PHYSICS-BASED APPROACH TO INVESTIGATE THE CHAOTIC DYNAMICS OF THE DOLLAR EXCHANGE RATE

Javier Montenegro JOO*

Abstract. *In order to investigate the chaotic behavior of a physical system, the evolution of its state space is ordinarily analyzed. Then, by following the same logic, in order to investigate the chaotic dynamics of the exchange rate of a given currency, its state space must be considered and, with the aim on constructing this state space, the displacement and velocity of the x-rate at each instant of time must be identified. Based on the elementary Physics definitions of position, displacement and velocity, the presence of these three magnitudes is identified –as a function of time- in the daily evolution of the dollar x-rate. In this way, by using these magnitudes, the state space of the dollar x-rate is re-constructed and, it is observed that it has the structure of the state spaces of the chaotic dynamics of physical models, hence it is concluded that indeed the behavior of the x-rate is chaotic, however, since the x-rate is a non-deterministic system, those fascinating and interesting curves –the attractors- typical of ideal chaotic models will never be generated from x-rate data. The method here proposed to reconstruct the state space of the x-rate may be applied in other real-life systems to analyze its dynamics, whether chaotic or not.*

Keywords: *Economy, Physics, Econo-Physics, Chaos, Exchange rate, State space reconstruction, Dollar*

1. Introduction

In Physics the chaotic behavior of a system is ordinarily investigated by analyzing the evolution of its dynamics and, this is customarily achieved by examining its state space, which is the 3D plotting of displacement and velocity versus time. Then for the exchange rate of any currency, if displacement and velocity can be both identified as a function of time during a rather long period, its state space may be reconstructed and, by analyzing this, its chaotic behavior if any, can be studied.

* Virtual Dynamics: Science & Engineering Virtual Labs. Edo. Astete Mendoza 574, Las Magnolias, Surco, Lima 33, Perú, jmj@virtualdynamics.org ---- www.virtualdynamics.org

By reviewing the literature dealing with chaos in the exchange rate, it results that the approach proposed in this paper to study the chaotic behavior of the x-rate has never been attempted before. Usually the sophisticated techniques used to analyze time series have the disadvantage of not being intuitive and, sophisticated approaches leave all the analysis to predesigned mathematical algorithms, the researcher limits its work to analyzing the outcome of those methods. The method proposed in this paper is based on elementary concepts of physics, commonly known by engineering students. Some of those concepts are detailed ahead and may be skipped by readers familiar with them.

In a previous research [1] this author reported an investigation on the chaotic behavior of the US dollar exchange rate in Perú along five years. The x-rate data this paper is based on spans along 12 years and, though this is not enough data for a research on chaotic behavior, this implies more trustable graphs and more solid interpretation of results. It is important to realize that the method here proposed to reconstruct [2] the state space of the x-rate may be applied to other systems, whether to investigate chaotic dynamics or some other dynamics.

2. Methodology

2.1. Chaotic Dynamics of the price of the dollar

Consider an economy where the price of the dollar changes from day to day. This is the case observed in Perú, where there is a daily official price of the dollar based on its flotation [1]. A floating (a.k.a. fluctuating or flexible) exchange rate is a regime in which the currency value is allowed to fluctuate in response to foreign-exchange market dynamics as well as some other factors; in Perú this official price is calculated and reported every day by the Peruvian Central Bank of Reserve (BCR). By simple observation, the daily ups and downs in the price of the dollar in a given currency can be considered as fluctuations (ideally: oscillations) about a reference value. This reference value however, is not necessarily constant; during some days the tendency of the price of the dollar may also fluctuate between increasing and decreasing, but in general the reference price of the US coin either increases or decreases during some periods of time. In a chaotic oscillating pendulum its equilibrium position is observed to fluctuate while it definitively rotates as the pendulum oscillates.

In this way the above described behavior of the x-rate resembles an oscillating pendulum whose equilibrium position slowly rotates and flickers the direction of this rotation, while the pendulum vibrates, a behavior commonly observed in a chaotic pendulum [3-5]. Thus, it can be concluded that there are strong indicatives that the behavior of the dollar exchange rate in Perú has a chaotic behavior, it only remains to verify it and, to achieve this, it is necessary to study the state space of the x-rate dynamics.

As already mentioned, Chaos in physical systems is studied in the state spaces of the dynamics of those systems; in order to achieve this for the case of dollar x-rate it is necessary to construct and analyze its state space, this being the essence of this paper. The next section includes some definitions from elementary physics and from chaos theory; these may be skipped by readers already acquainted with those themes; however readers not familiar with physics might find the next section very interesting and instructive.

2.2. Position, Displacement and Velocity of the Dollar Exchange Rate

In a country where the price of the dollar changes every day, its price – the x-rate – may be considered as the position of the US currency for that day. Ever since the price of the dollar changes from one day to the next, the dollar x-rate displaces daily – either ahead or backwards – and sometimes there is no displacement, when the x-rate does not change. Then the velocity of the x-rate is the quickness of that change and, the larger the change, the higher the velocity. Note that the velocity may be positive, negative or even null. As an example of displacement and velocity in the x-rate, consider the currencies of two countries, the currency that devaluates more in a given period of time (let's say one year), is the one that has displaced more with a higher velocity. In an ideal situation, where the x-rate remains invariable, the displacement and velocity of the x-rate is null.

2.3. Some definitions from Physics

Both, displacement and velocity are vector magnitudes; this means that they have not only a numerical value, but also a direction. Displacement is the distance a body has moved in a given direction. This implies that though the distance between two fixed positions is always the same, the displacement between them will differ in the direction of the motion. Additionally, after each displacement the body has a new position and, the velocity to accomplish that displacement is the quickness of that motion.

When adapting these concepts to the daily price of the dollar, its price yesterday can be considered as its position yesterday, and its price today as its position today; thus the associated displacement from yesterday to today is the difference in price and, the direction of the motion is given by the sign of this difference. Obviously this difference is computed by subtracting yesterday's price from today's price, and, the resulting sign indicates the direction of the displacement.

In order to compute the velocity of the exchange rate, the physics definition of velocity may be used [6]. In physics the velocity is the change in displacement $\Delta x,$ during the shortest interval of time Δt , in this way:

$$Position: \quad x = Price = P$$

$$Displacement: \quad \Delta x = P_{final} - P_{initial}.$$

Since the price of the dollar is reported every day:

$$Velocity: \; v = \frac{\Delta x}{\Delta t} = \frac{P_{final} - P_{initial}}{time} = \frac{P_{Today} - P_{Yesterday}}{1 \; day}$$

$$\therefore \; v = P_{Today} - P_{Yesterday}.$$

Therefore it can be seen that the velocity with which the dollar displaced from yesterday to today is obtained by subtracting the yesterday's price from that of today. Note that this velocity may be positive, negative or zero.

As soon as the daily positions and velocities are obtained for a given interval of time – the longer the better – the state space representing the dynamics of the exchange rate during that interval may be constructed. Note that since velocity is not part of the initially available data and, it is obtained from the known positions, then technically speaking, this is a reconstruction of state space. Usually in chaotic physical systems modeled by a time-dependent equation (a mathematical model), displacements and associated velocities are available data and it is neither necessary to compute the velocity nor to reconstruct the state space.

2.4. Deterministic and Random Systems, Dynamical Systems

A deterministic system is governed by precise laws [7], this means that during its evolution only one thing can take place after another. A random system has no determinism [7], so that one of several things can occur after another, though not necessarily anything.

Dynamical systems evolve deterministically as time elapses, for example the randomness-free evolution of any physical system, like a non-perturbed pendulum. Obviously the best example of a dynamical system is a mathematical model, as long as it is fed with deterministic data. Evidently a mathematical model fed with random data does not evolve deterministically. Economy makes use of mathematical models, which supposedly are deterministic; otherwise they would not be models. If these models are fed with nondeterministic data, those models lose their deterministic quality.

2.5. Deterministic Chaos

Deterministic chaos or simply chaos is the apparent disordered behavior observed in the evolution of deterministic systems, this is, in systems where the current state defines its future states.

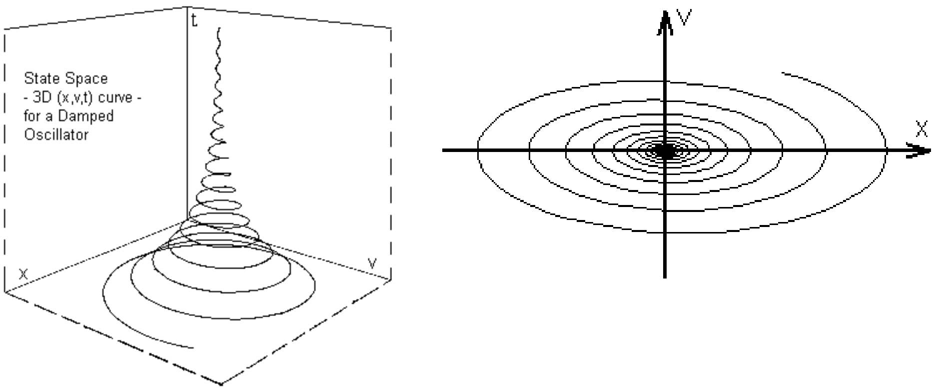

Figure 1. Left: Tridimensional state space of a damped oscillator, the plotting of position x and velocity v versus time t, the latter usually appearing along the vertical. Right: Bi-dimensional projection of the state space on the XV-plane. Usually 2D projections of state spaces are shown. It can be seen that due to the constant damping the oscillator is experiencing, the curve gradually contracts as time goes by.

2.6. The State Space

In order to analyze the dynamics of a physical process, its tridimensional 'state space' (figure 1) must be constructed [3,8]. The most common state space in physics is the 3D graph of position 'x' and velocity ‘v’ versus time ‘t’ and, this is just the sequence of (x, v, t) points, generated by the dynamics of the system under investigation, during the time this takes place. Since 3D state spaces of non-trivial systems are difficult to interpret, usually those 3D graphs are presented as its 2D projections (Figure 1) on the XV-plane.

2.7. Dynamics of the Exchange Rate

The ups and downs in the exchange rate of a given currency with respect to the USA-dollar, this is, the fluctuations in the price of the dollar in a given country, may ideally be seen from the point of view of physics, as oscillations and, these are gradual rises and falls [1]. The oscillations are described among other physical magnitudes, by its position and by its velocity and, when adapting these magnitudes to the case of the exchange rate, the price of the dollar today may be regarded as its position today. In this way, if the dollar price today has not changed with respect to yesterday, if it keeps constant, there is no displacement, but if the price of the US currency goes up, there is a positive displacement and, if that price goes down, there is a negative displacement. In countries like Perú, whose economy is strongly tied to the US-dollar, and its exchange rate continuously changes, this displacement is assessed every day by the BCR, by entrepreneurs and also by common people wanting to protect their money [1].

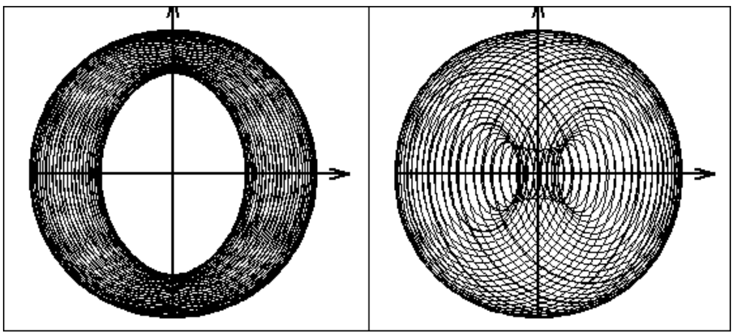

Figure 2. State spaces of the oscillations of two coupled springs with constant coupling. In this example notwithstanding the space is completely occupied in the right-side case, neither of the two springs vibrates chaotically. The curve depicted in the state space of a chaotic model from physics visits the entire space in a rather unpredictable and literally chaotic way, usually displaying a dark non-uniform spot.

2.8. The data used in this investigation

In order to carry out this investigation the author has processed the daily prices of buy and sell of the US-dollar in Perú, these refer to the price the banks pay when they buy dollars and to the price they sell those and, obviously the price of selling is always higher than the price of buying; these referential rates are reported everyday by the SUNAT, the Peruvian institution equivalent to the IRS in USA, this institution gets the information from the BCR. The time interval comprised in this study spans along 12 years, from 2007 to 2018. More accurate results would have been obtained with the hourly evolution of the x-rate, it is known that it is assessed by the BCR, but this is not available. In this research, by averaging the daily prices of buy and sell a unique daily price for the US Dollar was obtained.

3. Results

3.1. State Space of the USD exchange rate for the years 2007-2008

Seeing that the behavior of the US currency in the lapse 2007-2008 was memorable due to the USA financial crisis, figure 3 shows – at the top – the evolution of its x-rate in Peru during that period. It can be seen that the price of the USD was going downwards during 2007 until early 2008 when it reached its lowest value. It was then – when, with the purpose of avoiding a disaster in the Peruvian economy – the BCR came to the rescue of the x-rate, by purchasing enormous amounts of dollars so as to artificially foster its scarcity and thus to elevate its price and, keep it within a safety band. This means that the apparent recovery of the US currency in Peru after its catastrophic downfall is non-natural. It is worthwhile to note at this point, that the X-rate never behaves like a natural magnitude, it is always either politically controlled or its value is organized by non-natural factors, like the balance of payments; the above mentioned situation in which the BCR comes to the x-rate rescue is extreme and, it is not a common practice when the x-rate is floating.

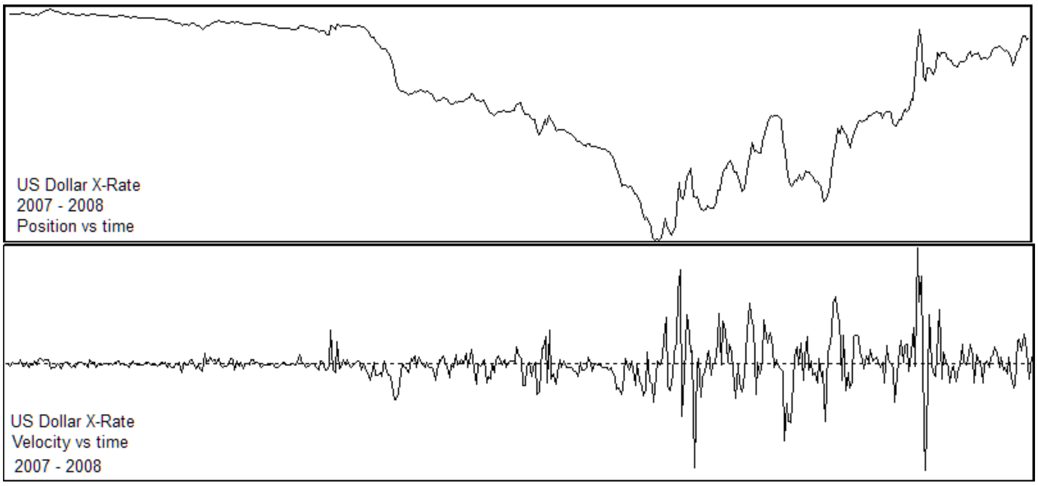

Figure 3. Evolution of the US dollar x-rate in Peru during the financial crisis 2007-2008. The top graph shows the position of the x-rate versus time, this graph reflects the financial crisis occurred in early 2008 and, it is evident that the price of the US dollar went catastrophically down during 2007. The bottom graph shows the associated velocity. Note that large changes in the x-rate are associated to notable changes in its velocity and vice versa. Top: The apparent recovery of the US currency after its downfall was artificial.

The graph of the state space of the exchange rate during years 2007 and 2008 is shown in figure 4. This graph would be softer, i.e., it would show less sudden changes, if more frequent x-rate data were available, for example, if the price of the dollar were known every hour. With hourly data, between every two points in the curve (one day of difference), would appear the information of each of several hours throughout the day and, the curve would definitively turn soft.

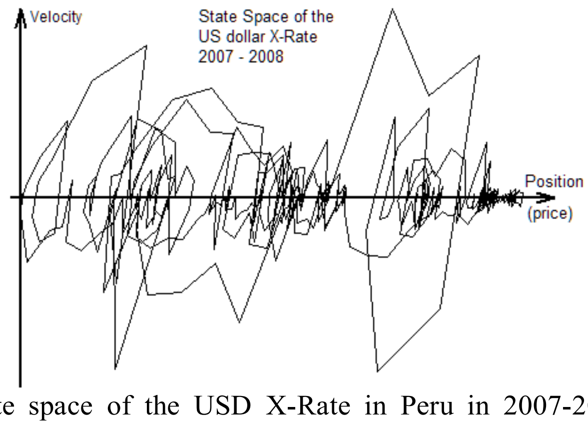

Figure 4. State space of the USD X-Rate in Peru in 2007-2008. The price started at the rightmost extreme, then it went backwards and, as a consequence of the artificial rise in its price, the curve went rightwards. This unpredictable back and forth motion in the state space is typical of oscillating chaotic systems.

Due to its own nature, prices have always positive values, this implies that the curve in the state space (figure 4) must always appear in the positive region of the position (price) axis and never visit its negative region. When the x-rate increases, displacement and velocity are positive and, the curve moves to the right, when the rate of exchange drops, displacement and velocity are negative and, the curve displaces to the left.

3.2. Dynamics of the exchange rate between 2007 and 2018

Bearing in mind that the data of the exchange rate for one year, are very few, in order to carry out a statistics with more data, twelve years in a row were considered, between 2007 and 2018. The results are shown in figures 5 and 6. It can be seen in figure 5 that the price of the USD went down in Perú, until 2013 when it started to recover. As the price of the USA-dollar increased in Peru between 2013 and 2018, the curve on the state space (see figure 6), although many times moving backwards – a characteristic of chaotic motion – it moves on average towards the right.

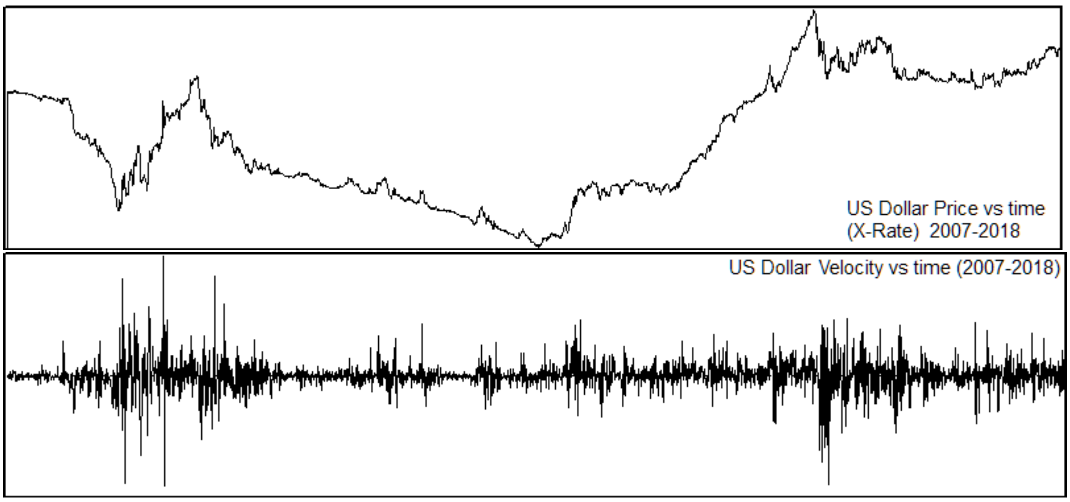

Figure 5. Evolution of the US Dollar exchange rate in Peru from 2007 to 2018. The top graph shows the position (price) vs time and the bottom one shows the corresponding velocity. It can be seen that when the fluctuations in price are rather small, the velocity fluctuates a little, but if the price changes significantly from one day to the next, the velocity also makes abrupt changes. Until early 2013 the x-rate went downwards, and then it started to recover.

In figure 6, the existence of abrupt changes (vertices) in the graph is due to the fact that the available data are too few. If the time evolved between measurements of position and velocity was shorter, the curves would be smoother.

3.3. Evidence of chaotic behavior in the X-Rate

Bearing in mind that there is no exchange rate on holidays, the total number of daily x-rate values for the 12 years from 2007 to 2018 is 3028; these values have been used in figure 6 to show the state space of the US Dollar x-rate for the mentioned period. By comparing figure 6 with the graphs in figure 7 of the state spaces of chaotic events in the Duffing and in the Van der Pol equations [9], it can be seen that, in both cases (figures 6 and 7), the behavior is similar, being it impossible to predict what will be the next position, i.e., the next state.

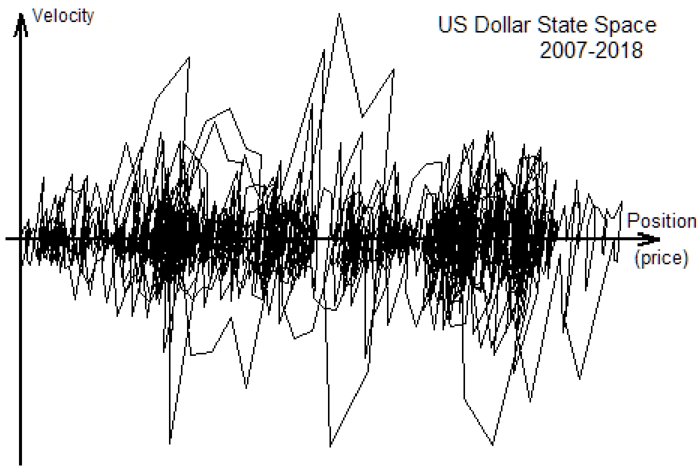

Figure 6. Bi-dimensional projection of the 3D state space (velocity versus price) of the exchange rate for the period from 2007 to 2018. This graph shows the typical behavior observed in the state spaces of chaotic physical systems, resulting from the fact that sometimes the curve goes backwards, and other times it goes forward, with no particular order, which implies a rather chaotic and unpredictable dynamics. Since by definition, prices are never negative the curve in the state space of the x-rate is restricted to the positive region.

The apparent difference between the state space of the x-rate of the US-dollar in Perú (figure 6) and that of the Duffing and Van der Pol equations (figure 7), is that the x-rate curve moves inexorably to the right, because the dollar – even though it raises and lowers its price – in the very long run it increases its exchange rate. In the case of a more or less stable dollar, its displacement would be negligible or nil, and the state space of the x-rate would center on a fixed value, such as in the state spaces shown in figure 7. For this reason this seeming difference between the x-rate state space and those shown in figure 7 must not be regarded as dissimilarity in behavior.

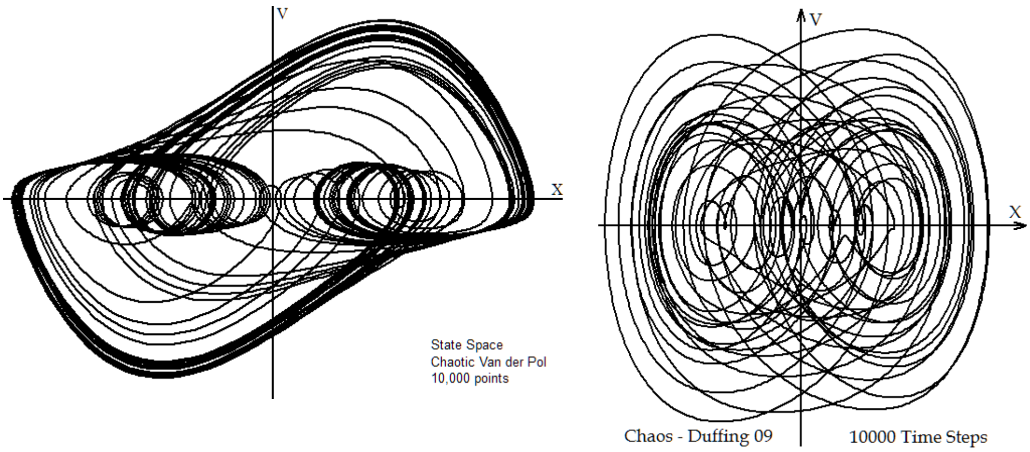

Figura 7. State spaces of chaotic events observed in two mathematical models from physics, the Van der Pol oscillator (left) and the Duffing oscillator (right). In both cases, 10000 pairs of velocity versus position have been plotted. The smoothness of the curves is a consequence of the closeness of data, this is, the short time spent between assessments of (x,v) pairs. As it can be seen the oscillators go back and forth in a literally chaotic way.

In this study, in order to conclude that the exchange rate has a chaotic behavior, only the similarity between the state space of chaotic events from Physics (Duffing and Van der Pol oscillators) and the state space of the x-rate can be considered and taken as reference (figures 6 and 7). It is not possible to apply the crucial test of chaos, namely, the numerical estimate of the Butterfly effect [7] by means of the detection of the maximum Lyapunov exponent [10-16], because the only available data are the empirical data of the exchange rate. In order to determine the maximum Lyapunov exponent – which in chaotic system measures the sensitive dependence on initial conditions – it is required to operate on an equation, which when re-started on a slightly different position, generates a time series whose divergence with respect to a another time series, taken as a reference is extremely large[10-16].

3.4. Absence of attractors in the chaotic dynamics of the x-rate

The attractors – the astonishing and interesting curves – characterizing chaotic events are obtained from deterministic chaotic systems, among them, the Duffing equation, the Van der Pol equation and the Nonlinear damped and forced oscillator (NLDFO), etc [3,5]. Attractors – usually strange ones – are gotten by taking transversal cuts or tomographies of state spaces of chaotic systems. Since due to its own nature, non-deterministic systems cannot display attractors, then the x-rate – which is one of those – will never show any attractor. By simply knowing the origins and behavior of the data used to compute the x-rate, it can be predicted that, if transversal sections of the state space of the exchange rate were extracted, neither order nor structure would be found, in other words, no attractor of the x-rate would be detected.

By experience it is known that in order to detect any pattern in a Poincaré's transversal tomography of the state space, it is necessary to plot at least some 10000 (x,v,y) – points of that space, a number of data corresponding to some 40 years of daily x-rates, an impossible mission.

3.5. The missing detail: Physicists do it with models

The main reason behind the fact that real life data will never generate the curves of behavior observed in Physics, is that as people say, "physicists do it with models", to be more precise, ideal models, then it is worthwhile recalling at this point that the reference systems used to compare the exchange rate behavior, namely those shown in figure 7 are ideal physical models, and it is very unlikely that a real-life system like the x-rate, eventually behave like any of these models. Those physical models must be considered as what they are, referential ideal models.

Even if a polynomial formula (a deterministic mathematical model) were used to set the everyday price of the dollar, this equation would be fed with real-life data of unpredictable (non-deterministic) nature, thus the results could not be deterministic and, if there were some sort of chaotic behavior in the exchange rate this could not behave like an ideal system, hence no attractors would exist.

It is known that not all chaotic systems possess attractors; usually (strange) attractors are encountered in dissipative chaotic systems [7]. Since the rate of exchange is renewed every day by the inclusion of updated data, it cannot be considered a dissipative system, which explains the absence of attractors in the dynamics of the x-rate. It is known that in the very long run the curve in the state space of a dissipative mathematical model will wind up in an attractor [10], then it can be inferred that since the x-rate is not a dissipative system it will never display any attractor.

4. Conclusions

Using real-life data consisting of the x-rate along the latter twelve years (from 2007 to 2018), an investigation of the chaotic dynamics of the US-dollar exchange rate in Perú has been carried out. In order to accomplish this, the state space of the x-rate was reconstructed with the cited data by making a parallel between the daily price of the US-dollar and the physics definitions of position, displacement and velocity. It has been found that in fact, the behavior of the x-rate is disordered and erratic and, the x-rate state space displays all the features observed in the dynamics of chaotic models from physics, however the smoothness of the curves from physical models is not achieved when using x-rate data, because the latter are non-deterministic data, while the former are ideal data from deterministic models. Additionally, the chaotic dynamics of the x-rate do not get to reveal any traces of the presence of an attractor in state space, which is justified by the fact that the factors influencing the daily price of the US-dollar neither have a regular comportment nor are dissipative.

Strange attractors have been observed in computer simulations dealing with ideal deterministic models, besides this, the presence of attractors is associated with dissipative systems. Since the daily x-rate shows unexpected variations, the x-rate system becomes a non-deterministic one and, as the x-rate is renewed every day with updated data, it is not a dissipative system either, due to these reasons the dynamics of the x-rate behaves chaotically but will never display the characteristic attractors of ideal chaotic models from physics.

REFERENCES

[1] Montenegro Joo, J. (2018), *Chaotic dynamics of the dollar exchange rate in Peru*, Chaos and Complexity Letters, Vol. 12, No. 2

[2] Packard N. H., Crutchfield J. P., Farmer J. D., Shaw R. S. (1980), *Geometry from a Time Series*. Physical Review Letters 45(9)

[3] Montenegro Joo, J. (2018), *Anatomía del Caos*, Editorial Académica Española, Beau Bassin, Mauritius.

[4] Montenegro Joo J. (2009), *A pragmatic introduction to Chaos Theory for engineers*, Industrial Data, Vol. 12, No 2

[5] Montenegro Joo J. (2018), *Tomographies of the State-Space of the Chaotic NLDFO,* Chaos and Complexity Letters, International Journal of Dynamical Systems Research, Vol. 12(1).

[6] Alonso M., Finn E. J. (1967), *Fundamental University Physics*, Vol. 1, Addison Wesley Pub. Co., Mass., USA.

[7] Lorenz E. (1993), *The essence of chaos*, University of Washington press.

[8] Hagedorn P. (1981), *Non-linear oscillations*, Oxford University Press.

[9] Montenegro Joo J. (2010), *Chaos Virtual Lab*, Virtual DynamicsSoft.

[10] De Souza-Machado S, Rollins R. W., Jacobs D. T, Hartman J. K. (1990), *Studying chaotic systems using microcomputer simulations and Lyapunov exponents*, Amer. Journal of Phys. 58(4)

[11] Wolf A., Swift Jack B, Swinney Harry L., Vastano John A. (1985), *Determining Lyapunov exponents from a time series,* Physica 16D

[12] Farmer J. Doyne, Ott Edward, Yorke James A. (1983), *The dimension of chaotic attractors*, Physica 7D

[13] Sprott J. C. (2003), *Chaos and Time-Series Analysis*. Sprott J. C., Oxford University Press

[14] Rosenstein Michael T., Collins James J., De Luca Carlo J. (1993), *A Practical method for calculating the largest Lyapunov exponents from small data sets*, Physica D-65

[15] Packard N. H., Crutchfield J. P., Farmer J. D., Shaw R. S. (1980), *Geometry of a Time Series*, Physical Review Letters, 45-9

[16] Crutchfield James P., Farmer J. Doyne, Packard Norman H., Shaw Robert S. (1986), *Chaos. Scientific American*, Vol. 255, No 6.