THE CHAOTIC GROWTH MODEL OF THE ARMS-PRODUCING MONOPOLY OUTPUT

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Abstract. *Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems. The chaotic behavior is not to quantum-mechanical-like uncertainty. Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaotic systems exhibit a sensitive dependence on initial conditions. The basic aim of this paper is to construct a relatively simple chaotic arms-producing monopoly output growth model that is capable of generating stable equilibrium, cycles, or chaos. It is supposed that the price of the arm-producing monopoly is set to achieve productive efficiency. The important factors that affect the local stability of the arms – producing monopoly output growth are: (i) the coefficient of the average cost function of the arms-producing monopoly firm; (ii) the coefficient of the inverse demand function, and (iii) the coefficient growth of the arms – producing monopoly price.*

Keywords: *Arms – Producing Monopoly, Regulation, "Fair-Return Price", Growth, Chaos.*

1. Introduction

Arms production is the production of military equipment. The national government policy determines its size, structure, and trade. Also, the national government policy controles and regulates the arms industry.

"Furthermore, the increasing concentration in national arms industries is leading toward oligopolistic and even monopolistic positions of suppliers in certain product areas, which are protected from foreign competition. Thus, there exists no real competitive "market" for weapon systems, since this "market" by tradition is monopsonistic, and is gradually moving toward increasing interdependence between government and industry". (Sköns E. and J. P. Dunne, 2006 : 1).

"The arms industry has a unique place in the political economy: because weapons technology is subject to economies of scale, and because innovation plays a major role in firms' strategies, market failures abound in the arms industry, because of the industry is direct ties to national security, politians and

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bureaucrats carefully scrutiniye arms production, because governments are the only customers, lobbying stregth joins price and quality as a key variable in business strategy. The ratio of development cost to production cost of weapons is high, which yields static scale economies. The production process also benefits from learning effects (dynamic scale economies). Both drive the arms industry toward monopoly. The arms industry is heavily regulated because of the industry's monopolistic tendencies and governments' interest in national security. Governments often seek direct control through nationalizaton. Fixed-price contracts with private firms often fail because of political and technological uncertaints, but ill-monitored cost-plus contrats that mitigate those risks are subject to rent seeking. Some minimum level of public expertise is vital for monitoring contractor performance and for designing incentive regulation". (R. J. B. Jones, 2001: 37).

Dune and Smith (2016) examine the evolution of concentration in the global arms market, or industry, over the period 1990-2013. They use data from the Stockholm International Peace Research Institute (SIPRI) list of the largest 100 arms producing firms. They find that within the international arms industry, there has been change but also continuity, particularly in the nature of the markets. Also, they conclude that countries do not like monopoly arms producers, but there is no western country other than the United States that can currently support more than one competitor, although in the near future Russia could and China may provide serious international competition to the U.S. Economic forces pushing for increased competition, but the final outcome will be determined by political forces.

The general characteristics of arms production are (Dunne, 1995): (i) An emphasis on the performance of high technology weapon rather than on cost. (ii) The bearing of risks by governments who often finance R&D and in some cases provide investment in capital and infrastructure. (iii) Elaborate rules and regulations on contracts, as a result of the lack of a competitive market and to assure public accountability. (iv) Close relations between the contractors, the procurement executive and the military. (v) Outside of the US many companies will be national monopolies or close to it.

There are market, technological and procedural barriers to enter and to leave the defence industry. The trend of rising costs of research and development (R&D) has created a pressure in the arms industry toward market concentration. (Dunne et al, 2005).

If demand fall, then the government has to decide whether to allow mergers and acquisitions which would reduce competition and in particular whether to allow mergers and acquisitions which involved foreign partners. They were also in a situation where the change in the security environment made it harder to justify previous levels of support for the industry and ‘competitive procurement policies’ aimed at value for money were introduced in a number of countries (Dunne et al, 2005).

The basic aim of this paper is to construct a relatively simple chaotic growth model of the arm-producing monopoly output that is capable of generating stable equilibria, cycles, or chaos. It is supposed that the government use average cost pricing rule. Average cost pricing rule is a pricing strategy that regulators impose on the arms-producing monopoly to decrease price to where the monopoly’s average total cost intersecs the market demand curve.

2. Methodology

Monopoly is the market structure in which there is a single seller of a good or service for which there are no close substitutes. A natural monopoly exists for a product for which there are sufficient economies of scale such that the product can be produced or supplied by a single firm at lower cost than by two or more firms. A natural monopoly is the largest supplier that creates the lowest production prices through economies of scale .

Monopolists: (i) restrict output and raise price of their products; (ii) make supernormal profits and increase inequalities in income distribution; (iii) cause deadweight losses (inefficiency in the allocation of resources of the society); (iv) generate a lower advance in the development and application of new technology.

Monoply is regulated to improve income distribution and to ensure economically efficient allocation of resources. The ways in which governments can intervene to reduce the problem of monopolies can be classified into three broad groups: (1) increasing competition (breaking up a monopoly; the antitrust laws); (2) regulating the behavior of the monopolies; and (3) public ownership.

What price should be set for the monopoly?

1. For a competitive firm, profit is maximized when marginal cost (MC) equals market price. However, since the average total cost of a natural monopoly continually declines, the marginal cost will always be less than the average total cost (ATC). In this case, a natural monopoly will continually lose money. If price equals marginal cost, then the allocation of resources will be efficient. The price that achieves allocative efficiency is called “socially optimal price”. Economic efficiency or maximum social welfare is reached if marginal cost pricing is adopted. However, marginal cost pricing requires provision of subsidy by the Government.

2. Firms may establish a "fair-return price" (P = ATC). If price equals to the monopolist's average total cost, the normal profit will be obtained. Monopoly will have only a normal profit and this price is determined by intersection of ATC and demand curves. This price is less than the price charged by a profit-maximizing monopoly, which selects the price corresponding to the point where marginal cost equals marginal revenue (MR). The problem with average cost pricing as a regulatory approach is that it gives the monopolist no incentive to reduce costs.

This paper suppose that the price of the arm-producing monopoly is set to achieve productive efficiency (P = ATC).

Chaos theory attempts to reveal structure in unpredictable dynamic systems. Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.

Chaos theory can explain effectively unpredictable economic long time behavior arising in a deterministic dynamical system because of sensitivity to initial conditions. A deterministic dynamical system is perfectly predictable given perfect knowledge of the initial condition, and is in practice always predictable in the short term. The key to long-term unpredictability is a property known as sensitivity to (or sensitive dependence on) initial conditions.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behavior. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2011, 2013, 2016), among many others.

3. The Model

In the model of a profit-maximizing arms-producing monopoly, take the inverse demand function:

$$P_t = \alpha - \beta Q_t \qquad (1)$$

Or

$$P_{t+1} = \gamma - \delta Q_{t+1}. \tag{2}$$

Where: P – the arms-producing monopoly price; Q – the arms-producing monopoly output; α, β, γ, and δ – the coefficients of the inverse demand function.

A shift in the market demand is supposed:

$$P_{t+1} = (1+\mu)P_t. \tag{3}$$

Where: μ – the coefficient of the arms-producing monopoly price growth.

Further, suppose the quadratic total-cost function for the arms-producing monopoly is

$$TC_t = \rho + \omega Q_t - \eta Q_t^2 + \xi Q_t^3. \tag{4}$$

TC – total cost; Q – the arms-producing monopoly output; ρ, ω, η, ξ – the coefficients of the quadratic total-cost function.

In this case, the average cost functiona is

$$ATC_t = \omega - \eta Q_t + \xi Q_t^2. \tag{5}$$

It is supposed that $\gamma = 0$ and $\omega = 0$.

It is supposed that the governmetn use the average pricing rule:

$$P_t = ATC_t. \tag{6}$$

The average cost pricing for regulating the product-price of arms-producing monopolist is applied. In this case, the arms-producing monopolist is just covering his average cost of production. His average cost includes normal profits or fair return on his capital investment. This normal profits or fair return on capital is the opportunity cost of his capital. In other words, economic inefficiency will be reduced under average cost pricing due to the expansion in output and lowering of price. With average cost pricing, the producer will be earning only normal profits and fair return on his capital investment. When the price is set for productive efficiency ($P = ATC$) arms-producing monopolists can cover its costs.

By substitution (3) and (5) in (6) we obtain

$$\frac{P_{t+1}}{(1+\mu)} = -\eta Q_t + \varepsilon Q_t^2. \tag{7}$$

By substitution (2) in (7) we obtain

$$Q_{t+1} = \left[\frac{\eta\,(1+\mu)}{\delta}\right] Q_t - \left[\frac{\varepsilon\,(1+\mu)}{\delta}\right] Q_t^2. \tag{8}$$

Further, it is assumed that the arms-producing monopoly output is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the arms-producing monopoly output growth rate depends on the current size of the arms-producing monopoly output, Q, relative to its maximal size in its time series Q^m. We introduce q as $q = Q / Q^m$. Thus q range between 0 and 1. Again we index q by t, i.e., write q_t to refer to the size at time steps t = 0,1,2,3,... Now, growth rate of the arms-producing monopoly output is measured as

$$q_{t+1} = \left[\frac{\eta\,(1+\mu)}{\delta}\right] q_t - \left[\frac{\varepsilon\,(1+\mu)}{\delta}\right] q_t^{\,2}. \tag{9}$$

This model given by equation (9) is called the logistic model. For most choices of η, μ, ε, and δ there is no explicit solution for (9). Namely, knowing η, μ, ε, and δ and measuring q_0 would not suffice to predict q_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (9) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of q_t. This difference equation (9) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point q_0 the solution is highly sensitive to variations of the parameters η, μ, ε, and δ; secondly, given the parameters η, μ, ε, and δ, the solution is highly sensitive to variations of the initial point q_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

4. The Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. These quadratic logistic map demonstrates chaos. Chaos is used to describe a system which behaves in an apparently random way, even when the system itself is deterministic. Very simple system behave in very complex ways. It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t(1 - z_t)\pi \in [0,4], z_t \in [0,1]. \tag{10}$$

is equivalent to the iteration of growth model (9) when we use the following identification:

$$z_t = \left(\frac{\varepsilon}{\eta}\right) q_t \text{ and } \pi = \left[\frac{\eta(1+\mu}{\delta}\right]. \ z_t = \left(\frac{\varepsilon}{\eta}\right) q_t \tag{11}$$

Using (9) and (11) we obtain

$$z_{t+1} = \left(\frac{\varepsilon}{\eta}\right) q_{t+1} = \left(\frac{\varepsilon}{\eta}\right)\left\{\left[\frac{\eta\,(1+\mu)}{\delta}\right] q_t - \left[\frac{\varepsilon\,(1+\mu)}{\delta}\right] q_t^{\,2}\right\} =$$

$$= \left[\frac{\varepsilon\,(1+\mu)}{\delta}\right] q_t - \left[\frac{\varepsilon^2\,(1+\mu)}{\eta\delta}\right] q_t^{\,2}.$$

On the other hand, using (10), and (11) we obtain

$$z_{t+1} = z_t(1 - z_t) = \left[\frac{\eta\,(1+\mu)}{\delta}\right]\left(\frac{\varepsilon}{\eta}\right) q_t \left[1 - \left(\frac{\varepsilon}{\eta}\right)\right] q_t =$$

$$= \left[\frac{\varepsilon\,(1+\mu)}{\delta}\right] q_t - \left[\frac{\varepsilon^2\,(1+\mu)}{\eta\delta}\right] q_t^{\,2}.$$

Thus we have that iterating (9) is really the same as iterating (10) using (11). It is important because the dynamic properties of the logistic equation (10) have been widely analyzed by Li and Yorke (1975), and May (1976)).

It is obtained that:

(i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;

(ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;

(iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$;

(iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;

(v) For $3 < \pi < 4$ all solutions will continously fluctuate;

(vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Important parameter π values “0, 1, 1, 2, 3“ are part of the Fibonacci sequence. The Fibonacci Sequence is the series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ... Namely, each number is the sum of the two numbers before it. If we make squares with those widths, we get a nice spiral. Because the Fibonacci sequence starts small and “grows” outwardly proportionately, it was given a visual representation of a spiral, The Golden Spiral. Also, if we take any two successive, important values of parameter π, (“2, 3“), their ratio is very close to the Golden ratio which is approximately 1.618034... For example 3/2 is 1.5. The golden ratio that has approximate value of 1.618. The golden ratio and the golden rectangle are connected. This is because the ratio of the longer side of a golden rectangle to the shorter side is equal to the golden ratio ($1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + \ldots$) (Jablanovic, 2016 : 49).

5. Conclusion

This paper creates a simple chaotic arms-producing monopoly output growth model. The model (9) has to rely on specified parameters parameters δ, η, μ, and ε, and initial value of the arms-producing monopoly output, q_0. But even slight deviations from the values of parameters parameters δ, η, μ, and ε and initial value of the arms-producing monopoly outpt, q_0, show the difficulty of predicting a long-term behavior of the arms-producing monopoly output. A key hypothesis of this work is based on the idea that the coefficient $\pi = \left[\frac{\eta\,(1+\mu)}{\delta}\right]$ plays a crucial role in explaining local stability of the arms-producing monopoly output, where, η – the coefficient of the average cost function of the arms-producing monopoly firm, δ – the coefficient of the inverse demand function, μ – the coefficient of price growth.

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