

# LESSON OF THE DEBT CRISIS IN GREECE: AMPLIFIED CREDIT GROWTH AND DEPLETED LIQUIDITY

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**Abstract.** *By linking a qualitative technique from a differential equation, we investigate the burst of the Greek bond. The low interest rate of the Greek bond was formed by the German bond. A series of shocks – the Chinese stock shock in 2007, the BNP Paribas shock, and the Lehman shock (crisis) – triggered the Greek debt crisis. Its climax was caused by a sudden stop of a capital inflow due to a loss of investor confidence. The ECB responded with the Outright Monetary Transactions (OMTs) to offset the abrupt lack of liquidity; however, its timing was late. We argue that Greece's economy is suffering low growth due to eternal debt.*

**Keywords:** *Outright Monetary Transactions (OMTs), External Debt, Financial Crisis, Factor Analysis – Differential Equation.*

**JEL Classification Numbers:** *C02, G01, E58, H63.*

*It is against this background that the Governing Council today decided on the modalities for undertaking Outright Monetary Transactions (OMTs) in secondary markets for sovereign bonds in the euro area. As we said a month ago, we need to be in the position to safeguard the monetary policy transmission mechanism in all countries of the euro area. We aim to preserve the singleness of our monetary policy and to ensure the proper transmission of our policy stance to the real economy throughout the area. OMTs will enable us to address severe distortions in government bond markets which originate from, in particular, unfounded fears on the part of investors of the reversibility of the euro. ... Let me repeat what I said last month: we act strictly within our mandate to maintain price stability over the medium term; we act independently in determining monetary policy; and the euro is irreversible – Mario Draghi, President of the ECB, Vítor Constâncio, Vice-President of the ECB, Frankfurt am Main, 6 September 2012.*

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## 1. Introduction

This paper presents and discusses the origins and causes of the Greek sovereign debt crisis and its implications for the German bond and the Euro. The debt crisis was created by credit growth, accelerated by the association of Greece's Euro-denominated bonds to the German bond, resulting in a sequence of crises from 2007 to 2009. On the other hand, the Outright Monetary Transactions (OMTs) enhanced liquidity, and the Greek bond could avoid an even worse collapse. However, the crisis left Greece with negative GDP growth followed by deflation. Influx of external capital did not increase the competitiveness of the economy.

The Greek economy had a golden age from 2001 to 2007. More precisely, GDP growth was 4.21%, and the inflation rate was 3.47% from 2001 to 2006. What caused these excellent conditions? Gibson et al. (2013) describes how adoption of the Euro led to sustainable growth. Martin and Waller (2012), however, show that the inflation rate in Greece converged to that of the rest of Europe, which is not surprising in a currency union, and the interest rate on long-term Greek debt converged to the rate paid by Germany and France. In fact, after Greece adopted the Euro, the interest-rate spread between bonds issued by Greece and by Germany became negative, due to an underestimate of the risk of the Greek bond. It seems that the past credit record was not working properly because following Greece's adoption of the Euro, investors associated the Greek bond with the German bond. As such, our empirical results show that the German bond served to anchor the Greek bond.

In the aftermath of the series of the global financial shocks (GFS) from 2007 to 2009, Greece became the center of the Eurozone debt crisis. We define the start of the GFS to BNP Paribas shock, which occurred on August 9, 2007, and the third shock is the collapse of Lehman Brothers, which happened on September 15, 2008. During these shocks, the market in the euro area was gradually scathed, increasing the risk of sovereign default. In fact, the interest rate on Greek bonds jumped with the increasing volatility of CDS and VIX after the Lehman crisis. The risk of default became critical after Greece's government published falsified data of the Greek debt in November 2009. We focus on how the Greek bond interacted CDS during this period. Our results show that the movement of the Greek bond is explained almost entirely by CDS.

To explain the crisis, we considered two time periods. Period I is a panic market time from 2010 to 2012, and period II is a severe economic

time with external political influence (via the Troika) after 2012. During period I, the bailout programs by the Eurozone and International Monetary Fund (IMF) provided a total of around €216 billion in 2010 and 2012. Despite these programs, the interest rate of Greece's 10-year bond could not be sustained. Instead, this movement was volatile and chaotic. The movement links credit to confidence. In fact, consolidated positions on counterparties in residence in Greece decreased abruptly by \$56.224 billion in 2012. On August 2, 2012, to manage the lack of liquidity, ECB President Draghi announced the OMTs: the unlimited purchase of short-term government bonds. Following this announcement, the Greek interest rate decreased. Bordo (2014) concludes that the liquidity support provided by the OMTs led to a reduction of the spreading of sovereign debt.

Period II was characterized by severe economic conditions: deflation, negative GDP, and an unstable political situation. (The consumer price is minus 0.13% in 2016, and the GDP is minus 1.3% in 2016.) The IMF (2016) reported that the Greek deficit expanded by 2% of GDP after adopting euro compared with the 1% of GDP in the period prior to the crisis. Why was Greece unable to recover quickly from a moribund economy while the ECB supported liquidity? Although the timing of the support was late, it seems that this issue is connected to external debt. A series of shocks shifted the curve relating external borrowing and economic growth. Although the volume of external borrowing did not change drastically, the GDP shifted from a high to low growth rate. As a result, the high level of external debt stalled the economy's recovery.

The remainder of the paper is organized as follows. In Section 2, we describe the data. Section 3 explains the methodology and the empirical results. Section 4 presents a discussion. The last section contains concluding remarks.

## **2. Data**

In this paper, we mainly analyze daily data from the 10-year Greek government bonds, 10-year Germany government bonds, 5-year Greek Credit Default Swaps (CDS), and the VIX (CBOE Volatility Index). All data were accessed via Bloomberg.

Our data sample covers the period from January 1, 1999 to December 31, 2016, except for the CDS data, which are sampled from February 1, 2005 to December 30, 2016 except for the period of suspended trading.

### 3. Methodology and Empirical Results

#### 3.1. Methodology

Factor analysis is a standard statistical tool. The core part of the factor analysis is to approximate the target series  $f(t)$  as a linear combination  $f(t) = A \cdot a(t) + B \cdot b(t) + \dots - (3-1)$ . To measure the approximate degree, one uses the  $L^2$  norm  $\| \cdot \|$  :

$$\|f(t) - (A \cdot a(t) + B \cdot b(t) + \dots)\| = \sum |f(t) - (A \cdot a(t) + B \cdot b(t) + \dots)|^2 - (3-2) .$$

Thus factor analysis turns out to be a type of variation problem for the evaluation function  $\|f(t) - (A \cdot a(t) + B \cdot b(t) + \dots)\|$ , namely, to minimize the sum

$$\begin{aligned} V(f(t), a(t), b(t), \dots : A, B, \dots) &= V(A, B, \dots) \\ &= \sum |f(t) - (A \cdot a(t) + B \cdot b(t) + \dots)|^2 - (3-3) \end{aligned}$$

by a suitable choice of the constants  $A, B, \dots$

If we use the inner product  $\langle, \rangle$  of vectors, the function above can be rewritten as

$$\begin{aligned} V(A, B, \dots) &= \langle \{f(t) - (A \cdot a(t) + B \cdot b(t) + \dots)\}, \\ &\quad \{f(t) - (A \cdot a(t) + B \cdot b(t) + \dots)\} \rangle - (3-3) \end{aligned}$$

and expanded into a quadratic form in  $A, B, \dots$  as follows:

$$\begin{aligned} V(A, B, \dots) &= A^2 \langle a(t), A(t) \rangle + B^2 \langle b(t), b(t) \rangle + \dots + 2AB \langle a(t), \\ &\quad b(t) \rangle + \dots + 2A \langle a(t), f(t) \rangle + 2B \langle b(t), f(t) \rangle + \dots + \langle f(t), \\ &\quad f(t) \rangle - (3-4) . \end{aligned}$$

Differentiating  $V(A, B, \dots)$  in  $A, B, \dots$ , and setting the result equal to zero, we find that the answer to the variation problem; it is given by the simultaneous linear equations in  $A, B, \dots$  with the coefficients  $\langle a(t), A(t) \rangle, \langle b(t), b(t) \rangle, \langle a(t), b(t) \rangle, \langle a(t), f(t) \rangle, \langle b(t), f(t) \rangle, \dots, \langle f(t), f(t) \rangle$ . This indicates once we choose the time series  $a(t), b(t), \dots, f(t)$ , then the minimal value of  $V(A, B, \dots)$  is determined automatically.

For ease of computation, we introduce an order into the polynomial algebra generated on the basis  $g(t), \frac{d}{dt}g(t), \frac{d^2}{dt^2}g(t), \dots$  for a differentiable function  $g(t)$ , then the differentials  $\frac{d}{dt}g(t), \frac{d^2}{dt^2}g(t) \dots$  are of order 2, 4 etc.

Choose a basis of the algebra of order not greater than 5,

$$B(g; 4) = \{1, g(t), g^2(t), \dots, \frac{d}{dt}g(t), \frac{d}{dt}g(t) \cdot g(t), \frac{d}{dt}g(t) \cdot g^2(t), \frac{d^2}{dt^2}g(t)\} - (3 - 5)$$

as a vector space, then define

$$B(g; 4) = B(4) - \{g(t)\} = \{\text{exclude } g(t) \text{ from } B(4)\} - (3 - 6) .$$

For a target function  $f(t)$  and for the functions in  $B(f; 4)$ , we can minimize the variation  $V(f(t), B(f; 4): A, B, \dots)$ .

This is the case of the sequence  $f(t)$ , for a variable

$$x = GY - 0.85GK^1 - (3 - 7)$$

using factor analysis, it is easy to deduce the answer of the variation  $V(f(t); B(f(t), 4)$  attaining the minimum on the set of functions in  $B(f(t))$ , so that the  $L^2$  norm indicates the error of the function from the data. That is, we require that the  $L^2$  norm of the error term is less than 5% of the  $L^2$  norm of the original data, because the differential equation compresses the original data.

Using Fourier analysis, we can show the frequency component of the data  $f(t)$  over entire the range. The point is that the Fourier transform which is the design of electronic circuit (see Appendix A) shows one clear peak near around 10 on the frequency axis, which indicates both high-cut over 10 and low-cut under 10 mechanisms should exist (see Fig. 1). Accordingly, we show the differential equation is to be described by a 2<sup>nd</sup>-order differential equation:

$$f(t) = 5 \frac{d^2x}{dt^2} - 1.3x^2 + x - 0.15 - (3 - 8) .$$

The data show that in the period from March 13, 2000 to November 19, 2007 – February 2, 2009, the period is described by the following differential equation

$$5 \frac{d^2x}{dt^2} - 1.3x^2 + x - 0.15 = 0 - (3 - 9) .$$

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<sup>1</sup> The coefficient 0.85 of Equation 3-1 was determined by observation. From January 1999 to November 2009, Fig. 2-1 and 2-2 show the stacked line of the interest rate of the Greek bond and German bond. Notice the phases are the same. On the other hand, they are out of phase after November 2009. We suspect that the out-of-phase period might express the recovery of the German bond's role as anchor.

In contrast with Equation (3-9), consider

$$5 \frac{d^2x}{dt^2} - 0.05x^2 + x - 0.15 = 0 \quad - (3-10)$$

where we have changed only the coefficient of the  $x^2$  term. Using the real data, we find that Equation (3-10) does not approximate zero well in the period from March 13, 2000 to November 19, 2007 – February 2, 2009, as did Equation (3-9). However, during the crisis period from November 20, 2007 to May 31, 2016, Equation (3-10) approximates zero better than (3-9).

Therefore, we assume that the behavior of the period is described by the differential Equation (3-9) during the stable period and by (3-10) during the crisis period.

### 3.2. Empirical Result

There is a direct relationship between the Greek CDS and the interest rate of the Greek bond. Fig. 3 illustrates the relationship between Greek bond and CDS, lagged 50 days; the coefficient of determination ( $R^2$  value) is 0.867. We explain this as follows. Suppose that investors trust the Greek government, from the CDS index. Then they buy Greek bonds. This behavior accelerates the fall of interest rates. The interest rate embodies a belief that default risk is low as judged from the CDS. Regarding Greek bond and VIX, on the other hand, we cannot find a specific relationship.

Our next result is that the German bond diminished in its function as a peg for the Greek bond. Figures 4.1, 4.2 and 4.3 display the 10-days moving average of the differential coefficients for the German bond, the Greek bond and the Euro. Figure 4.1 shows the out-of-phase movement of the Euro compared to the movement of the German and Greek bonds. The phases gradually decouple after February 2007. The Euro depreciates when the difference of the interest rates rises. With the emerging crisis, on the other hand, the phase will be chaotic, because investors ignore reasonable behavior – they are likely to sell the bond quickly.

As shown in Figure 4.2, the Greek differential coefficient exceeded 100 on March 31, 2010, and its amplitude swelled sharply the day after. In the panic market, there is snowballing pressure on investors to sell the Greek bond when they noticed high risks such as the financial crisis.

Figure 4.3 illustrates that the Greek differential coefficient plummeted sharply on February 21, 2012<sup>2</sup>, reaching its maximum negative point (−1309.14) on March 2, 2012.

The range of extreme values narrowed significantly after March 12, 2012 following 1) the statement by ECB President Draghi that the central bank has done enough to combat the sovereign debt crisis and 2) debt restructuring, in particular, Greece closing a €200 billion restructuring deal with its creditors. After March 12, 2012, the Greek differential coefficient shrank, except in June and July 2015 when the media was reporting whether Greece would adopt a new currency in place of the Euro.

During the phase-decoupling of the bonds and Euro (see Fig. 4.1 and 4.2), the German bond's role of anchor for the Greek bond was being diminished by the chain of shocks (see the beginning of this section), and it is supported by the data of the monthly volumes of Greek government bonds trade in the HDAT electric platform (see Fig. 5).

## 4. Discussion

### 4.1. The Timing of the OMTs

We now discuss the timing of the OMTs by the ECB. To do this, we introduce our method. The method reveals qualitatively how such interactions can bring about a new structure.

Consider the 2nd-order differential equation  $\frac{d^2x}{dt^2} + ax^2 + bx + C = 0$ . To trace the solution in the  $(x, \frac{dx}{dt})$  plane, the behavior of the real roots of the quadratic equation  $ax^2 + bx + C = 0$  is crucial. The solution can increase up to the larger of the two roots, after which it turns back. This indicates that this root functions as a gate, i.e. a point at which the

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<sup>2</sup> A new Greek debt deal was finally agreed to by Greece, its creditors and Eurozone finance ministers. Details of the plan included: current creditors agreed to lose 53.5% of the face value of their debt to satisfy the IMF; the ECB and other European central banks take no loss on debt holdings with any profit made on the holdings transferred to Greece; a lower interest rate on new debt; and the oversight of a debt-servicing account by official creditors. The next step was getting a large percentage of the debt holders to agree to the debt swap that would allow for new bailout money to be given to Greece. Particularly controversial was the Greek legislature's retroactive change to debt covenants executed by the passage of a new law called "collective action clauses." The new debt deal triggered fear about whether the agreement constituted a default, which would trigger massive payouts on credit default swaps (CDS).

<https://blogs.cfainstitute.org/investor/2011/11/21/european-sovereign-debt-crisis-overview-analysis-and-timeline-of-major-events/>

trajectory reverses. Monetary policy, political power, etc. sets this gate low or high, according to the time period. Note that the sharpness of the quadratic polynomial determining the roots decreases as the larger root increases.

From our results, the Equation is

$$5 \frac{d^2x}{dt^2} - 1.3x^2 + x - 0.15 = 0$$

(*March 13, 2000 to November 19, 2007 (February 2, 2009)*)  
 $-(4 - 1)$

$$5 \frac{d^2x}{dt^2} - 0.05x^2 + x - 0.15 = 0$$

(*November 20, 2007 (February 2, 2009) to May 31, 2016*)  $-(4 - 2)$   
 $x = GY - 0.85GK - (4 - 3) .$

The coefficient of the quadratic term in Equation (4-1) is  $-1.3$ ; in Equation (4-2), it is the comparatively very small  $-0.05$ . The  $-0.05$  (Equation (4-2)) stands for the weak power to push back the sustainable condition compared to  $-1.3$  (Equation (4-1)). The economic implication is that the interest rate of the Greek bond was not controlled by the German bond. Figures 6-1, 6-2 and 6-3<sup>3</sup> illustrate the fit of Equations (4-1), (4-2), and (4-3) to the original data. Figure 8.1 shows the total period from January 1999 to December 2016. Figure 6.2 shows the period over which the fit of Equation (4-1) becomes worse and worse; it ceases to apply after November 2011. On the other hand, Equation (4-1) fits the data well in the sustainable period (see Fig. 6.3).

The Figure 7 displays the ECB interest rate and the credits for each country and zone: we compare Germany, Greece and the Euro zone to the private non-financial sector, taking the average over the preceding four quarters. The ECB started hiking interest rates in December 2005 and finished in July 2008. Over this period, the main official reference rate (Variable Rate Tenders Minimum Bid Rate) was hiked from 2.25% to 4.25%. In the midst of the series of GFS, the ECB hiked the rate from 4.00% to 4.25% on July 9, 2008. This action was taken because the Harmonised Index of Consumer Prices (HICP) surpassed 4%, compared to

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<sup>3</sup> The position of small peaks in the 1st differential of the German bond's interest rate is similar to those of the Greek bond. By subtraction they may be flattened to yield a simpler differential equation. In fact, the variable  $x$ , defined as "German bond interest minus 0.85 times Greece bond interest," satisfies a simple Duffing-type equation, describing a rather simple mechanism. The interest rates of German and Greek bonds are linked by a simple mechanism, expressed by a differential equation. This may be tied to the currency.

the ECB's targeted of 2%, and despite the fact that the credit of Greece was declining. (The credit growth of Germany and the Euro zone was increasing at this time). From the primary objective of the ECB's monetary policy<sup>4</sup>, the policy cannot be criticized; it seems, however, that the timing of the interest-rate hike was wrong, seeing that the growth of German credit was based on the flight to quality from the flight of Greek risk. On the other hand, the main official reference rate (Fixed Rate Tenders Fixed Rate)<sup>5</sup> was cut 50 basis points (3.75%) on October 15, 2008 and brought down 1% on May 13, 2009 brought down to 1%. Additionally, during this period the ECB also adopted credit enhancement measures to mitigate interbank markets through main refinancing operations (MROs) and long-term refinancing operations (LTROs)<sup>6</sup>. If the ECB had quickly taken a non-traditional monetary policy, then Greece could have alleviated the liquidity problem.

In the next period, the Governing Council decided to initiate the covered bond purchase program (CBPP) in May 2009. These purchases started on 6 July 2009 and ended on 30 June 2010. The purpose of the CBPP was to improve funding conditions for financial institutions that issue covered bonds. Despite the collapse of the German bond's function as anchor, the ECB made their decision to increase the interest rate 25 basis points on April 13, 2011. It seems that this timing was wrong. Greek credit was declining with increasing Germany credit, and the ECB again started to cut the interest rate on November 9, 2011. In our view, their monetary policy should have focused on peripheral economic conditions such as those in Greece. The problem of a single monetary policy is that policy makers would need to take into account countries with particularly good or bad economic situations, even if they have an amalgamated fiscal policy.

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<sup>4</sup> The primary objective of the ECB's monetary policy is to maintain price stability. The ECB aims at inflation rates of below, but close to, 2% over the medium term. <https://www.ecb.europa.eu/mopo/html/index.en.html>

<sup>5</sup> On 8 October 2008 the ECB announced that, starting from the operation to be settled on 15 October, the weekly main refinancing operations would be carried out through a fixed-rate tender procedure with full allotment at the interest rate on the main refinancing operations. This change overrode the previous decision (made on the same day) to cut by 50 basis points the minimum bid rate on the main refinancing operations conducted as variable rate tenders. [https://www.ecb.europa.eu/press/pr/date/2008/html/pr081008\\_2.en.html](https://www.ecb.europa.eu/press/pr/date/2008/html/pr081008_2.en.html)

<sup>6</sup> MROs serve to steer short-term interest rates, to manage the liquidity situation, and to signal the monetary policy stance in the euro area. LTROs provide additional, longer-term refinancing to the financial sector.

<https://www.ecb.europa.eu/mopo/implement/omo/html/index.en.html>

## 4.2. External Debt and Economic Growth

Rajan (2010) shows how developing countries that borrow money externally have changed drastically since the 1950s. In the 1950s and 1960s, the money source was multilateral institutions like the World Bank; in the 1970s and 1980s, it was Western banks; and in the 1990s, it was foreign arm's-length investors. These changes have two implications: (1) Developing countries are now able to more easily access external money through international financial markets. (2) Developing countries do not need to consider the strong correspondence between their investment and savings. Accordingly, these countries could grow their economies faster than domestic savings would allow.

To discuss the relationship between debt and growth in Greece, we distinguished between external debt and domestic debt. This enabled us to determine which financing source leads to faster growth. The external debt in Greece was 366,909 million Euros in Q4 2008 (See Fig. 8). In addition, 71% of long-term Greek bonds issued from 2005 to 2010 were owned by foreigners. Thus, Greece's budget is financed heavily by nonresidents. We discuss the relationship between external borrowing amount and economic growth. Prasad, Rajan, and Subramanian (2007) shows that successful developing countries have limited absorptive capacity for foreign resources.

Although Greece is not a developing country, it does share certain features that make the analysis of financing source enlightening as a developing member of the European Union. We show that external debt tends to trigger long-term recession, resulting in more difficulty for investors to recover their confidence more quickly.

The economic growth rate  $G$  is given by

$$G = -\alpha E^2 + \beta E - (4 - 4) \\ (\alpha > 0)$$

where  $E \geq 0$  is the real external borrowing amount,  $\alpha$  is the speed of growth rate, and  $\beta < 0$  is a measure of the foreign investors' animal spirits. Note that  $G$  increases with  $\beta$ .

Equation (5-4) expresses the growth rate  $G$  as a convex quadratic function of external borrowing  $E$ : increasing external borrowing beyond equilibrium generates a decreasing growth rate.

The external borrowing rate  $E$  is determined by the difference between the foreign interest rate, the domestic interest rate, and the real exchange rate (see Equation (5-5)):

$$E = (i_d - i_f)E' + AE' - (4 - 5)$$

where  $E'$  is the external borrowing amount,  $i_f$  is the foreign interest rate,  $i_d$  is the domestic interest rate, and  $A$  is the real interest rate.

In the Greek case, we discuss how this convex function  $G$  shifted. Krugman (2013) investigates the reason for the Greek sovereign debt shock. His explanation is a sharp decline in the capital account due to let a loss of confidence by foreign investors. This shock shifted the function from (1) to (2) (see Fig. 9). Figure 9 shows that the external borrowing point did not change drastically in the shift of equilibrium from  $A$  to  $A'$ .

From this function, the creditworthiness assigned by foreign investors gives

$$\frac{d}{dE}G = -2\alpha E + \beta - (4 - 6)$$

$$C = \gamma - \frac{d}{dE}G \left( \frac{d}{dE}G > 0 \right) - (4 - 7)$$

$$C = \gamma + \frac{d}{dE}G \left( \frac{d}{dE}G < 0 \right) - (4 - 8).$$

Equation (4-6) expresses the effect of the velocity of external borrowing on the growth rate. Equations (4-7) and (4-8) describe the creditworthiness assigned by foreign investors. Here,  $C$  is the creditworthiness given external borrowing amount  $E$ , and  $\gamma$  is the maximum creditworthiness. In Figure 11, the debt shock shifts positive creditworthiness (1)' to a new value (2)' with negative slope. As such, Greece's economy was susceptible to slowdown caused by a loss of investor confidence and a subsequent outflow of foreign investment.

Our economic model shows that it is difficult for nonindustrial countries like Greece to continue to grow their economies using foreign capital, because of a loss in foreign confidence (i.e. a decline in creditworthiness) leading to capital outflow, resulting in recession and a lower potential growth rate.

## 5. Conclusion

The main conclusion of this paper is that the Euro, by serving as an anchor for the Greek bond, helped stabilize the interest rate of the Greek bond, and that the Greek bond bubble was created by the credit of German bond rather than by the Euro. In particular, the collapse of the bubble of the Greek bond was not caused by the single currency. Rather, the trigger of the collapse was a series of shocks: the Chinese stock shock, the BNP Paribas shock and the Lehman shock. Directly contributing to the collapse

was an abrupt stop of capital inflow by investors, due to a loss of confidence. The OMTs were useful in controlling the interest rate, although the ECB should have taken the action earlier. Due to the level of external debt, the Greek economy continues to experience low growth.

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### Appendix. A

In electronic circuits, both high-cut and low-cut mechanisms are expressed by CR circuits by linear differential equations. We show a high-cut filter (see Equation 1) a low-cut filter (see Equation 2)<sup>7</sup>.

$$f(t) = \frac{d}{dt}g(t) + CR \cdot g(t) - 1$$

$$f(t) = \frac{d}{dt}\{f(t) - G(t)\} + CR\{f(t) - G(t)\} - 2 .$$

For general input  $f(t)$ , the equation for the high-cut filter has the solution

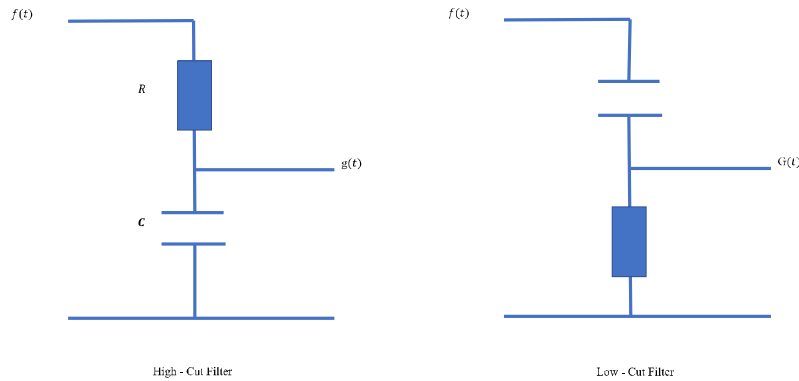
$$g(t) = \frac{1}{CR} \cdot \exp\left(-\frac{t}{CR}\right) \int \exp\left(\frac{x}{CR}\right) \cdot f(x) dx - 3$$

which is the output from the mechanism if the input is  $\cos(kt)$ , then the solution is

$$g(t) = \frac{\{kCR \sin(kt) + \cos(kt)\}}{1 + k^2 C^2 R^2} - 4 .$$

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<sup>7</sup> The Fig shows high-cut filter and low-cut filter.



The square of the amplitudes of  $f(t)$  and  $g(t)$  are 1 and  $1/(1 + k^2 C^2 R^2)$ , respectively.

The situation is quite similar for the low-cut mechanism for the input  $f(t) = \cos(kt)$ , In this case, the equation for the low-cut filter has the solution

$$\begin{aligned} G(t) &= \cos(kt) - \frac{\{kCR \sin(kt) + \cos(kt)\}}{1 + k^2 C^2 R^2} \\ &= \frac{\{kCR \sin(kt) - kCR \cos(kt)\}}{1 + k^2 C^2 R^2} - 5. \end{aligned}$$

The square of the amplitude of  $G(t)$  is  $k^2 C^2 R^2 / (1 + k^2 C^2 R^2) = 1 - \{1/(1 + k^2 C^2 R^2)\}$ , indicating that the output by the low-cut mechanism. The output  $G(t)$  is satisfied with the second order differential equation as it has a single peak around  $CR$  as well as mathematics<sup>8</sup>.

If we apply the Fourier transform, we can identify the long-term signal  $d(t)$  as it is constructed mainly from waves with low frequency (slow changes over time), while the noise signal is constructed from waves with high frequency (fast changes over time). In this paper, a proposed modification of the real discrete Fourier transform

(RDFT) and its inverse operation (IRDFT) is used to pre-process the raw data. The  $n$ -point ( $n$  = power of 2) Fourier transform of a signal  $\vec{x} = [x_t]$ ,  $t = 0, 1, \dots, n-1$  is defined to be a sequence  $\vec{X}$  of  $n/2 + 1$  complex numbers  $X_f$ ,  $f = 0, 1, \dots, n/2$ , given by  $X_f = R_f + iI_f$  in which,

$$R_f = \sum_{t=0}^{n-1} x_t \cos(2\pi f t / n)$$

and

$$I_f = \sum_{t=0}^{n-1} x_t \sin(2\pi f t / n), \quad f = 0, \dots, n/2 - 1$$

where  $i$  is the imaginary unit. The signal  $\vec{x}$  can be recovered by the inverse transform:

$$\begin{aligned} x_t &= (R_0 + \frac{R_n}{2} \cos \frac{\pi t}{2}) + \sum_{f=1}^{\frac{n}{2}-1} R_f \cos\left(\frac{2\pi f t}{n}\right) + \sum_{f=1}^{\frac{n}{2}-1} I_f \sin\left(\frac{2\pi f t}{n}\right), \quad t \\ &= 0, \dots, n-1. \end{aligned}$$

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<sup>8</sup> Equation 3 can be read as a correspondence of a functional space into a functional space inverse to the differential equation, written by an integral operator. It indicates that the composition of two such operators has twice of the integral operator, so that the inverse is the second differential equation.

To efficiently filter noise and perform reduction on data, we add a parameters  $m$  to Equation 2 to control the reduction rate. Equation 2 is then changed to Equation 3.

$$x_t = (R_0 + R_{n/(2m)} \cos(\pi_t)/2 + \sum_{f=1}^{n/(2m)-1} R_f \cos(2\pi f t m/n) + \sum_{f=1}^{n/(2m)-1} I_f \sin\left(\frac{2\pi f t m}{n}\right), t = 0, \dots, n/m - 1$$

– 8

## REFERENCES

- [1] Alogoskoufis, George, 2012, *Greece's Sovereign Debt Crisis: Retrospect and Prospect*. Hellenic Observatory Papers on Greece and Southeast Europe, GreeSE Paper No. 54.
- [2] Bank of Greece, 2014, *The Chronicle of the Great Crisis – The Bank of Greece 2008-2013*.
- [3] Bordo, Michael, 2014, *The Eurozone Crisis? Will the Euro Survive?* The conference “Political Economy of International Money” by the Federal Reserve Bank of Dallas and Southern Methodist University.
- [4] Eichengreen, Berry, 1992, *Is Euro an Optimum Currency Area?* Greece and the Euro: Proceedings of the 7<sup>th</sup> International Conference of the Economics Society of Thessaloniki.
- [5] European Parliament, 1998, *EMU and Greece*, Prepared by the Directorate General for Research Economic Affairs Division, PE 166.453/rev.1.
- [6] Frankel, Jeffry, 2007, *Responding to Crises. In Federal Reserve Policy in the Face of Crises*. Cato Journal 27, No. 2, Spring/Summer.
- [7] Gibson, Heather., Theodore Palivos, and George Tavlas, 2013, *The Crisis in the Euro Area: An Analytic Overview*. Bank of Greece, Special Conference Paper: The Crisis in the Euro Area, May 23-24, 2013.
- [8] International Monetary Fund, 2016, *Greece: Preliminary Debt Sustainability Analysis – Updated Estimates and Further Considerations*. IMF Country Report No. 16/130.
- [9] Gowers, Timothy, 2008, *The Princeton Companion to Mathematics*. Princeton University Press.
- [10] Jopp, Tobiasa, 2016, *Contemporaries' Opinions of the Allied and Central Power's performance during the First World War: Measuring Turning Points in Perceptions with Sovereign Debt Prices*. European Review of Economic History, 20, 242-273.
- [11] Krugman, Paul, 2013, *Currency Regimes, Capital Flows, and Crisis*, the 14<sup>th</sup> Jacques Polak Annual Research Conference by International Monetary Fund.
- [12] Martin, Fernando., and Christopher Waller, 2012, *Sovereign Debt: A Modern Greek Tragedy*. Federal Reserve bank of St. Louis Review, Vol. 94, No. 5.

- [13] Minister of Finance, 1999, *Government Budget Report 2000 Executive Summary*. Hellenic Republic, Athens.
- [14] Pan, Jun., and Kenneth Singleton, 2008, *Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads*, The Journal of Finance, Vol. **64**, Issue 5, 2345-2384.
- [15] Prasad, Eswar., Raghuram Rajan and Arvind Subramanian, 2007, *Foreign Capital and Economic Growth*. NBER Working Paper Series, Working Paper 13619.
- [16] Rajan, Raghuram (2010), *"Fault Lines"* Princeton University Press.

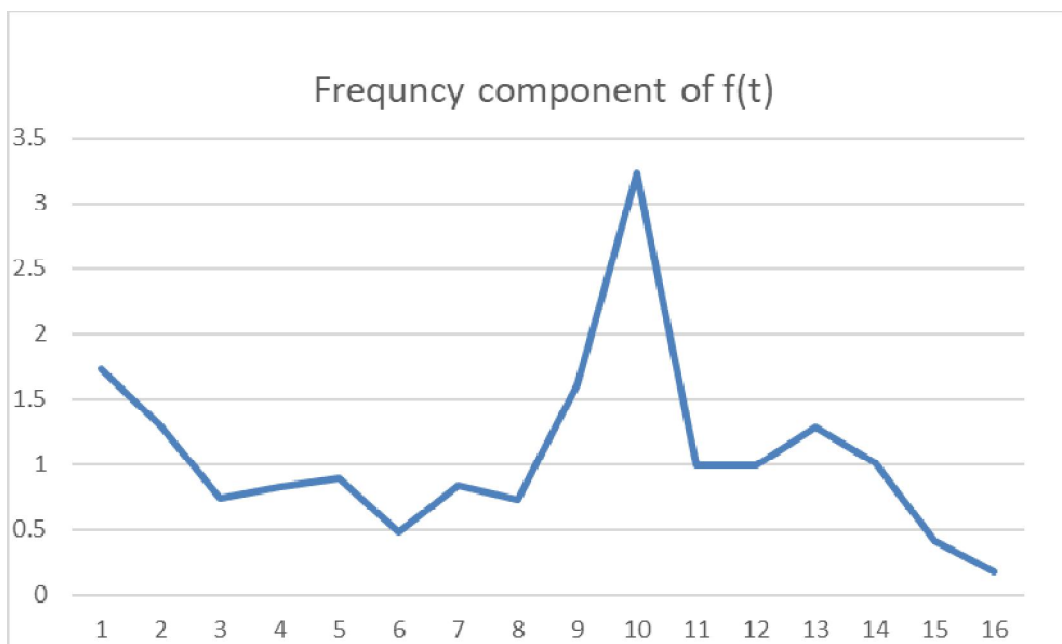


Figure 1.

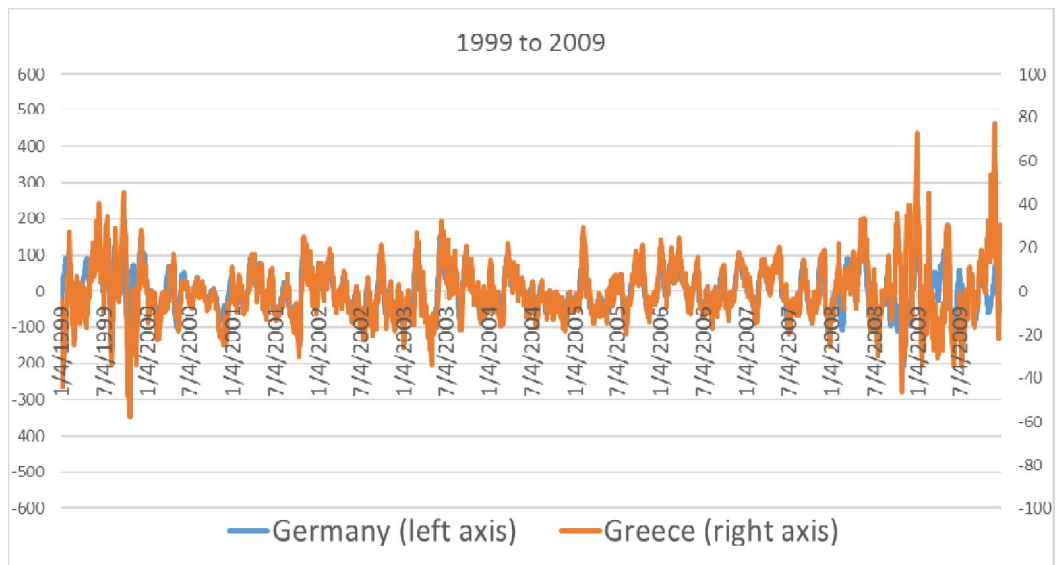


Figure 2.1.

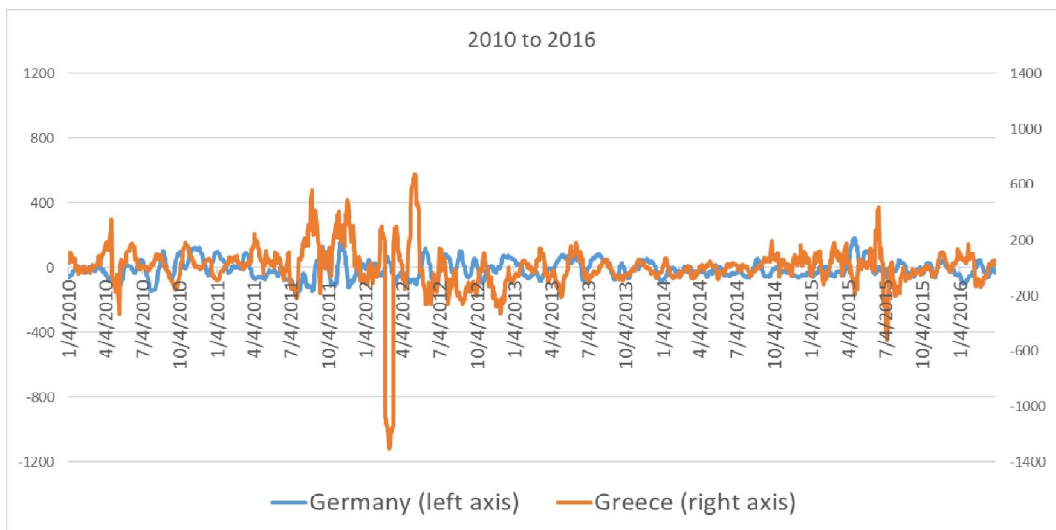


Figure 2.2.

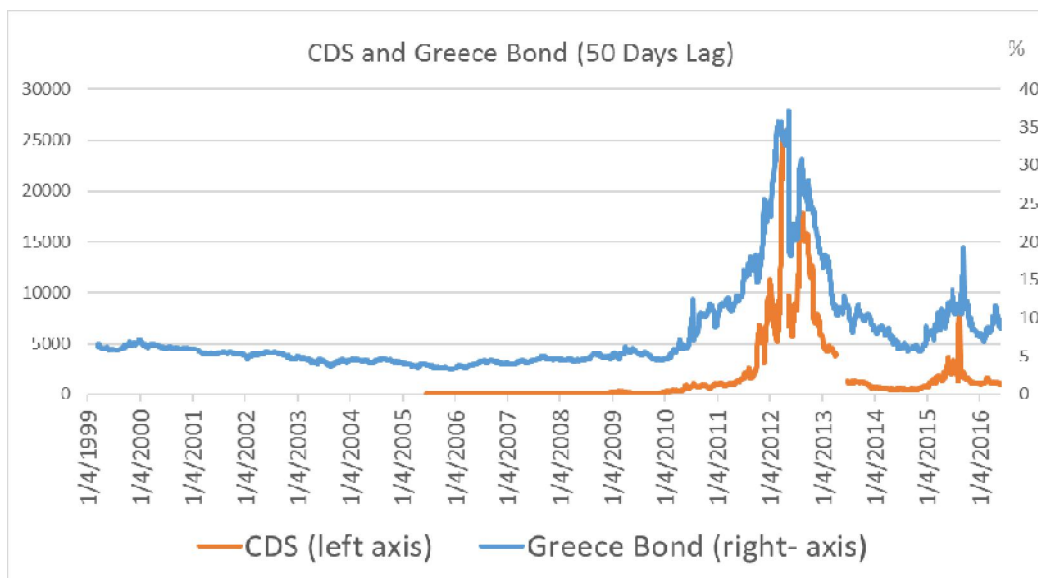


**Figure 3.**

Source: Bloomberg.

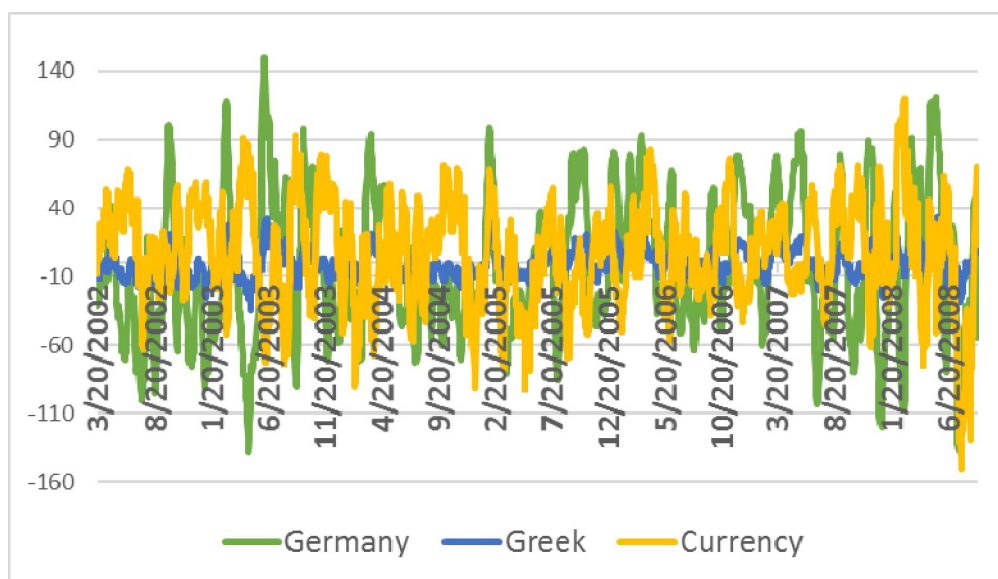


**Figure 4.1.**

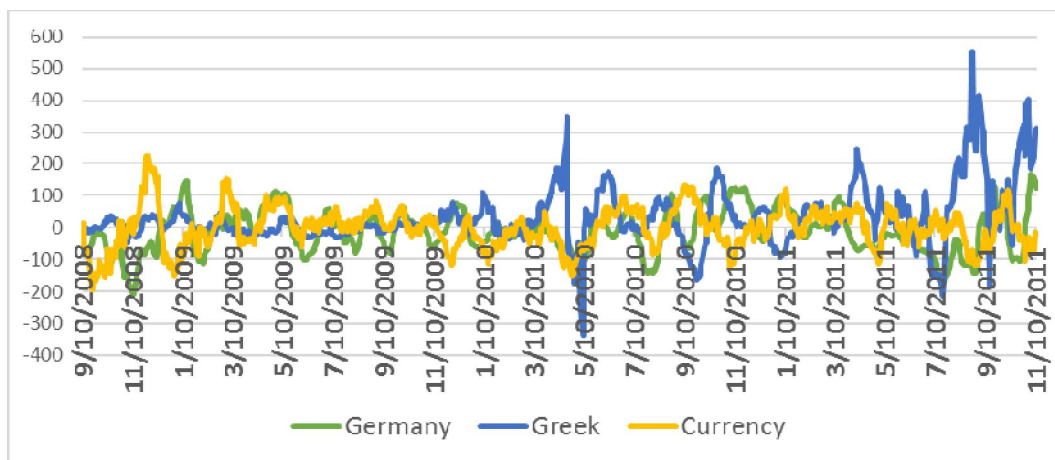


Figure 4.2.

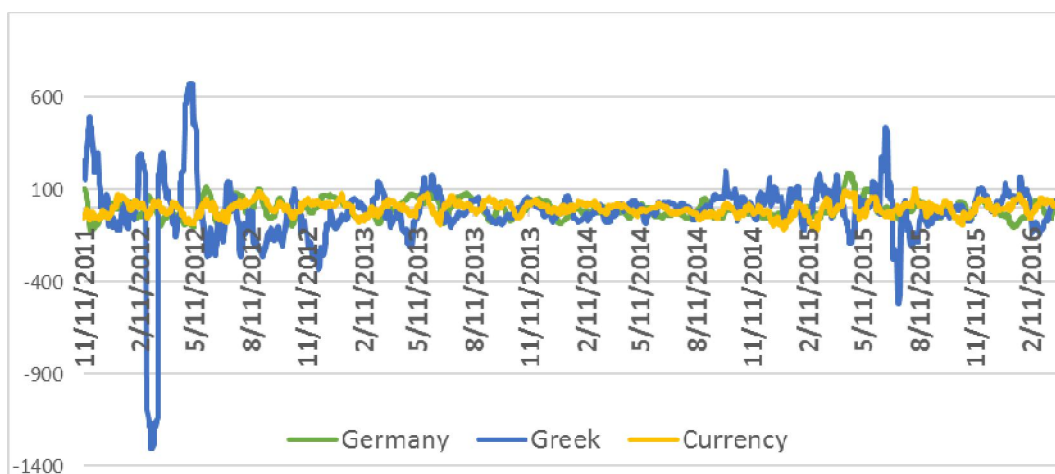


Figure 4.3.



**Figure 5.**

Source: Bank of Greece.

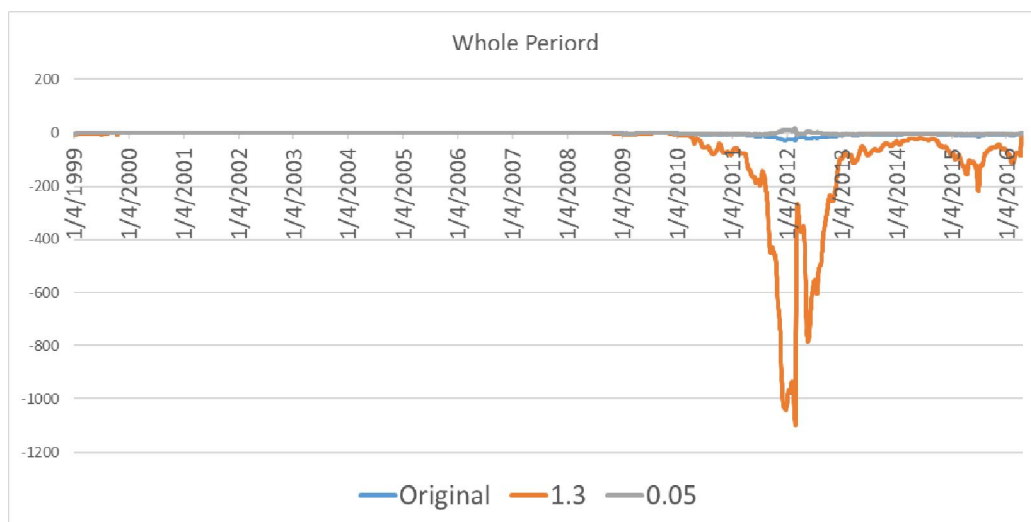


**Figure 6.1.**

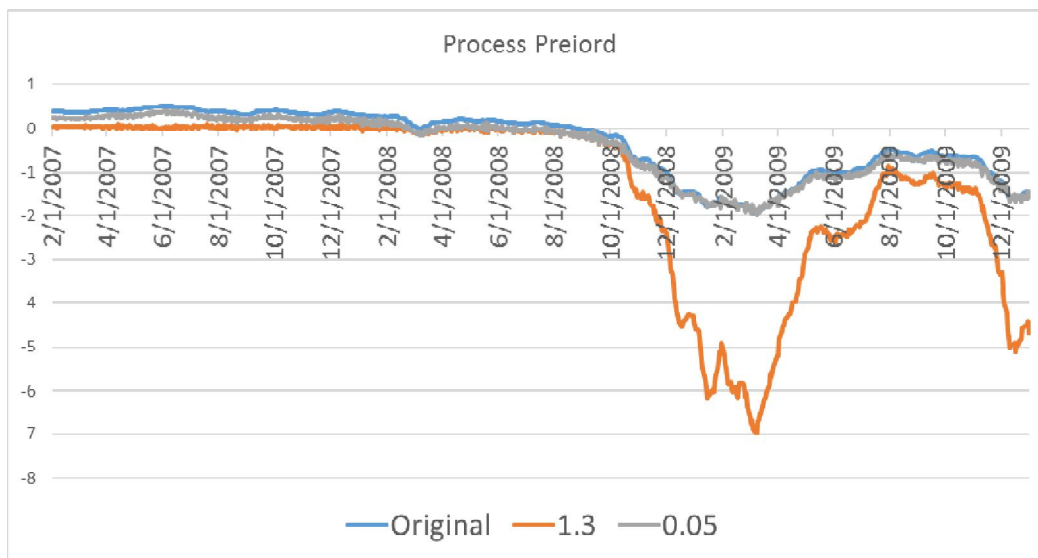


**Figure 6.2.**

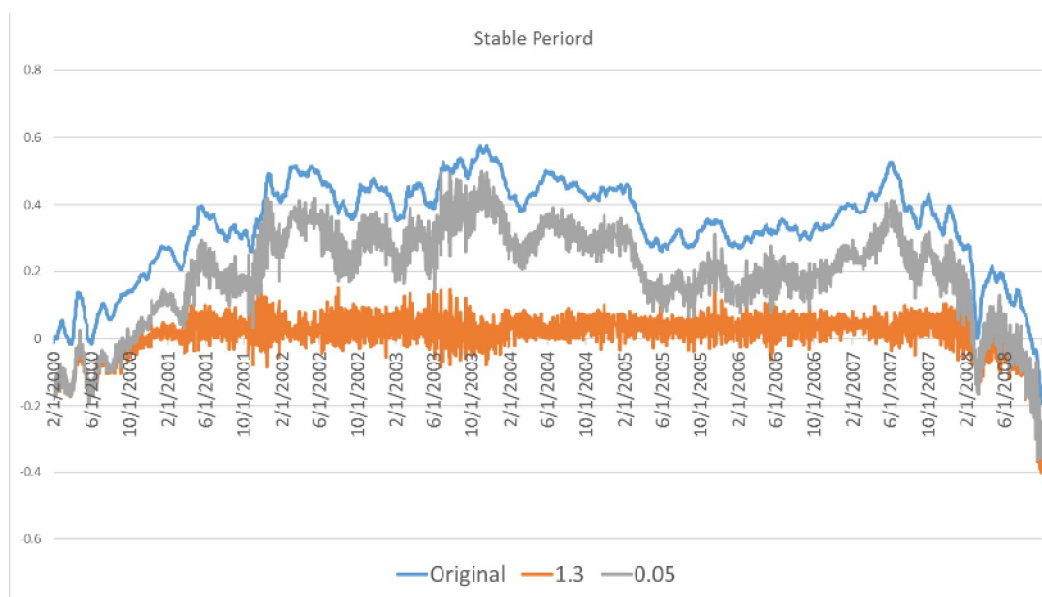


**Figure 6.3.**

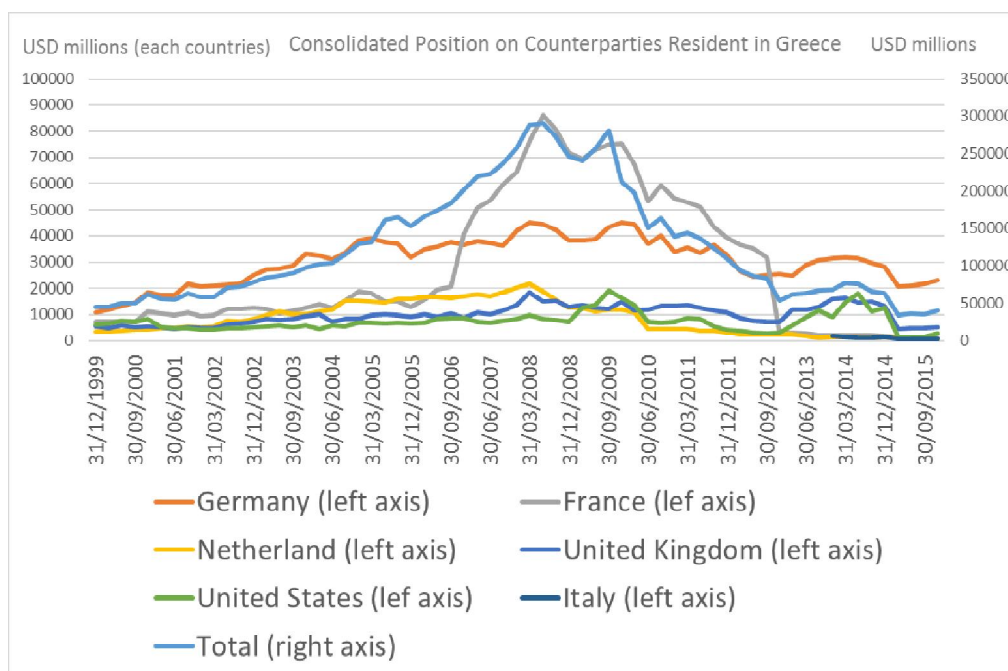


**Figure 7.**

Source: BIS (Consolidated Banking Statistics)



**Figure 8.**

Source: Bank of Greece.

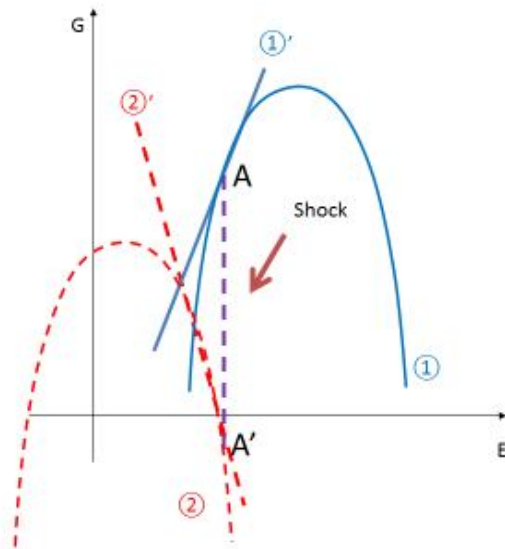


**Figure 9.**