplication of the sizes of all pieces by hundred leads to collapsing. By ignoring this limitation, the servum pecus, involved in zeta-management but practicing usual cognitive dissonance, will be naturally tempted to bring the project management back to its traditional features, namely able to be depicted in the vectorial framework of separate cyclic states. Let us examine what mathematically results from such a resistance to change on the base of the $\alpha$-exponantielle. This resistance leads a transformation of Legendre upon curve space, which manifests itself physically by a primary need for energy [44] for the local actor. The work to be implemented is then expresses through a thermodynamics force associated to a change of the curvature of the variety whose dynamics is a geodesic. This calculation would be without heuristic value if it did not underscore that it appears an optimum for the $\alpha$ value associated to the curvature. Thanks to its internal entropic ability, the intrinsic forcing capabilities of the system can be used to lock the internal optimization (self-internal forcing). Due to the fact that this energy is limited it can be prove that the optimal value is reached for $\alpha = 3/4$. The development of the
TEISI model by using the Riemann zeta functions leads to attribute to the factor $\alpha + 1 = 7/4$ a universal role directly associated with the presence in our universe of a time arrow and resulting from a coincidence of overall and the local symmetries, both being ruled by a Integro-Differentiation of power laws.

5.3. Pedagogic and entrepreneurial testing

Although we had not yet a well-founded management model, the basic ideas of this model have been applied both in pedagogy and in creative actions for decades. Thus, in the late seventies, the design and the pre-development of the lithium-ion batteries, owes their foundations to the setting up of an informal zeta-engineering team around (i) Michel Armand (CNRS Grenoble) Jean Rouxel and Raymond Brec (University of Nantes) and one of the authors of this note (ALM, CGE Research Center in Marcousis). In the early 1980s the ALM design for “Addive Layer Manufacturing” and Rapid Prototyping has been performed in the framework of what we call now the zeta-management. It gives birth to several industrial applications. Later many applications were dedicated to the pedagogy [45-50] in the framework of education at the Advanced Institute for Materials Design (ISMANS) in Le Mans [51-54]. Between 1992 and 2010 new methods were implemented with efficiency without any comparison with the usual methodologies of management for teaching in such engineering schools (tenfold the number of students, creation of start-ups, access to progressive financial autonomy, pedagogical shift towards e-pedagogy etc.). These methodologies were gradually generalized, unfortunately at that time without strong theoretical basis, giving rise to very significant institutional resistance because the criteria of qualities required by the zeta-management enter in conflict with the criteria imposed by the academic doxa of additive systems (determinist mechanistic and/or stochastic points of view).

The incompleteness of our human actions is mainly associated with the existence of an arrow of time (and vice versa). It is clearly formalized in the framework Topos-based management models (zeta management), while stochastic management appears as a degenerate form of this zeta management. These degenerated forms is characterized by a maximization of randomness for exchanging between well-defined states because it is of course easier to think the world like a static universe built on immutable states (determinist and stochastic) whose exchanges are ruled only by digit (in energy, in matter, in currency units etc.). But such a world is not robust either because the games
too well balanced are not productive in the long term (leading the system to its obsolescence) or because the life naturally opens, cannot match a world having fences. The life can of course be just feed from such quantitative world but it cannot expand from itself. There are a “before” and an “after” only if action is partly a dissipation. The meaning of the rose is the meaning of a lost instant (Saint-Exupery)! Nothing justifies a priori the initialization of an action. This initialization must only be an act of will. Arthur Schopenhauer’s incredibly prophetic title, *The World as Will and as Representation*, find in the realm of management an amplified acuity if henceforth we read this title in the above mathematical and physical framework of fractal Topos.

6. Conclusion

By working analytically on the categories the authors formalized few months ago a representation of complexity that has been called zeta-complexity. When facing the complexity this new tool based on Riemann's zeta function properties, appears as a possible lever for understanding and acting rationally. Any dynamic system which requires (i) a choosing, (ii) mimetic learning or (iii) use analogies, tends to behave as a “universal black box” giving birth to self-similar emerging features. In fact the “universal black box” behaves as a mathematical trellis (lattice). Nevertheless, if the categorical analysis is useful, it is not sufficient. The theory of Topos introduces a genuine cognitive improvement which resides in a logical counterpart added to the geometrical properties. As Grothendieck noticed, the sheaves on a site possess all the properties of the sets. Nevertheless, after the demonstration of Godeland Cohen’s theorem about the absence of closure of the arithmetic, set theory *stricto sensu* can no longer be univocally defined and several non-equivalent models of the set theory may be designed. We owe Benhamou, Lawvere and Tierney [55] for having underscored that the Topos gives birth to a set of models that are only relevant in intuitionist logic. This logic conceived by Luitzen E. J. Brouwer is characterized by the absence of validity of the excluded middle law [56]. This extended logic imposes itself herein by virtue of the fact that the interior of the mathematical adherence of an open system is not equal to it [19], thus, the set] – infinite, a [open to the future is different from its closure, that shuts the past] – infinite, a]. The arrow of time expresses precisely this fundamental feature of the open-sets. Therefore, an open system always possesses a point of leakage: the present instant. This last is irreducible and it has dual asymptotes (toward the past and toward the future) which are neither material nor abstract; the dual limit whose
consciousness is responsible comes from a representation of our environment which depends arbitrarily from the will.

From a general point of view, a Topos carries within it a modal logic that must necessarily be contextualized. The value of truth depends on cultural location and on historical time. Nevertheless, this assertion does not justify any relativism. Indeed, in intuitionist logic, a proposition is valid if and only if, its interpretation in the framework trellis or Heyting’s algebra is true. It is true for any open lattice over topological space because the inclusion in it defines an order of priority [19]. Therefore, both features openness and order can guide our choices. Moreover, – thanks to the inversion procedure –, in any Topos, the trellis of the sub-objects of an object A builds naturally such a lattice \([a, 1/a]\).

The lattice structures thus appear universal and lead to major thermodynamic consequences: the freedom of being, the binding between the individual wills and the putting up of the common representations of the universe. The coherence of these wills requires the matching between the quantum character of the materiality of the beings and their macroscopic scales of environment (Pascal’s infinities). The main root for the coincidence which has to mesh the overall and the local aspects of our representations, is the absence, in the universe, of any external source of energy (as thermodynamic constant) and at best its spatial dispersion. This “object”, the energy, being an extensible variable, the thermodynamics assures us that it can be shared between both items: the entropy and the work (free energy / loss). Entropy is based on the existence of a lowest limit in scale of length. This limit is fixed by a thermostat which is the universe itself, situation which involves quantification via the Rovelli & Connes thermal time. In this case the time unit defines a statistical incompleteness normed by the energy. Thence the significant variable is the velocity and the time variable is intrinsically reversible. The time then appears in physics as a mere accessory variable. As Rovelli asserted it we can assume at this stage of reasoning that the time does not exist in physics [57].

But energy is just the cognitive fixed point of a representation that ensures the stability of our mental universe [59, 60]. At human scales, the cognitive and the logical universe remain open and we can easily be conceived from an anti-factual philosophy. As a result, human representations can be associated to a human scaling which can be distinct of the statistical hierarchies \((d = 2)\) on which the universal physical laws are based. Therefore the need to take into account the middle scales requires also the taking into account of the none-linear relationship between space and time. Thus, the incompleteness becomes metric and an
arrow of time emerges. Using the constant overall energy like a lever this incompleteness becomes a creative potential [60, 61] oriented towards the future: a force. This force is balanced through an adjustment of both metrics (statistic and hierarchical) with exchanges of extensities, namely exchange of objects. It has been assumed above that the basic object subject of exchange is the “delay”, the set of time constants outlining all possible. The resulting creative momentum meshes overall available symmetries onto the existing local symmetries. This gearing builds the degree of freedom of the being in accordance with the environment. In this context the work for renovating project management may be called zeta-management. Any return to a deterministic and Taylorism management or any extension to stochastic management (Uberism) that suppresses individuation (being) and the social relationships (beings), might lead a world which could not be sustainable. At least for these reasons the project management methodologies must be urgently revisited as well as the defective analogies used in econophysics. The human being is not an agent and still less a component of a machine [62].

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