CONSTRUCTIVE AND NUMERICAL STUDY OF THE SPECIFIC ELECTRODYNAMIC PROCESS

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Abstract. *The specific case of the differential Maxwell equations is considered in the Cartesian coordinate system. Basing on this statement, as on the initial mathematical model of the spatial electrodynamic process, the electromagnetic wave propagation is analyzed here. The general wave PDE (partial differential equation) is got including all scalar components of the electromagnetic field vector intensities. Those above mentioned results imply the explicit solution, numerical implementation and computer modeling of the corresponding boundary value problem regarding the aforesaid PDE, and whose boundary conditions differ from the generally accepted.*

Keywords: *the spatial electromagnetic phenomenon, differential Maxwell system, specific boundary conditions, exact results, computer modeling.*

1. Introduction

The differential Maxwell system in the Cartesian coordinates is given

$$\begin{cases} \mathbf{rot}\vec{H} = \partial_0\vec{D} + \vec{i}, \, \mathrm{rot}\vec{E} = -\partial_0\vec{B}, \vec{i} = \sigma\vec{E}, \\ \mathrm{div}\vec{D} = \rho, \, \vec{D} = \varepsilon_a\vec{E}, \, \mathrm{div}\vec{B} = 0, \, \vec{B} = \mu\vec{H}. \end{cases} \tag{1}$$

$$\vec{i} = \vec{i}\,(x, y, z, t), \quad \rho = \rho(x, y, z, t); \quad \partial_0 = \partial / \partial t. \tag{2}$$

In equation (2):

– the first line of vectors represents the unknown electric, magnetic intensities and the given electric, magnetic inductions respectively;

– two functions in the second line determine the known current (charges) and the charge density.

In (1): σ, ε, μ – are the positive real constants denoting specific conductivity, electric and magnetic permeability of the medium; **rot**, div describe the classical differential field operators.

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The unknown electromagnetic field vector intensities are harmonic regarding the time argument, i.e.

$$\vec{E},\vec{H}=\vec{E},\vec{H}(x,y,z)\exp(i\omega t),\, i=\sqrt{-1}, \tag{3}$$

where ω is the vibration frequency, and σ in (1) is assumed to be equal to zero at first.

2. Analytic results

Under those mentioned restrictions (3), system (1) is reduced to the following

$$\begin{cases}\mathbf{rot}\,\vec{E}+i\mu_a\omega\vec{H}=0\\ i\varepsilon_a\omega\vec{E}-\mathrm{rot}\,\vec{H}=0.\end{cases} \tag{4}$$

Then, (4) is transformed into the general wave PDE (partial differential equation) regarding all scalar components of the electromagnetic field vector intensities

$$(\Delta+\varepsilon_a\mu_a\omega^2)F_{kj}=0;\ (k=1,2;\ j=\overline{1,3}), \tag{5}$$

where

$$F_{1j}=E_j=E_j(x,y,z),\ F_{2j}=H_j=H_j(x,y,z),\ (j=\overline{1,3});$$

$$\Delta=\sum_{j=1}^{3}\partial_j^2,\ \ \partial_1=\partial/\partial x,\partial_2=\partial/\partial y,\ \partial_3=\partial/\partial z. \tag{6}$$

Equations (4) to (5), (6) are based on [2].

Rejecting the zero value for σ and leaving assumption (3), system (1), (2) is reduced to the following general wave PDE, including (5), (6) as the particular case

$$(\Delta+\mu\omega(\varepsilon\omega-i\sigma))\vec{F}_k=\vec{f}_k^*;\ \vec{f}_k^*=-\vec{f}_k,\ (k=1,2). \tag{7}$$

In (7),

$$\begin{aligned}&\vec{F}_1=\vec{E},\ \vec{F}_2=\vec{H};\ \vec{f}_2=\vec{f}_2(x,y,z)=\vec{h}(x,y,z);\vec{f}_1=\\&=\vec{f}_1(x,y,z)=-(1/\varepsilon)\cdot\mathbf{grad}\,\rho.\end{aligned} \tag{8}$$

Equations (1) – (3) $\Leftrightarrow$ (7), (8) are based on the technique of [1, 2].

<u>The aim of the present article</u> is the detailed analytic solution and numerical implementation of (7), (8) in the spatial statement under the specified physical conditions.

The proposed numerical values of ε, μ are the following:

($\mu = \mu_a = 12{,}56\cdot 10^{-7}$ H/m), ($\varepsilon = \varepsilon_a = 8{,}85\cdot 10^{-7}$ F/m) determine medium as an air and $\sigma \neq 0 = 10^{-11}$ S/m while $t = 20^0$ C;

$\omega = 1{,}5\cdot 10^9\cdot 2\,\pi$ Hz is the vibration frequency.

Mathematical modeling of the electromagnetic field study for the rectangular cavity resonator is based on the boundary value problem statement regarding the general wave equation (7), (8) in $\mathbf{R}_3$ and respective boundary conditions

$$\begin{gathered} {}^{0,l}g_{kji} = F_{kj}(x_i = 0, l_i),\ (i = \overline{1,3}); \\ F_{kj}(x_i,\ (i = \overline{1,3})) = F_{kj}(x, y, z),\ (k = 1, 2;\ j = \overline{1,3}), \end{gathered} \tag{9}$$

where

$$x_i \in [0, l_i],\ \ (i = \overline{1,3});\ \ x_1 = x, x_2 = y, x_3 = z. \tag{10}$$

In (9), $g\ldots$ are the known continuous functions, and (10) determines the sizes of the resonator.

The unified explicit solution of (7) – (10), using operator generalization of the integral transform method [3]

$$S_i = (l_i/\pi)\int_0^{l_i} \sin\left(\frac{\pi n_i}{l_i} x_i\right) \mathrm{d}x_i,\ \ (i = \overline{1,3}), \tag{11}$$

is given by the following formula

$$\begin{aligned} F_{kj}(x_i,\ (i = \overline{1,3})) &= \left(\prod_{i=1}^{3} S_i^{-1}\right)_{tr} F_{kj} = \\ &= (2/\pi^2)^3 \left(\prod_{i=1}^{3} l_i \sum_{n_i=1}^{\infty} \sin(\pi n_i x_i / l_i)\right)_{tr} F_{kj},\ (k = 1, 2;\ j = \overline{1,3}). \end{aligned} \tag{12}$$

The general design formula for the specific linear boundary conditions (9), (10), basing on (7), (8) looks like

$$\begin{aligned} F_{k\nu}(x_\nu; (\nu = \overline{1,3})) = l_\pi \cdot \sum_{n_1=1}^{N_1} \sum_{n_2=1}^{N_2} \sum_{n_3=1}^{N_3} ({}_{tr}f_{k\nu}^{*} + {}_{k\nu}C_{n_1 n_2 n_3}) \cdot (D_0 - i\sigma\mu_a\omega) \times \\ \left(D_0^2 + (\sigma\mu_a\omega)^2\right)^{-1} \cdot \sin\left(\frac{\pi n_1}{l_1} x_1\right) \cdot \sin\left(\frac{\pi n_2}{l_2} x_2\right) \cdot \sin\left(\frac{\pi n_3}{l_3} x_3\right),\ (k = 1, 2) \end{aligned};$$

$$\operatorname{Re} F_{k\nu}(x_\nu;(\nu=\overline{1,3}))=l_\pi\cdot\sum_{n_1=1}^{N_1}\sum_{n_2=1}^{N_2}\sum_{n_3=1}^{N_3}({}_{tr}f_{k\nu}^{*}+{}_{k\nu}C_{n_1n_2n_3})\cdot D_0\times$$

$$\left(D_0^2+(\sigma\mu_a\omega)^2\right)^{-1}\cdot\sin\left(\frac{\pi n_1}{l_1}x_1\right)\cdot\sin\left(\frac{\pi n_2}{l_2}x_2\right)\cdot\sin\left(\frac{\pi n_3}{l_3}x_3\right),\ (k=1,2); \tag{13}$$

$$\operatorname{Im} F_{k\nu}(x_\nu;(\nu=\overline{1,3}))=-l_\pi\sigma\mu_a\omega\cdot\sum_{n_1=1}^{N_1}\sum_{n_2=1}^{N_2}\sum_{n_3=1}^{N_3}({}_{tr}f_{k\nu}^{*}+{}_{k\nu}C_{n_1n_2n_3})\times$$

$$\times\left(D_0^2+(\sigma\mu_a\omega)^2\right)^{-1}\cdot\sin\left(\frac{\pi n_1}{l_1}x_1\right)\cdot\sin\left(\frac{\pi n_2}{l_2}x_2\right)\cdot\sin\left(\frac{\pi n_3}{l_3}x_3\right),\ (k=1,2);$$

where:

$$D_0=\sum_{j=1}^{3}\left(\frac{\pi n_j}{l_j}\right)^2-\varepsilon_a\mu_a\omega^2;\ l_\pi=\left(\frac{2}{\pi^2}\right)^3\cdot l_1l_2l_3;$$

$${}_{tr}f_{1\nu}^{*}=\frac{l_\nu\cdot(1+(-1)^{n_\nu+1})\cdot(-1)^{\left(\sum\limits_{m=1}^{3}n_m\right)-n_\nu}\cdot\prod\limits_{\substack{m=1\\m\neq\nu}}^{3}l_m^2}{\varepsilon_a\pi^6\cdot\prod\limits_{m=1}^{3}n_m};\ {}_{tr}f_{2\nu}^{*}=\frac{l_\nu\cdot\prod\limits_{\substack{m=1\\m\neq\nu}}^{3}l_m^2\cdot\sum\limits_{p=1}^{3}\left((-1)^{n_p}l_p\cdot\prod\limits_{\substack{m=1\\m\neq p}}^{3}((-1)^m-1)\right)}{\pi^6\cdot\prod\limits_{m=1}^{3}n_m};$$

$${}_{k\nu}C_{n_1n_2n_3}=\left(\frac{l_2l_3}{l_1}\right)^2\cdot\frac{n_1}{\pi^3\cdot n_2\cdot n_3}\times \tag{14}$$

$$\Big((l_1\cdot(-1)^{n_1+1}\cdot((-1)^{n_2+1}-1)\cdot((-1)^{n_3+1}+1)+\nu\cdot l_3\cdot(-1)^{n_3+1}\cdot((-1)^{n_2+1}-1)\cdot((-1)^{n_1+1}+1)+$$

$$+\ k\cdot l_2\cdot(-1)^{n_2+1}\cdot((-1)^{n_3+1}+1)\cdot((-1)^{n_1+1}+1)\Big)+\left(\frac{l_1l_3}{l_2}\right)^2\cdot\frac{n_2}{\pi^3\cdot n_1\cdot n_3}\times$$

$$\Big(l_2\cdot(-1)^{n_2+1}\cdot((-1)^{n_1+1}-1)\cdot((-1)^{n_3+1}+1)+\nu\cdot l_3\cdot(-1)^{n_3+1}\cdot((-1)^{n_1+1}-1)\cdot((-1)^{n_2+1}+1)+$$

$$k\cdot l_1\cdot(-1)^{n_1+1}\cdot((-1)^{n_3+1}+1)\cdot((-1)^{n_2+1}+1)\Big)+\left(\frac{l_1l_2}{l_3}\right)^2\cdot\frac{n_3}{\pi^3\cdot n_1\cdot n_2}\times$$

$$\Big(l_3\cdot(-1)^{n_3+1}\cdot((-1)^{n_1+1}-1)\cdot((-1)^{n_2+1}+1)+\nu\cdot l_2\cdot\left(-1\right)^{n_2+1}\cdot((-1)^{n_1+1}-1)\cdot((-1)^{n_3+1}+1)+$$

$$+\ k\cdot l_1\cdot(-1)^{n_1+1}\cdot((-1)^{n_3+1}+1)\cdot((-1)^{n_2+1}+1)\Big),\ (\nu=\overline{1,3};\ k=1,2).$$

In all expressions from (13), the coefficient (14) between the sum symbol and the sine's product is called as the coefficient of the electromagnetic field modification, since just this expression is responsible for the description of the field behavior under specific linear boundary conditions (9), (10).

While all three upper sum limits are infinite in (13), the explicit solution of (7)-(10) appears, and the respective functional series converges absolutely and uniformly. It is enough as for the mathematical explicit results, as for the experimental / numerical conclusions.

3. Discretization procedure

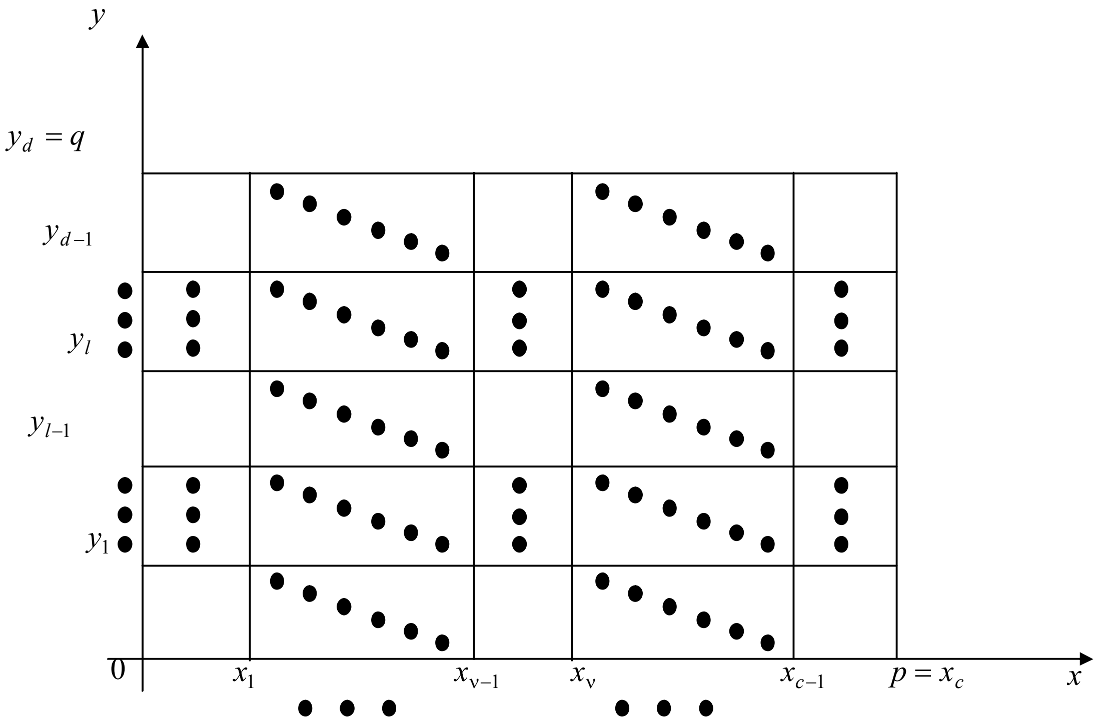

Figure 1. The discretization procedure scheme.

Numerical implementation of (7) – (10) is done applying the simple discretization procedure Figure 1 to (13), (14) with the knots,

$$(x_\nu, y_l),\ x_\nu = \frac{p}{c}\nu,\ y_l = \frac{q}{d}l,\ (\nu = \overline{1, c};\ l = \overline{1, d});\ c, d \in \boldsymbol{N}; \tag{15}$$

$$x_{\nu s} = \frac{l_\nu}{m_\nu} \cdot s,\ (s = \overline{1, m_\nu};\ \nu = \overline{1, 3}),$$

and creating at first the cut set by the axis 0*z* which is shown below

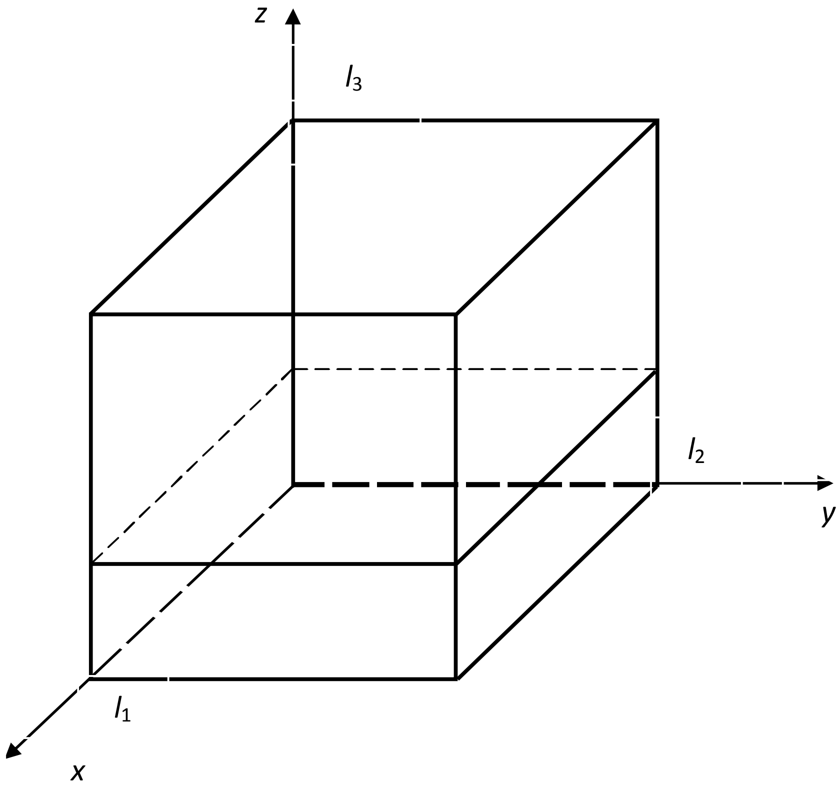

Figure 2. The cut set of the resonator by the axis 0*z*.

4. Computer modeling

F11Re x / y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	-7.08E-33	-2.3E-32	-4E-32	-6E-32	-8E-32	-9.5E-32	-1.1E-31	-1.1E-31	-7.8E-32
0.002	-2.25E-32	-7.4E-32	-1.3E-31	-2E-31	-2.7E-31	-3.2E-31	-3.5E-31	-3.5E-31	-2.5E-31
0.003	-3.88E-32	-1.3E-31	-2.4E-31	-3.6E-31	-4.8E-31	-5.7E-31	-6.3E-31	-6.1E-31	-4.2E-31
0.004	-5.61E-32	-1.9E-31	-3.5E-31	-5.3E-31	-7.1E-31	-8.4E-31	-9.1E-31	-8.8E-31	-6.1E-31
0.005	-7.29E-32	-2.5E-31	-4.6E-31	-6.9E-31	-9.2E-31	-1.1E-30	-1.2E-30	-1.1E-30	-7.9E-31
0.006	-8.41E-32	-2.8E-31	-5.3E-31	-8E-31	-1.1E-30	-1.3E-30	-1.4E-30	-1.3E-30	-9.1E-31
0.007	-9.05E-32	-3E-31	-5.6E-31	-8.5E-31	-1.1E-30	-1.3E-30	-1.5E-30	-1.4E-30	-9.8E-31
0.008	-8.98E-32	-2.9E-31	-5.3E-31	-8E-31	-1.1E-30	-1.3E-30	-1.4E-30	-1.4E-30	-9.8E-31
0.009	-6.37E-32	-2E-31	-3.6E-31	-5.4E-31	-7.2E-31	-8.5E-31	-9.5E-31	-9.8E-31	-7.1E-31

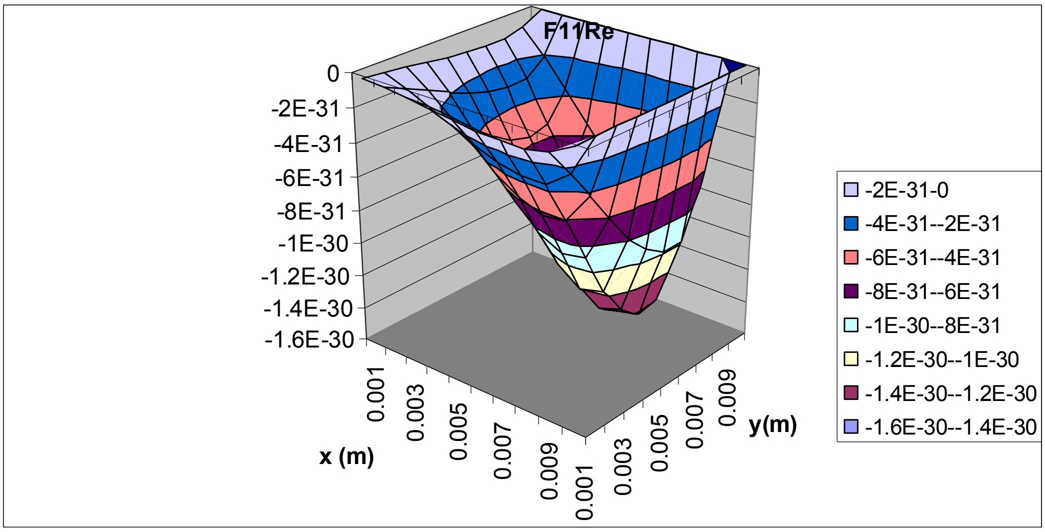

Figure 1. Re E_1.

F12Re y \ x	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	-7.08E-31	-1.1E-30	-1.3E-30	-1.4E-30	-1.5E-30	-1.4E-30	-1.3E-30	-1.1E-30	-7.1E-31
0.002	-1.13E-30	-1.8E-30	-2.2E-30	-2.4E-30	-2.5E-30	-2.4E-30	-2.2E-30	-1.8E-30	-1.1E-30
0.003	-1.34E-30	-2.2E-30	-2.7E-30	-2.9E-30	-3.1E-30	-2.9E-30	-2.7E-30	-2.2E-30	-1.3E-30
0.004	-1.5E-30	-2.5E-30	-3E-30	-3.3E-30	-3.5E-30	-3.3E-30	-3E-30	-2.5E-30	-1.5E-30
0.005	-1.6E-30	-2.7E-30	-3.2E-30	-3.5E-30	-3.7E-30	-3.5E-30	-3.2E-30	-2.7E-30	-1.6E-30
0.006	-1.58E-30	-2.6E-30	-3.2E-30	-3.5E-30	-3.6E-30	-3.5E-30	-3.2E-30	-2.6E-30	-1.6E-30
0.007	-1.51E-30	-2.5E-30	-3E-30	-3.3E-30	-3.4E-30	-3.3E-30	-3E-30	-2.5E-30	-1.5E-30
0.008	-1.36E-30	-2.2E-30	-2.5E-30	-2.8E-30	-2.9E-30	-2.8E-30	-2.5E-30	-2.2E-30	-1.4E-30
0.009	-8.71E-31	-1.4E-30	-1.6E-30	-1.7E-30	-1.7E-30	-1.7E-30	-1.6E-30	-1.4E-30	-8.7E-31

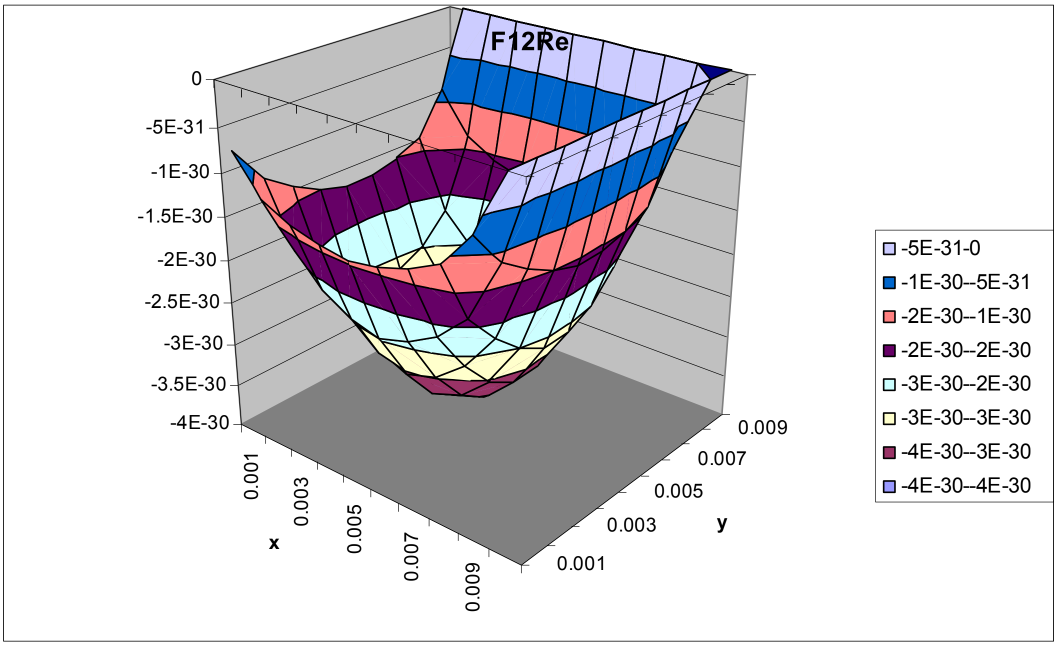

Figure 2. Re E_2.

F13Re

x \ y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	7.31E-32	6.66E-32	-2.7E-32	-1.4E-31	-2.3E-31	-3E-31	-3.7E-31	-4E-31	-2.7E-31
0.002	6.66E-32	3.1E-32	-1.2E-31	-3E-31	-4.4E-31	-5.4E-31	-6.4E-31	-6.4E-31	-4.3E-31
0.003	-2.66E-32	-1.2E-31	-2.9E-31	-4.8E-31	-6.4E-31	-7.4E-31	-8.2E-31	-7.8E-31	-5E-31
0.004	-1.44E-31	-3E-31	-4.8E-31	-6.8E-31	-8.3E-31	-9.2E-31	-9.7E-31	-8.9E-31	-5.7E-31
0.005	-2.32E-31	-4.4E-31	-6.4E-31	-8.3E-31	-9.8E-31	-1.1E-30	-1.1E-30	-9.7E-31	-6.1E-31
0.006	-3.01E-31	-5.4E-31	-7.4E-31	-9.2E-31	-1.1E-30	-1.1E-30	-1.1E-30	-9.8E-31	-6.2E-31
0.007	-3.75E-31	-6.4E-31	-8.2E-31	-9.7E-31	-1.1E-30	-1.1E-30	-1.1E-30	-9.6E-31	-6E-31
0.008	-3.98E-31	-6.4E-31	-7.8E-31	-8.9E-31	-9.7E-31	-9.8E-31	-9.6E-31	-8.6E-31	-5.5E-31
0.009	-2.72E-31	-4.3E-31	-5E-31	-5.7E-31	-6.1E-31	-6.2E-31	-6E-31	-5.5E-31	-3.6E-31

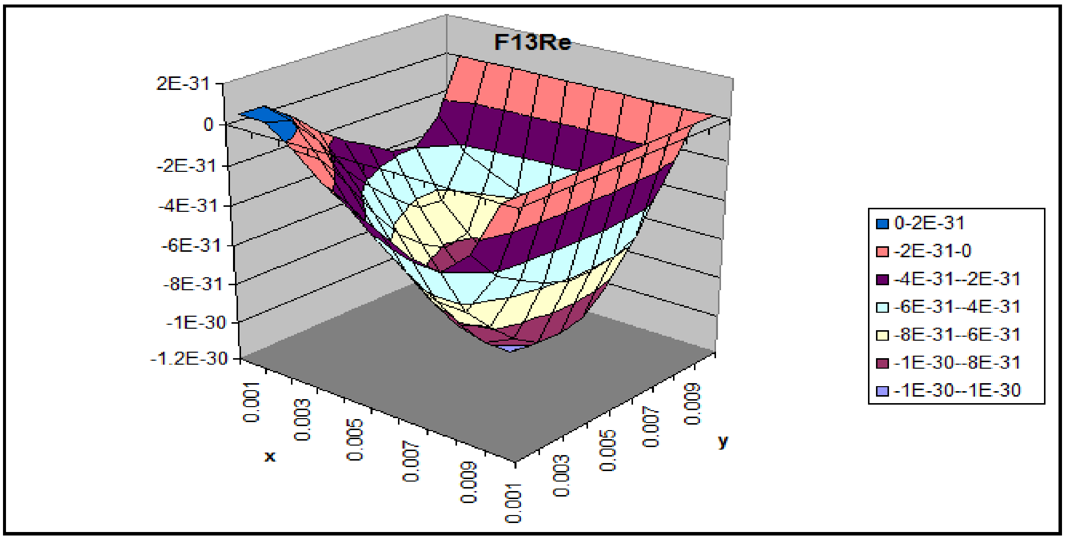

Figure 3. Re E_3

H21Re

x \ y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	-4.63E-37	-8E-37	-6.8E-38	8.25E-37	1.59E-36	3.35E-36	5.48E-36	5.58E-36	3.25E-36
0.002	-7.91E-37	-1.1E-36	3.82E-37	1.65E-36	2.78E-36	6.18E-36	9.02E-36	6.35E-36	1.37E-36
0.003	-1.26E-38	5.04E-37	1.41E-36	1.68E-36	2.34E-36	4.26E-36	3.82E-36	-1.3E-36	-4.6E-36
0.004	9.8E-37	2.06E-36	2.06E-36	3.28E-36	5.75E-36	6.65E-36	7.1E-36	9.48E-36	8.92E-36
0.005	1.87E-36	3.54E-36	3.24E-36	6.32E-36	1.15E-35	1.31E-35	1.76E-35	3.02E-35	3.05E-35
0.006	3.75E-36	7.33E-36	5.78E-36	7.99E-36	1.4E-35	1.44E-35	1.75E-35	3.31E-35	3.6E-35
0.007	6.06E-36	1.07E-35	6.13E-36	9.45E-36	1.97E-35	1.88E-35	2.34E-35	5.37E-35	6.25E-35
0.008	6.32E-36	8.45E-36	1.59E-36	1.26E-35	3.32E-35	3.56E-35	5.49E-35	1.2E-34	1.32E-34
0.009	3.84E-36	3.03E-36	-2.3E-36	1.14E-35	3.3E-35	3.81E-35	6.36E-35	1.33E-34	1.42E-34

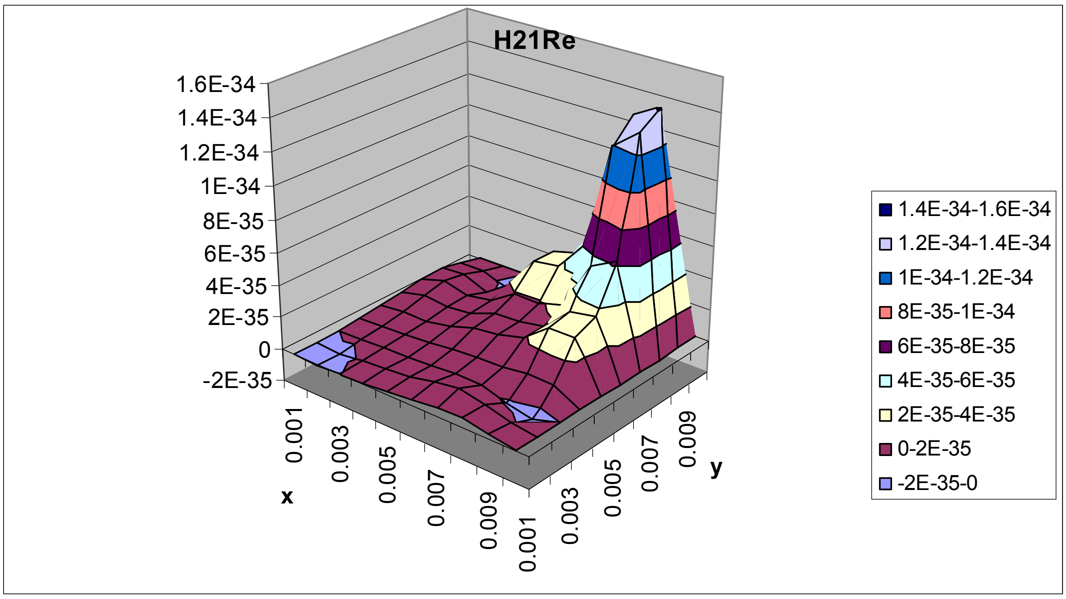

Figure 4. Re H_1

H22Re

x \ y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	-1.07E-36	-2E-36	-8.7E-37	3.57E-37	1.12E-36	3.88E-36	7.32E-36	6.9E-36	3.18E-36
0.002	-2E-36	-3.4E-36	-9.6E-37	7.1E-37	1.54E-36	6.94E-36	1.16E-35	6.11E-36	-2E-36
0.003	-8.03E-37	-8.2E-37	8.35E-37	9.51E-37	9.98E-37	4.16E-36	3.59E-36	-6.4E-36	-1.2E-35
0.004	5.36E-37	1.17E-36	1.39E-36	2.61E-36	4.99E-36	6.54E-36	6.83E-36	6.89E-36	5.27E-36
0.005	1.44E-36	2.42E-36	2.03E-36	5.64E-36	1.15E-35	1.42E-35	2.05E-35	3.47E-35	3.43E-35
0.006	4.35E-36	8.26E-36	5.9E-36	8.07E-36	1.51E-35	1.6E-35	2E-35	3.86E-35	4.21E-35
0.007	7.99E-36	1.36E-35	6.23E-36	9.53E-36	2.29E-35	2.15E-35	2.68E-35	6.66E-35	7.94E-35
0.008	7.74E-36	8.52E-36	-3.1E-36	1.04E-35	3.82E-35	4.14E-35	6.79E-35	1.57E-34	1.76E-34
0.009	3.86E-36	-7.2E-38	-9.6E-36	8.06E-36	3.72E-35	4.45E-35	8.08E-35	1.77E-34	1.91E-34

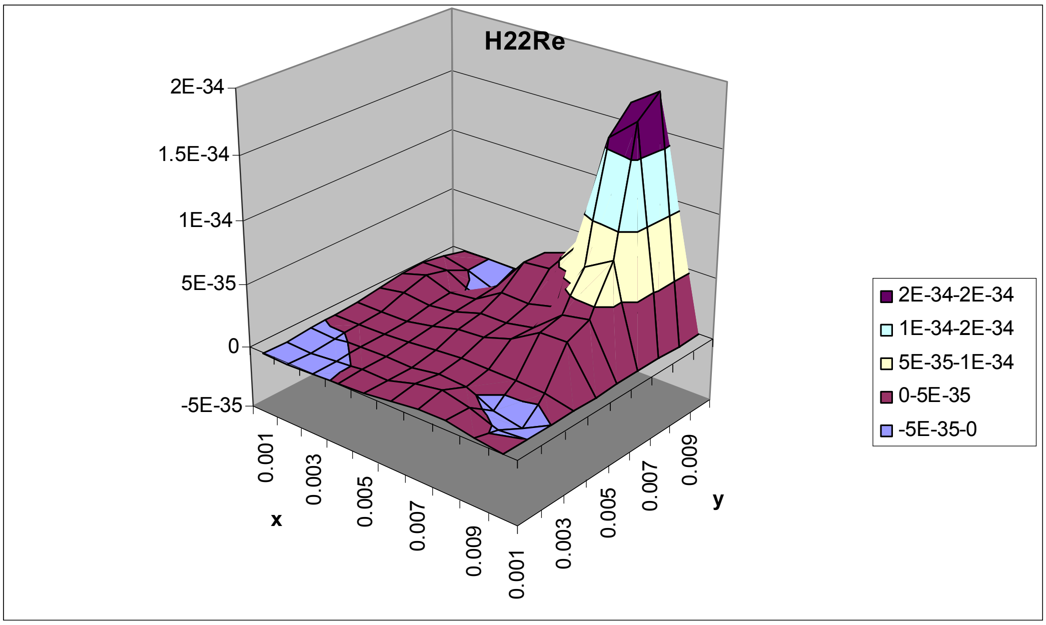

Figure 5. Re H_2

H23Re

X \ y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	-1.67E-36	-3.2E-36	-1.7E-36	-1.1E-37	6.57E-37	4.42E-36	9.16E-36	8.21E-36	3.11E-36
0.002	-3.21E-36	-5.7E-36	-2.3E-36	-2.3E-37	3.06E-37	7.7E-36	1.43E-35	5.87E-36	-5.3E-36
0.003	-1.59E-36	-2.2E-36	2.56E-37	2.2E-37	-3.4E-37	4.06E-36	3.36E-36	-1.2E-35	-2E-35
0.004	9.12E-38	2.92E-37	7.11E-37	1.93E-36	4.22E-36	6.42E-36	6.57E-36	4.31E-36	1.61E-36
0.005	1.02E-36	1.29E-36	8.18E-37	4.96E-36	1.15E-35	1.52E-35	2.33E-35	3.92E-35	3.82E-35
0.006	4.94E-36	9.19E-36	6.02E-36	8.16E-36	1.63E-35	1.75E-35	2.24E-35	4.4E-35	4.82E-35
0.007	9.91E-36	1.64E-35	6.34E-36	9.61E-36	2.6E-35	2.41E-35	3.03E-35	7.94E-35	9.64E-35
0.008	9.17E-36	8.59E-36	-7.8E-36	8.3E-36	4.31E-35	4.72E-35	8.09E-35	1.95E-34	2.21E-34
0.009	3.87E-36	-3.2E-36	-1.7E-35	4.76E-36	4.14E-35	5.1E-35	9.79E-35	2.21E-34	2.41E-34

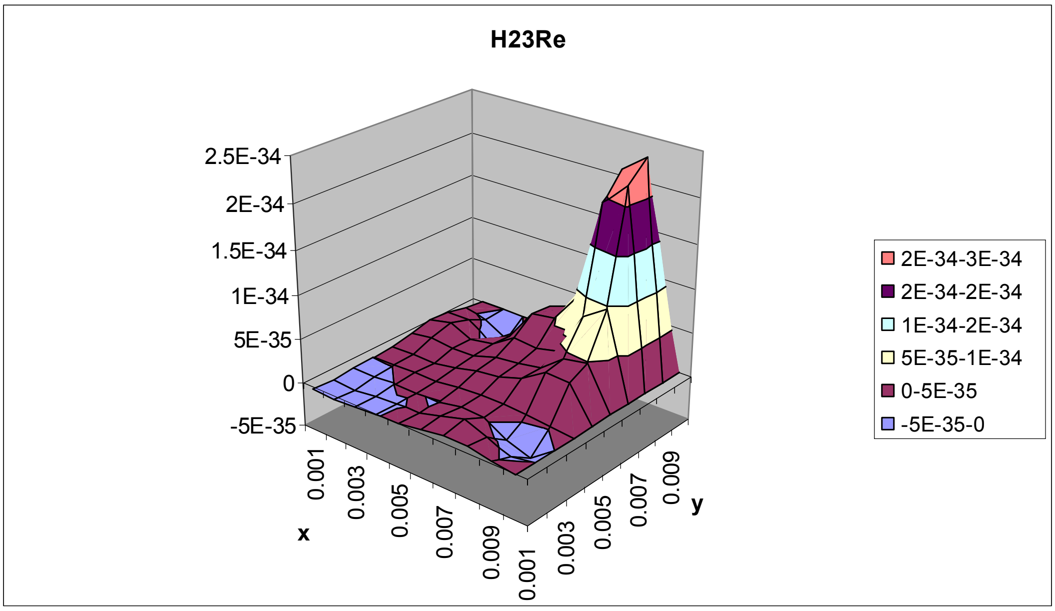

Figure 6. Re H_3

F11Im x \ y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	1.81E-45	6.65E-45	1.33E-44	2.07E-44	2.74E-44	3.2E-44	3.31E-44	2.91E-44	1.81E-44
0.002	6.6E-45	2.44E-44	4.92E-44	7.63E-44	1.01E-43	1.18E-43	1.22E-43	1.06E-43	6.59E-44
0.003	1.31E-44	4.86E-44	9.82E-44	1.53E-43	2.02E-43	2.36E-43	2.43E-43	2.11E-43	1.3E-43
0.004	2E-44	7.44E-44	1.51E-43	2.35E-43	3.11E-43	3.62E-43	3.72E-43	3.22E-43	1.98E-43
0.005	2.61E-44	9.71E-44	1.97E-43	3.06E-43	4.06E-43	4.73E-43	4.85E-43	4.2E-43	2.58E-43
0.006	3E-44	1.12E-43	2.26E-43	3.52E-43	4.66E-43	5.43E-43	5.58E-43	4.84E-43	2.97E-43
0.007	3.05E-44	1.13E-43	2.29E-43	3.57E-43	4.72E-43	5.51E-43	5.67E-43	4.93E-43	3.03E-43
0.008	2.64E-44	9.77E-44	1.97E-43	3.05E-43	4.04E-43	4.72E-43	4.87E-43	4.26E-43	2.64E-43
0.009	1.63E-44	5.99E-44	1.2E-43	1.86E-43	2.46E-43	2.88E-43	2.98E-43	2.62E-43	1.63E-43

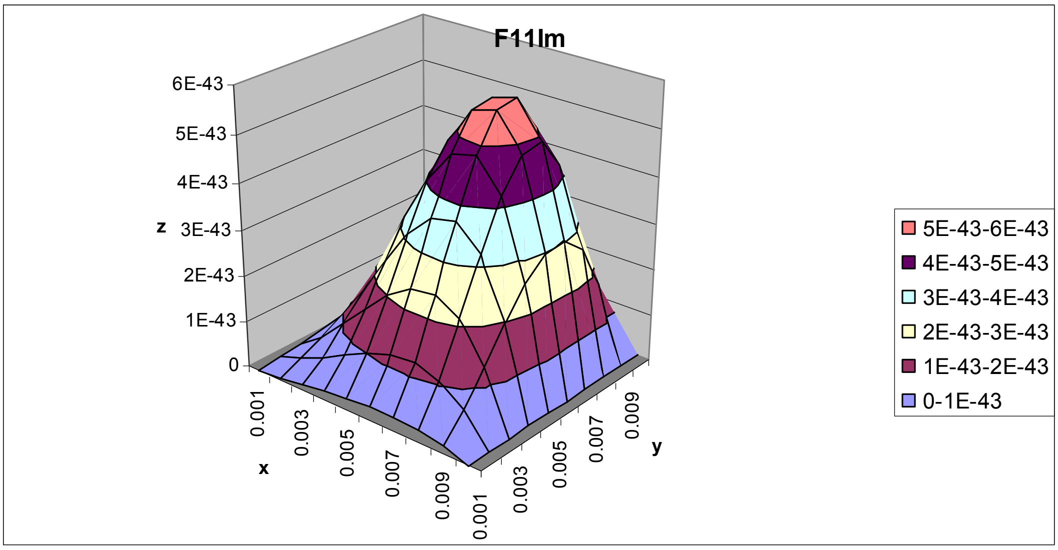

Figure 7. Im E_1

F12Im

x \ y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	1.81E-45	6.6E-45	1.31E-44	2E-44	2.61E-44	3E-44	3.05E-44	2.64E-44	1.63E-44
0.002	6.65E-45	2.44E-44	4.86E-44	7.44E-44	9.71E-44	1.12E-43	1.13E-43	9.77E-44	5.99E-44
0.003	1.33E-44	4.92E-44	9.82E-44	1.51E-43	1.97E-43	2.26E-43	2.29E-43	1.97E-43	1.2E-43
0.004	2.07E-44	7.63E-44	1.53E-43	2.35E-43	3.06E-43	3.52E-43	3.57E-43	3.05E-43	1.86E-43
0.005	2.74E-44	1.01E-43	2.02E-43	3.11E-43	4.06E-43	4.66E-43	4.72E-43	4.04E-43	2.46E-43
0.006	3.2E-44	1.18E-43	2.36E-43	3.62E-43	4.73E-43	5.43E-43	5.51E-43	4.72E-43	2.88E-43
0.007	3.31E-44	1.22E-43	2.43E-43	3.72E-43	4.85E-43	5.58E-43	5.67E-43	4.87E-43	2.98E-43
0.008	2.91E-44	1.06E-43	2.11E-43	3.22E-43	4.2E-43	4.84E-43	4.93E-43	4.26E-43	2.62E-43
0.009	1.81E-44	6.59E-44	1.3E-43	1.98E-43	2.58E-43	2.97E-43	3.03E-43	2.64E-43	1.63E-43

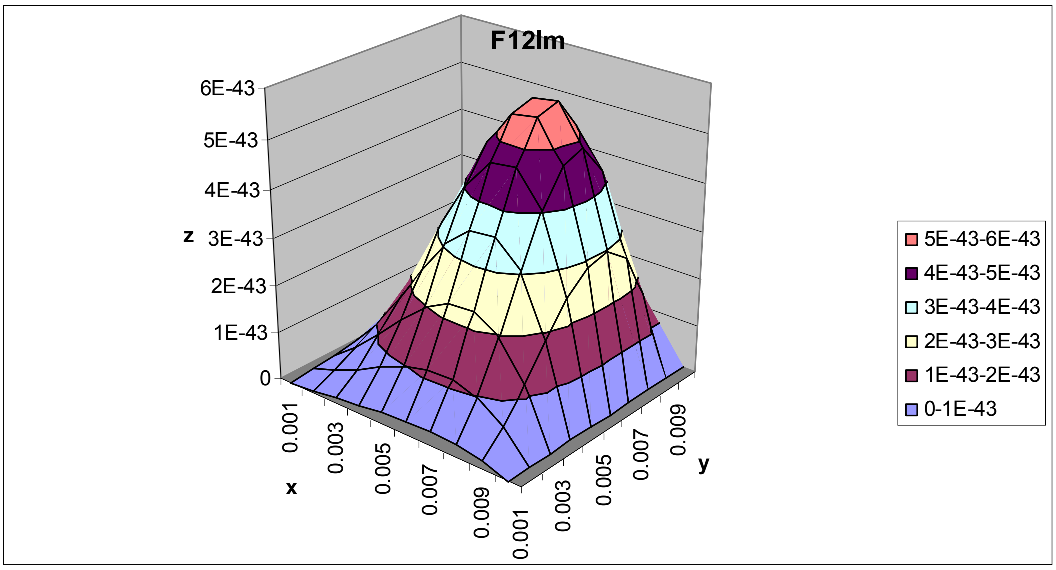

Figure 8. Im E_2

F13Im x \ y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	2.04E-46	8.96E-46	2.19E-45	3.98E-45	5.96E-45	7.69E-45	8.67E-45	8.13E-45	5.26E-45
0.002	8.96E-46	3.78E-45	8.86E-45	1.57E-44	2.31E-44	2.94E-44	3.27E-44	3.02E-44	1.94E-44
0.003	2.19E-45	8.86E-45	1.99E-44	3.42E-44	4.93E-44	6.18E-44	6.77E-44	6.17E-44	3.91E-44
0.004	3.98E-45	1.57E-44	3.42E-44	5.74E-44	8.12E-44	1E-43	1.08E-43	9.77E-44	6.15E-44
0.005	5.96E-45	2.31E-44	4.93E-44	8.12E-44	1.13E-43	1.38E-43	1.48E-43	1.32E-43	8.3E-44
0.006	7.69E-45	2.94E-44	6.18E-44	1E-43	1.38E-43	1.67E-43	1.78E-43	1.58E-43	9.91E-44
0.007	8.67E-45	3.27E-44	6.77E-44	1.08E-43	1.48E-43	1.78E-43	1.88E-43	1.67E-43	1.05E-43
0.008	8.13E-45	3.02E-44	6.17E-44	9.77E-44	1.32E-43	1.58E-43	1.67E-43	1.49E-43	9.39E-44
0.009	5.26E-45	1.94E-44	3.91E-44	6.15E-44	8.3E-44	9.91E-44	1.05E-43	9.39E-44	5.93E-44

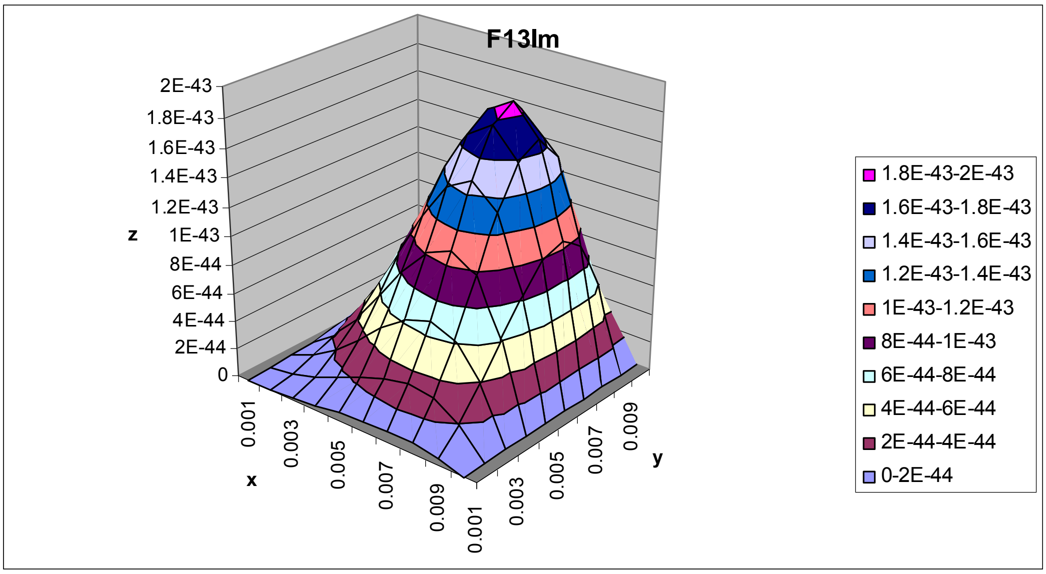

Figure 9. Im E_3

H21Im

y / x	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	1.04E-50	-2.8E-51	-8.8E-50	-2.2E-49	-3.7E-49	-5.4E-49	-6.5E-49	-6E-49	-3.6E-49
0.002	-4.1E-51	-1.1E-49	-3.9E-49	-7.8E-49	-1.2E-48	-1.7E-48	-1.9E-48	-1.7E-48	-9.3E-49
0.003	-9.5E-50	-4.1E-49	-9.3E-49	-1.6E-48	-2.3E-48	-3E-48	-3.2E-48	-2.7E-48	-1.6E-48
0.004	-2.4E-49	-8.4E-49	-1.7E-48	-2.7E-48	-3.8E-48	-4.7E-48	-5.1E-48	-4.8E-48	-3.1E-48
0.005	-4.1E-49	-1.3E-48	-2.5E-48	-3.9E-48	-5.5E-48	-6.8E-48	-7.7E-48	-7.7E-48	-5.3E-48
0.006	-5.9E-49	-1.9E-48	-3.3E-48	-5E-48	-7E-48	-8.8E-48	-1E-47	-1.1E-47	-7.5E-48
0.007	-7.3E-49	-2.2E-48	-3.6E-48	-5.6E-48	-8.1E-48	-1E-47	-1.2E-47	-1.4E-47	-1E-47
0.008	-6.8E-49	-1.9E-48	-3.1E-48	-5.3E-48	-8.1E-48	-1.1E-47	-1.4E-47	-1.7E-47	-1.3E-47
0.009	-4.2E-49	-1.1E-48	-1.9E-48	-3.5E-48	-5.7E-48	-7.8E-48	-1.1E-47	-1.3E-47	-1.1E-47

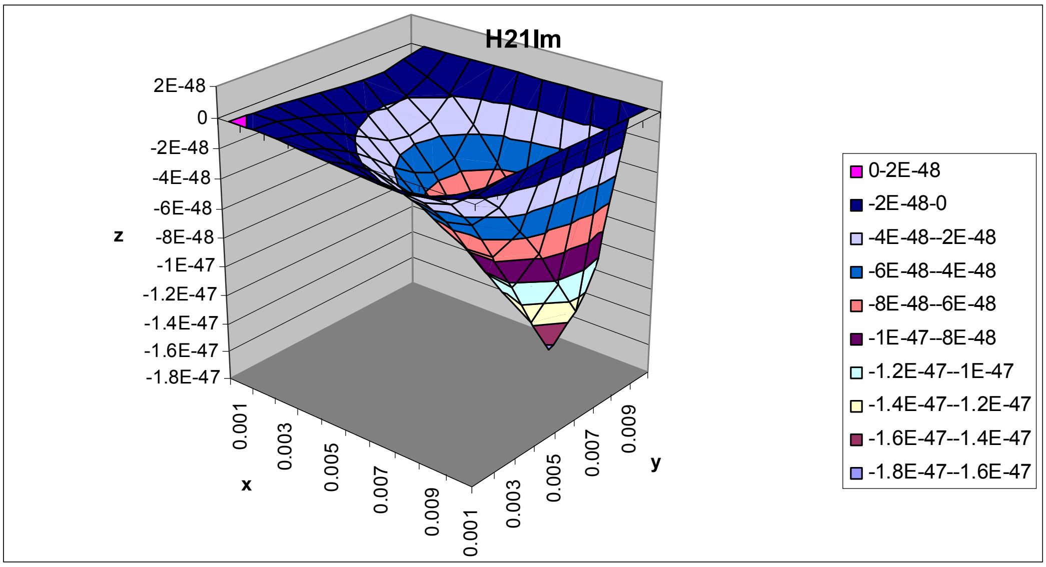

Figure 10. Im H_1

H22Im (y / x)	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	4.88E-50	9.35E-50	1.87E-50	-1.3E-49	-3.1E-49	-5.4E-49	-7.2E-49	-6.5E-49	-3.5E-49
0.002	9.2E-50	1.38E-49	-1.1E-49	-5.4E-49	-1E-48	-1.7E-48	-2E-48	-1.6E-48	-7.9E-49
0.003	1.03E-50	-1.3E-49	-5.9E-49	-1.3E-48	-2.1E-48	-2.9E-48	-3.2E-48	-2.5E-48	-1.3E-48
0.004	-1.5E-49	-6E-49	-1.3E-48	-2.4E-48	-3.6E-48	-4.7E-48	-5.2E-48	-4.8E-48	-3E-48
0.005	-3.5E-49	-1.2E-48	-2.3E-48	-3.7E-48	-5.5E-48	-7.1E-48	-8.2E-48	-8.3E-48	-5.8E-48
0.006	-6.1E-49	-1.9E-48	-3.2E-48	-5E-48	-7.3E-48	-9.5E-48	-1.1E-47	-1.2E-47	-8.7E-48
0.007	-8.1E-49	-2.3E-48	-3.6E-48	-5.7E-48	-8.7E-48	-1.2E-47	-1.4E-47	-1.6E-47	-1.3E-47
0.008	-7.4E-49	-1.9E-48	-3E-48	-5.3E-48	-8.8E-48	-1.2E-47	-1.7E-47	-2E-47	-1.7E-47
0.009	-4.2E-49	-1E-48	-1.6E-48	-3.4E-48	-6.2E-48	-9E-48	-1.3E-47	-1.7E-47	-1.4E-47

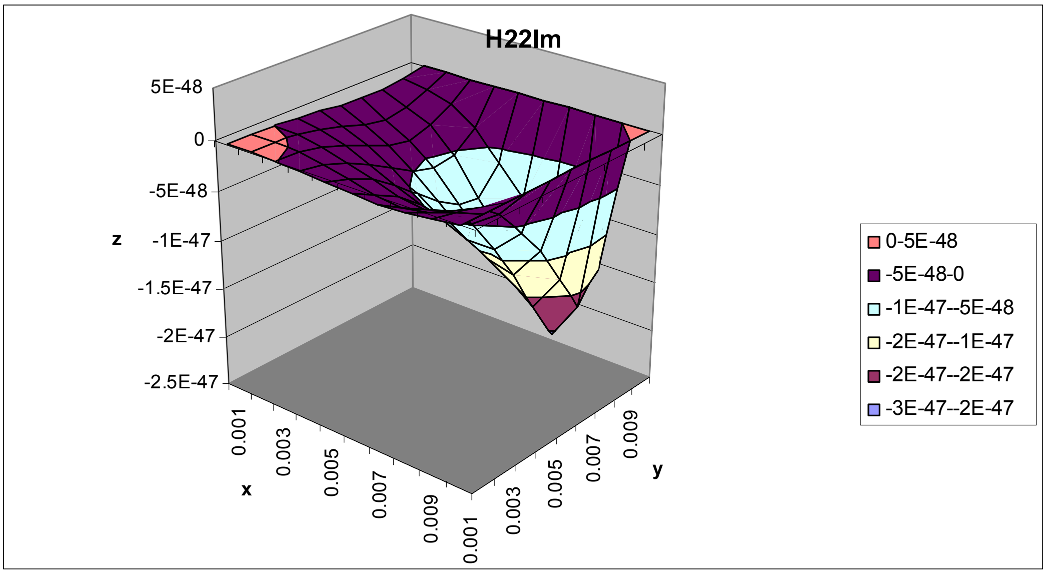

Figure 11. Im H_2

H23Im x \ y	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.001	8.73E-50	1.9E-49	1.25E-49	-4.3E-50	-2.5E-49	-5.5E-49	-7.8E-49	-6.9E-49	-3.5E-49
0.002	1.88E-49	3.82E-49	1.71E-49	-2.9E-49	-8.7E-49	-1.6E-48	-2.1E-48	-1.6E-48	-6.6E-49
0.003	1.16E-49	1.5E-49	-2.4E-49	-9.4E-49	-1.8E-48	-2.8E-48	-3.1E-48	-2.3E-48	-9.6E-49
0.004	-6.8E-50	-3.6E-49	-1E-48	-2.1E-48	-3.4E-48	-4.6E-48	-5.3E-48	-4.7E-48	-2.9E-48
0.005	-3E-49	-1E-48	-2E-48	-3.5E-48	-5.5E-48	-7.3E-48	-8.8E-48	-9E-48	-6.3E-48
0.006	-6.3E-49	-1.9E-48	-3.1E-48	-5E-48	-7.6E-48	-1E-47	-1.2E-47	-1.3E-47	-9.9E-48
0.007	-8.8E-49	-2.4E-48	-3.6E-48	-5.8E-48	-9.2E-48	-1.3E-47	-1.6E-47	-1.9E-47	-1.5E-47
0.008	-8E-49	-1.9E-48	-2.8E-48	-5.4E-48	-9.6E-48	-1.4E-47	-1.9E-47	-2.4E-47	-2E-47
0.009	-4.2E-49	-8.9E-49	-1.3E-48	-3.3E-48	-6.7E-48	-1E-47	-1.5E-47	-2E-47	-1.7E-47

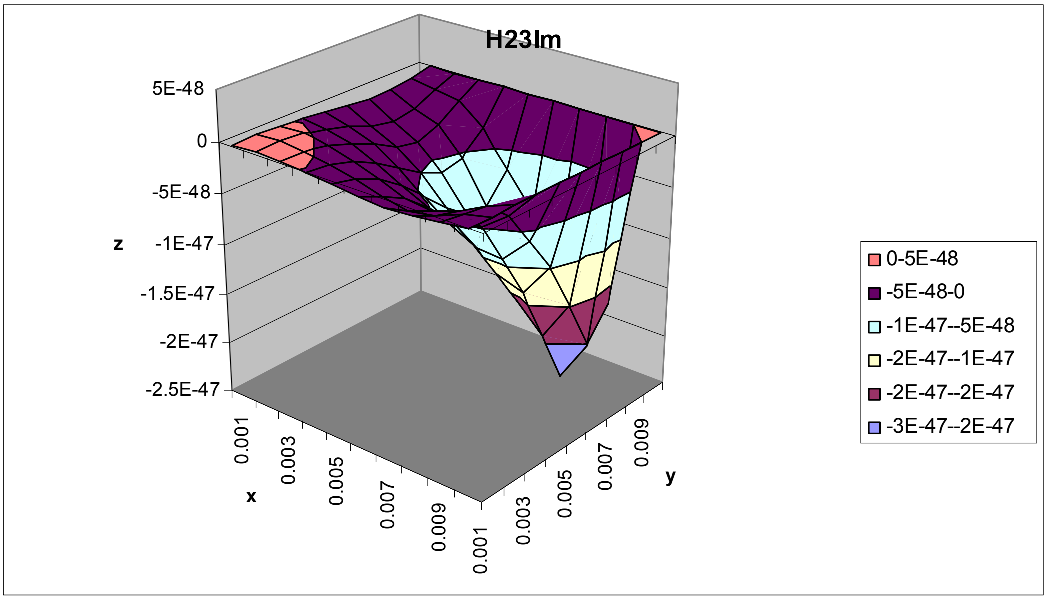

Figure 12. Im H_3

5. Conclusions

The general analytic technique [1] for the solution of the PDEs' systems is used in the present paper. This method has allowed to get the explicit results for the various versions of the differential Maxwell equations.

The obtained exact formulae have simplified substantially the comparative analysis between theoretical and experimental conclusions, the numerical implementation and the relevant computer modeling for the specific spatial electrodynamic phenomenon.

The aim of the paper is attained.

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