CAPITAL BUDGETING TECHNIQUE FOR BORROWING PROJECTS

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Abstract. *Every investment project needs to be financially viable in order to be accepted for implementation. Financial viability of a project is ascertained by employing a process termed as Capital Budgeting Technique. Among the various available Capital Budgeting Techniques, IRR and NPV methods appear to be the most widely used ones. The practicing managers, particularly those from multinational companies, have a marked preference for the IRR method. However, a few issues have been raised by some researchers against IRR – primarily on account of non-existence of IRR in certain situations and existence of multiple IRRs under some other circumstances. It also comes as a big surprise that practicing managers have such a humongous preference for a method fraught apparently with so many serious complications. While these issues have been dealt with in a consistent manner for investment projects and the condition for existence and uniqueness of IRR has been derived in a compact form, a similar task does not appear to have been undertaken for borrowing projects, where IRR is alleged to falter even more. In this article we attempt to deal with such issues and derive the condition for existence and uniqueness of IRR for borrowing projects. We also formulate certain modified decision criteria compatible with physical realities, which can help to resolve some apparent anomalies.*

Keywords*: Investment & Borrowing Projects; Parity for NPV & IRR; Modified Decision Criteria for NPV & IRR; Fischer's Intersection Point; Condition for Existence & Uniqueness of IRR; Ranking of Mutually Exclusive Projects*

Introduction

Financial viability of a project is a sine-qua-non for its acceptability. To examine financial viability, we need to compute, based on the cash flow pattern of the concerned project, one or more of the following parameters as part of the capital budgeting exercise:

a) Internal Rate of Return (IRR)

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b) Net Present Value (NPV)
c) Payback Period
d) Profitability Index (PI)

These four methods have been arranged in the order of their usage by practicing managers as the primary method [1]. Of these, all except the third are essentially based on the Discounted Cash Flow (DCF) approach. The third one may or may not use the process of discounting future cash flows. For the purpose of discounting, both IRR and NPV assume a uniform annual rate of return irrespective of the tenure. In other words, these assume a flat yield curve.

As persons from the field of Corporate Finance are aware, certain issues have been raised by some researchers against the applicability and usefulness of IRR as a consistent method, primarily due to possible non – existence of IRR in certain cases [1, 2, 3] and existence of multiple IRRs in some other cases [1, 2, 3, 4]. Certain prescription has also been suggested by some, based on the well known Descartes Rule of Signs for polynomials, to relate the number of IRRs to the number of changes of signs of the project cash flows [2]. There are also the scenarios where NPV and IRR apparently throw up difference in ranking of mutually exclusive projects [1, 2, 3]. Additionally, the apparent mystery about the large scale preference for IRR by the practicing managers also needs to be understood. For investment projects, these issues have been dealt with in detail, and the condition for existence and uniqueness of IRR has been derived [5]. These findings are summarized in a subsequent section.

However, the aforesaid issues for borrowing projects do not appear to have been undertaken so far. This is the aim of this article. Some small differences of opinion among the researchers also seem to exist about such basic issues as the definition of Profitability Index (PI) or the decision criterion for NPV. We shall explore the possibilities of reconciling such minor, almost trivial, differences, while dealing with the larger issues at length. At the time of dealing with such sensitive issues as possible non – existence of IRR or existence of multiple solutions for IRR, one has to clearly keep in mind that the parameters related to capital budgeting have physical implications and pertain to real life projects. Accordingly, these are physical entities and not merely some mathematical objects. All the necessary boundary conditions for the attendant mathematical equations/ solutions need thus to be set carefully to ensure a one – to – one correspondence between them and the physical realities on the ground. This approach is nothing extra ordinary and is followed as a standard practice for all scientific analyses/logical pursuits. Once we follow this course, some of the issues get resolved readily in a logically consistent manner.

Types of Projects

An investment project is characterized by an initial outflow i.e. at $t = 0$, when counting of time for the project commences. This initial investment in a project is followed by a stream of project related future cash flows that, in order to make the project viable is dominated by inflows, which ensure a positive rate of return for the initial investment.

A borrowing project, on the other hand, is characterized by an initial inflow. In the wake of this initial inflow comes a stream of project related future cash flows that, to make the project viable is dominated by outflows, which ensure a positive cost of borrowing/cost of capital.

Parity of IRR and NPV

As we shall observe from the illustrations below, if we were to reverse the signs of all the cash flows pertaining to a project (this will transform an investment project to a borrowing project and vice versa), then NPV of the project too changes sign. In other words, we can state that NPV has an odd parity with reference to such a transformation.

IRR of any project, on the other hand, remains unchanged in the face of such a transformation. In other words, we can state that IRR has an even parity because it remains unchanged when the signs of all the cash flows attached to a given project are reversed.

Accordingly, as we shall see below, while some problems may not surface for NPV, these may surface for IRR. However, such problems need not necessarily be insurmountable.

Modified Decision Criteria for IRR and NPV of Borrowing Projects

The usual decision criteria applicable to IRR and NPV for investment projects cannot be extended mechanically in case of borrowing projects. It is necessary that the decision criteria are compatible with the realities on the ground and also the basic maxim of maximization of shareholder wealth. To reiterate our earlier statement, all necessary boundary conditions for the attendant mathematical equations/solutions need to be set carefully to ensure a one – to –one correspondence between them and the physical realities on the ground. Once we follow this course, some of the issues get resolved automatically in a logically consistent manner.

For instance, for an investment project, the decision criteria in respect of IRR are:

(a) If IRR of a project is greater than some threshold rate, accept the project. Otherwise, do not accept it [1, 2]. This threshold rate would be linked to the rate of return currently available on similar projects (same risk category) and should never be lower than the cost of capital i.e. the rate of return required by the investors for the project.

(b) For mutually exclusive projects with similar risk profiles, among the projects having IRR above the threshold rate, select the one with the highest IRR [1, 2].

(c) For comparing projects with different risk profiles (may be on account of different temporal distribution of future cash flows from the project), (IRR – Cost of Capital), which represents the net rate of return, may be the appropriate parameter to compare [5].

For a borrowing project, IRR is a measure of the cost of borrowed/ debt capital. If we try to extend the above criteria mechanically to a borrowing project, it would be akin to committing a corporate hara-kiri. This would imply that cost of borrowing has always to exceed a threshold value. Cost of borrowing lower than such a threshold is to be strictly rejected. Further, higher the cost of borrowing better is the project. This is against the basic norms of common sense and directly militates against the tenet of maximization of shareholder wealth. It is thus unacceptable.

Logically, the appropriate condition for a borrowing project should be IRR>0 i.e. no financial reward or negative cost can be expected on account of borrowing. Also, absence of any positive solution for IRR would imply non-existence of IRR. However, at the same time, IRR should not unduly push up the existing cost of capital of the company. In other words, IRR ≤a ceiling rate decided by the company, which is linked to the existing discount rate/cost of capital of the company as well as current market conditions.

As for NPV, the decision criteria for an investment project are [1,2]:

(a) If NPV of a project is positive, only then accept the project. Otherwise, do not accept it.

(b) For mutually exclusive projects, among the projects with positive NPV, select the one with the highest NPV.

This is quite fine in so far as investment projects are concerned. However, this too cannot be mechanically extended to borrowing projects. We cannot expect that every act of borrowing by a company, on a standalone basis, would add positive NPV i.e. positive value to the company. If this were to be feasible, then merely a higher level of borrowing will make a project or a company more valuable. This, once again, is not consistent with either common sense or the maxim of maximization of shareholder wealth. It too is thus not acceptable. Assuming that additional borrowing may come only at a slightly higher cost or at best at the existing cost, NPV for borrowing projects should be ≤ 0 but not a large negative (to avoid unduly large borrowing costs).

As we shall observe in the next section, it is possible for a borrowing project to have a positive NPV only if every fresh borrowing comes at a cost lower than the existing cost of capital/discount rate. In other words, it will require the current discount rate to exceed the cost of fresh borrowing i.e. the IRR for the borrowing project (IRR < discount rate). It may be possible on rare occasions for a company to replace an existing high cost borrowing by a fresh loan raised at a cheaper rate. However, it is not possible to sustain such an activity as a chain event at the time of each and every mobilization of fresh resources to meet the company's additional requirement of finance.

Prior to deriving the condition for existence and uniqueness of IRR for borrowing projects, we now discuss some scenarios from [1] (the cases are presented quite succinctly there) in order to elaborate on whatever points we have discussed so far.

Certain Useful Illustrations:

Example 1: Apparently Incompatible Decisions from IRR and NPV

	CF at t=0	**CF at t = 1 year**	**Project Type**
Project 1	**- 100 units**	**125 units**	**Investment**
Project 2	**100 units**	**- 125 units**	**Borrowing**

The required rate of return/discount rate is 10% p.a.

Before considering the position using IRR, we note that:

(a) Project 1 is an investment project while project 2 is a borrowing project.

(b) If we reverse the signs of all the cash flows for project 1, we get project 2 and vice versa. However, IRR remains unchanged in the face of such reversal of signs of all the project related cash flows, and thus IRR has an even parity. So, both these projects have identical IRR.
(c) IRR for both the projects is 25% [$(1 + r) = 125/100$].
(d) We can apply the decision criteria for investment projects to project 1 in the usual manner. Since IRR of 25% is well above the cost of capital of 10%, project 1 is quite viable and acceptable. Another investment project with IRR of 30% would imply a higher rate of return on investment. With the cost of capital remaining unchanged, this project would be even better than project 1.
(e) If we were to apply the decision criteria for investment projects mechanically to project 2, then project 2 would also become equally acceptable. It would mean borrowing at 25% p.a. is quite acceptable even though the company's current cost is around 10% p.a. only. Moreover, in such a case, to borrow at 30% would be better than to borrow at 25%, and borrowing at 40% would be still better. This is quite clearly incompatible with the very basic physical requirements for borrowing projects.

That the rate of return from an investment has to exceed the cost of capital to enable a company to generate some business surplus is quite understandable. But, we cannot stipulate that the cost of a borrowing has to exceed the cost of capital in order that such a borrowing is financially beneficial for the borrowing company. Also, a higher borrowing cost can never be a better proposition for the borrowing company.
Therefore, the IRR related decision criteria for investment projects like project 1 above cannot be mechanically applied for borrowing projects such as project 2.

(f) Obviously, no practicing manager ever applies these criteria for the purpose of checking the acceptability of a borrowing project. So, they never come across these problems.
(g) According to the modified decision criteria related to IRR proposed in the previous section, an IRR of 25% for a borrowing project by a company with a required rate of return/discount rate of 10% p.a. is clearly unacceptable. So, there is no anomalous outcome from the IRR method in that case.

As for NPV, we note that:

(a) As we reverse the signs of all cash flows associated with the project without changing the discount rate, NPV of the project too changes sign. In other words, NPV has an odd parity.

(b) Project 1 has a positive NPV for a discount rate of 10% and is thus quite acceptable. As a matter of fact, NPV for project 1 is non negative as long as the discount rate does not exceed 25%.The discount rate must be less than or equal to the rate of return (IRR) for the investment project to be able to have a non-negative NPV.

(c) Project 2 has a negative NPV for a rate of return/discount rate of 10% p.a. and is not acceptable. So, investment at 25% is quite acceptable although borrowing at 25% is not. This is logically quite understandable and acceptable. However, stipulation of positive NPV as a criterion of decision making for all borrowing projects poses a problem because, as we shall see below, such a requirement would rule out borrowings not only at 25% but also at 10% or 10.1% or similar such rates close to but higher than 10% p.a.

(d) If the borrowing project were to have a borrowing cost/IRR of 10% (say, CF_0 = 100 units and CF_1 = (-) 110 units), then for the existing discount rate of 10%, NPV of the project would be zero. Obviously, one cannot, as a rule, reject such a borrowing at the existing cost of capital just because NPV is not positive. If the cost were to be slightly above 10% because of a higher cost associated with additional debt (say, CF_0 = 100 units and CF_1 = (-) 111 units), then for an existing discount rate of 10%, NPV would be slightly negative {(-) 0.91unit}. We cannot summarily reject such a proposal for borrowing, particularly if market interest rates are moving north, just because NPV < 0. However, a large negative value of NPV can also not be allowed because that would make borrowings at unduly higher borrowing costs (such as 25% or 30%) also acceptable. In other words, the realities on the ground can be accommodated by the modified decision criteria related to NPV as proposed in the previous section.

(e) Project 2 with IRR of 25% can have a positive NPV only if the discount rate exceeds 25%. This is understandable because, for a company with an existing cost of capital in excess of 25%, a fresh borrowing at 25% would effectively mean a reduction of overall cost and therefore a positive addition to value. Otherwise, for the discount rate to continue at 10%, a positive NPV would warrant an IRR lower than 10% i.e. borrowing at a cost lower than the existing cost.

Example 2: Non – Existence of IRR for some cases

	CF_0	CF_1	CF_2
Project 1	**(-) 105 units**	**250 units**	**(-) 150 units**
Project 2	**105 units**	**(-) 250 units**	**150 units**

No IRR exists for both the above 2 projects.

Here we note that:

(a) Project 1 is an investment project while project 2 is a borrowing project.

(b) If we reverse the signs of all the cash flows for project 1, we get project 2 and vice versa. However, since IRR has an even parity and remains unchanged in the face of reversal of signs of all the project related cash flows, both these projects have identical IRR.

(c) Project 1, an investment project, is characterized by an initial outflow of 105 units followed by two future cash flows of 250 units after year 1 and (-) 150 units after year 2. A study of the cash flow pattern for project 1 reveals that the algebraic sum of all future cash flows from the project falls short of the magnitude of the initial outflow $[(CF_1 + CF_2) <(-)CF_0$ i.e. $\{(CF_1 + CF_2)/(-)CF_0\}< 1]$. Hence, as per [5], existence of a unique IRR for this investment project is not possible.

(d) Further, since project 1 and project 2 are placed on identical footing in so far as IRR is concerned, absence of IRR for project 1 ensures absence of IRR for project 2 as well. Moreover, we shall derive below the condition for existence and uniqueness of IRR for borrowing projects. Once this is done, we shall cross check that the condition to be derived for borrowing projects also independently predicts absence of IRR for project 2.

(e) It thus appears that absence of IRR for the above investment and borrowing projects are as per the predictions of our theory.

(f) Since the algebraic sum of the future cash flows for the investment project, i.e. project 1 (250 units-150 units) falls short of the magnitude of the initial outflow {105 units}, such a project is not to be taken up by the practicing managers for consideration. As practicing managers leave out such projects ab initio, they do not come across any difficulty on this count and have no problem in using the IRR method as their analytical tool.

(g) Also, no borrowing by a corporate is practically feasible where the algebraic sum of future cash flows for the borrowing project is smaller in magnitude than that of the initial inflow. This would tantamount to an outgo of cash less than the amount initially received by way of disbursement of a loan. In other words, such projects are also beyond the realm of consideration of the practicing managers and do not cause them any concern.

Example 3: Incorrect Project Ranking for Mutually Exclusive Projects

	CF_0	CF_1	IRR	NPV at 10%
Project A	**-10000 units**	**20000 units**	**100%**	**8181.82 units**
Project B	**- 20000 units**	**36000 units**	**80%**	**12727.27 units**
Project C (B –A)	**-10000 units**	**16000 units**	**60%**	**4545.45 units**

Project A has a higher IRR but a lower NPV than project B.

Here we note that:

(a) All the projects in the above illustration viz. project A, project B and project C are investment projects. Still we examine this example because it will help us to understand how a proper appreciation of the physical realities on the ground may lead to a simple resolution of an apparent anomaly in a logically consistent manner.

(b) Project A has an IRR of 100% and a NPV of 8181.82 units (discount rate/required rate of return = 10%), while project B has an IRR of 80% and a NPV of 12727.27 units (discount rate/ required rate of return = 10%).

(c) It is argued that project A is better than project B in terms of IRR, although it is inferior to project B in terms of NPV. This is used to highlight an apparent anomaly between the IRR method and the NPV method.

(d) In this context, it is important to keep in view the fact that the scales of investment for projects A and B are different. Project A involves an initial investment of 10000 units whereas project B involves an initial investment of 20000 units. So, PI rather than

NPV should be the appropriate yardstick to go by at the time of comparing project A with project B. PI for project A is 1.82 and that for project B is 1.64. If we follow this approach, the anomaly would disappear inasmuch as project A is better ranked than project B in terms of both IRR and PI.

(e) Since the eventual goal of the NPV method is to maximize value/NPV, each and every project with a positive value (i.e. IRR > 10%) might have to be taken up for implementation provided unlimited fund is available to the company at the same cost (i.e. 10%).

This will mean that if there is a single project A with IRR of 100% and another single project C with IRR of 60%, we may accept both projects A and C, which together is equivalent to project B (project A + project C = Project B with combined IRR of 80%). Since both project A and project C have positive NPV, projects (A + C) have greater NPV than project A. Accordingly, project B too has higher NPV than project A. However, it does not mean that project (A + C) or project B is better than project A. If we had another project, marked as A*, having – like project A – an IRR of 100%, we would have certainly preferred (A + A*) to (A + C).

A 5 year Bond (B1, say) with YTM of 20% is better than another 5 year Bond (B2, say) with YTM of 12% if both belong to the same risk bracket. However, an investment of 20000 units of fund in B2 will generate a higher amount of aggregate interest income than an investment of 10000 units of fund in B1. But this is not to alter the relative ranking between B1 and B2. There is thus no conflict between: (a) B1 enjoying a higher yield as well as a higher rank than B2 and (b) a substantially higher investment in B2 yielding a higher aggregate interest income than a relatively low investment in B1.

Similarly, for the investment projects, project B (i.e. A + C) with an overall rate of return of 80% is not better than project A with a rate of return of 100%, although the former adds greater value when there is only a single project A and another single project C with IRR > 10%. An investor can maximize total value/NPV by selecting the most appropriate combination of available projects under a given scenario. But, that cannot alter the inter-se parity between the individual projects. The apparent anomaly is, therefore, due to an improper way of trying to interpret the scenario rather than to the capital budgeting techniques themselves. In sum, there appears to be no incorrect project ranking by either the NPV or the IRR method, and together they provide a cogent and consistent understanding of the overall situation.

Example 4: Incorrect Ranking of Projects with Identical Investments

	CF_0	CF_1	CF_2	CF_3	IRR	NPV at 10%
Project 1	**(-) 90 units**	**60 units**	**50 units**	**40 units**	**33%**	**35.92 units**
Project 2	**(-) 90 units**	**18 units ad-infinitum**			**20%**	**90.00 units**

Project 1 has higher IRR while project B has higher NPV, although both projects have identical initial investments.

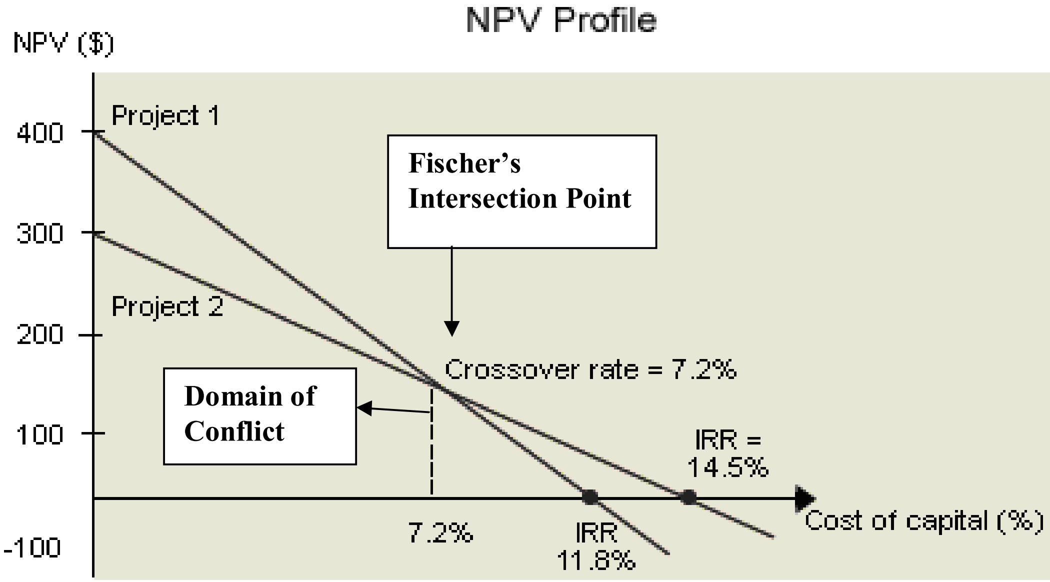
In this case, we note that:

(a) Both project 1and project 2 are investment projects. Still we examine this example because it will demonstrate how a proper appreciation of the physical realities on the ground may lead to a simple resolution of an apparent anomaly in a logically consistent manner.

(b) Project 1 has an IRR of 33% and a NPV of 35.92 units (discount rate/required rate of return = 10%), while project 2 has an IRR of 20% and a NPV of 90.00 units (discount rate/required rate of return = 10%).

(c) It is argued that project 1 is better than project 2 in terms of IRR, although it is inferior to project 2 in terms of NPV. This is used to highlight an apparent anomaly between the IRR method and the NPV method for two mutually exclusive projects.

(d) In this example, project 1 and project 2 have identical initial investments. However, the distributions of future cash flows are quite different for the 2 projects. Project 1 has future cash flows spread over only 3 years, where as project 2 has constant cash flows ad infinitum. It should also be noted that project 2, which is the project with an infinite stream of constant cash flows, has a higher NPV. As a matter of fact, in all such examples, the project with a higher NPV has invariably a greater share of distant cash flows. A reference to Fischer's Intersection Point would also corroborate this.



source: Fundamentals of Financial Management

(e) As we know, the uncertainty and risk associated with a distant cash flow (distant in time) is higher than that for a near term cash flow. Accordingly, the distant cash flows need to be discounted by a higher discount rate (risk adjusted discount rate). So, project 2 with an infinite time span needs to have a higher discount rate than project 1.

(f) If project 2 is to have a discount rate higher than that of project 1, there is no guarantee that NPV for project 2 would still continue to be higher than that for project 1. For instance, if the discount rate for project 2 becomes 15% instead of 10%, its NPV would become 30.00 units only. This is lower than the NPV for project 1, and the apparent anomaly would thus disappear. In any case, NPV cannot be a reliable yardstick for comparison in such circumstances where the cash flows from a project with higher NPV (computed on the basis of a uniform discount rate) would require to be discounted by a higher discount rate.

(g) In case of IRR, however, such a doubt cannot arise at all. If the discount rate for project 2 were to increase to 15% from 10%, the IRR related decision criterion would be fortified further, because (33%-10%) > (20%-10%) would mean that (33%-10%) >> (20%-15%), where the sign ">>" stands for "still greater than".

(h) The above analysis clearly shows that there is neither any sound basis for such an anomaly nor any deficiency of the IRR method in comparing mutually exclusive projects with identical initial cash outflows. If at all, IRR may enjoy an edge in this regard.

Resolution of Some Minor Difference: We may define PI for an investment project as [PV of Future Cash Flows/Initial Investment] = [PV/(-)CF_0] [1], although some other researchers consider the ratio [NPV/(-) CF_0][2]. PI as per the first definition equals (1 + PI) as per the second definition. This appears to be a trivial difference of nomenclature that need not lead to any serious discrepancy.

As we know, by definition, NPV has a value with monetary units. This is thus scale dependent, and changes sign with change of signs of all attendant cash flows (odd parity). IRR and PI, on the other hand, are ratios that are scale independent and do not change sign with change of signs of all attendant cash flows (even parity). So, for comparing projects with different scales of investment, IRR and PI may be compared rather than IRR and NPV.

Some researchers consider NPV > 0 as the condition for viability of investment projects [1,2], whereas some others consider NPV ≥ 0 as the appropriate criterion. This too is not likely to lead to any serious issue. As we know, NPV of a project can be viewed as the sum of the present values of the economic value added (EVAs) by the project over its life [6]. All companies in an industry cannot obviously have positive EVAs all along.

Condition for Existence and Uniqueness of IRR for Investment Projects – a Summary: For investment projects, the condition for existence and uniqueness of IRR has been derived in a compact form. This condition, for an investment project characterized by the stream of cash flows [(-) P_0, A_1, A_2, A_3,……………., A_n] with $P_0 > 0$, can be written down as [5]: $\sum a_i > 1$, where the summation is over the life of the project starting from year 1, and $a_i = A_i / P_0$, P_0 being the magnitude of the initial outflow. In other words, the algebraic sum of all future cash flows from an investment project has to exceed the magnitude of its initial outflow for an IRR to exist.

For $\sum a_i < 1$, there will be either no IRR or multiple IRRs. In other words, if the algebraic sum of all future cash flows from a project falls short of the magnitude of the initial outflow, either no IRR or multiple IRRs would exist. For project1 in Example 2 above, this is the situation with the investment project because [250/105 + (-) 150/105] < 1. This is a case where the algebraic sum of all future cash flows from the investment project [250 + (-) 150] falls short of the magnitude of the initial outflow (i.e. 105), and thus no IRR exists.

As such projects are not taken up for consideration by practicing managers, they do not come across any difficulty while dealing with the IRR method and it continues to be the most preferred method for them.

Condition for Existence and Uniqueness of IRR for Borrowing Projects:

Our goal here is to derive the general condition for existence and uniqueness of IRR for borrowing projects. However, we shall start by considering two simple cases of borrowing projects featuring a single outflow following the initial inflow.

1. Borrowing Project with a single outflow after year 1: Let the cash flow pattern be $[P_0, A_1]$, where $P_0 > 0$. This implies that an initial cash inflow of P_0 is followed by a single cash flow of A_1 at the end of year 1.

Then the equation for IRR is: $P_0 + A_1/(1 + r) = 0$

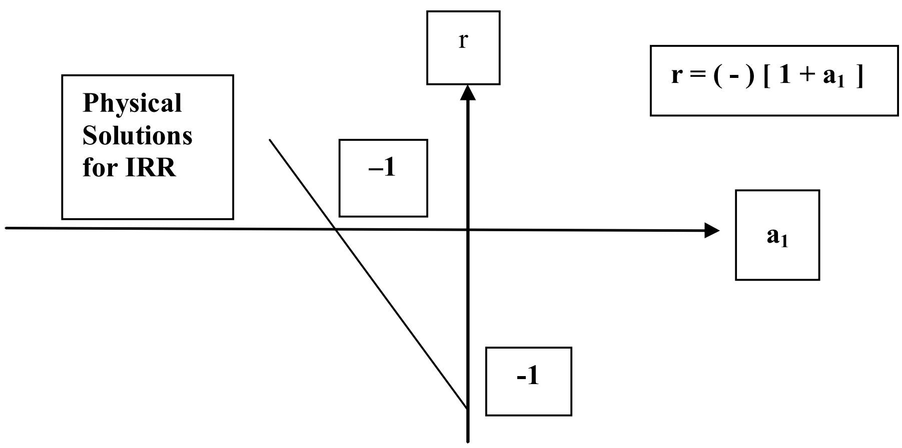
Or, $P_0(1 + r) + A_1 = 0$

Or, $(1 + r) + a_1 = 0$, where $a_1 = A_1 / P_0$

Or, $(1 + r) = (-)\ a_1$

Or, $r = (-)\ [1 + a_1]$

Thus, for r to have a real and positive value, i.e. for a physically acceptable root of IRR, $a_1 < (-)\ 1$. In other words, A_1 should have a negative sign (i.e. it must be an outflow) and a magnitude greater than P_0, the initial inflow. This is quite understandable. A loan amount of P_0 disbursed at $t = 0$, with a positive cost of borrowing, would require a future outgo of an amount greater in magnitude than P_0 towards repayment of the loan.



2. Borrowing Project with a single outflow after year 2: Let the cash flow pattern be [P_0, 0, A_2], where $P_0 > 0$. This implies that an initial cash inflow of P_0 is followed by a single cash flow of A_2 at the end of year 2. Then the equation for IRR is: $P_0 + A_2/(1 + r)^2 = 0$

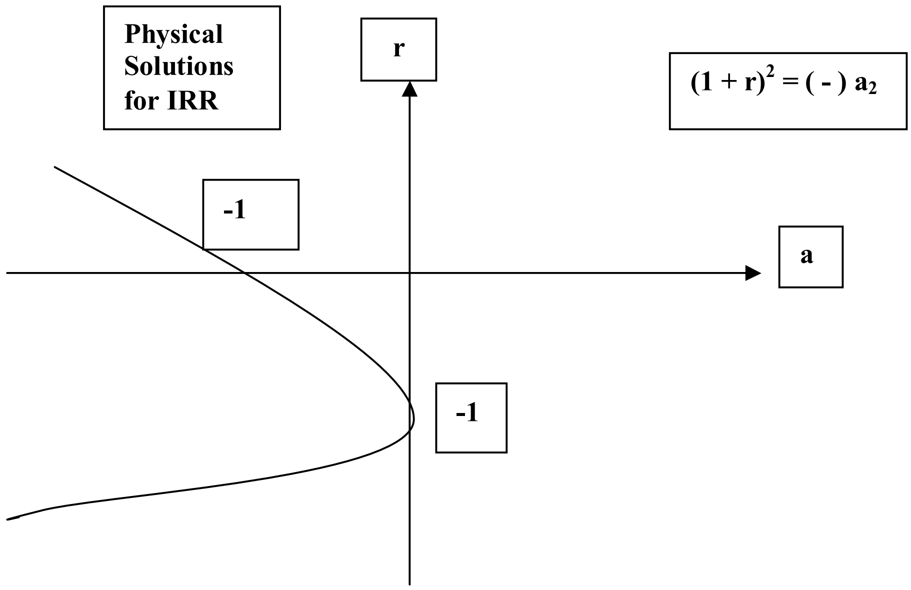
Or, $P_0 \ (1 + r)^2 + A_2 = 0$

Or, $(1 + r)^2 + a_2 = 0$, where $a_2 = A_2 / P_0$

Or, $(1 + r)^2 = (-) \ a_2$.

Thus, for r to have a real and positive value, i.e. for a physically acceptable value of IRR, $a_2 < (-) \ 1$. In other words, A_2 should have a negative sign (i.e. it must be an outflow) and a magnitude greater than P_0, the initial inflow. This is quite understandable. A loan amount of P_0 disbursed at t = 0 would require a future outgo of an amount greater in magnitude than P_0 towards repayment of the loan with a positive rate of interest.

For all such cases, IRR represents the cost of borrowed fund.



3. Borrowing Project with the Most General Cash Flow Pattern: Let us now consider a most general cash flow pattern for a borrowing project as follows: [P_0, A_1, A_2, ………… , A_n], where A_i is the cash flow at the end of the ith year; P_0 is the initial inflow ($P_0 > 0$) and $a_i = A_i / P_0$.

Then the equation for IRR is:

$P_0 + A_1 /(1 + r) + A_2 /(1 + r)^2 + \ldots\ldots\ldots\ldots + A_n / (1 + r)^n = 0$

Or, $P_0 (1 + r)^n + A_1 (1 + r)^{n-1} + A_2 (1 + r)^{n-2} + \ldots\ldots\ldots\ldots + A_n = 0$

Or, $(1 + r)^n + a_1 (1+ r)^{n-1} + a_2 (1 + r)^{n-2} + ….. + a_n = 0$ ………… (E1).

Here all the cash flows P_0, A_1, A_2, ……., A_n are real. Since the highest power of r in this equation is n, there can be n independent solutions for r. In general, some of these solutions may be real and positive, some may be real and negative and the rest may be complex (complex conjugate pairs). Since the complex roots come in pairs (complex conjugate pairs), the number of real roots would be even or odd depending on whether n is even or odd. Further, the product of 2 roots belonging to a complex conjugate pair is always positive. Hence complex roots can never affect the sign of the product of all the roots of r.

If the n roots for the above equation for r are designated by x_1, x_2, ……. , x_n, then we can write: $(r – x_1) (r – x_2) …… (r – x_n) = 0$ ……. (E2).

Equating the 2 polynomials in r as appearing in equations (E1) and (E2) and counting their r independent terms, we get:

$1 + a_1 + a_2 + ……… + a_n = (-1)^n x_1x_2 ………… x_n$
Or, $1 + \sum a_i = (-1)^n x_1x_2 ………… x_n$

Case A: For $\sum a_i< (-) 1$, $[1 + \sum a_i] < 0$
Then, $(-1)^n x_1x_2 ………… x_n$ = negative
For further analysis, let us now consider the 2 scenarios (i) n is even, and (ii) n is odd separately.

Case A Scenario (i) (n is even): In such a situation, $(- 1)^n = 1$.
Thus, $x_1x_2 ………… x_n$ = negative.

In other words, there is odd number of negative real roots of r. Since n is even, the number of real roots of r is also even. So, odd number of negative real roots for r would imply odd number of positive real roots for r. In other words, the number of physically acceptable solutions of IRR [only positive real solutions of r are acceptable as IRR] is odd and cannot thus be zero. Non existence of IRR is thus ruled out in this case.

Case A Scenario (ii) (n is odd): In such a situation, $(-1)^n = (-) 1$.
Thus, $x_1x_2 ………… x_n$ = positive.

In other words, there is even number of negative real roots of r. Since n is odd, the number of real roots of r is odd. So, even number of negative real roots for r would imply odd number of positive real roots for r. In other words, the number of physically acceptable solutions of IRR [only positive real solutions of r are acceptable as IRR] is odd and cannot thus be zero. Non existence of IRR is thus ruled out in this case also.

Case A i.e. $\sum a_i < (-) 1$, therefore, completely rules out non- existence of IRR irrespective of whether n is even or odd. In other words, $\sum a_i < (-) 1$ is a sufficient condition for existence of IRR.

Case B: $\sum a_i > (-) 1$ i.e. $[1 + \sum a_i] > 0$

Then, $(-1)^n x_1 x_2 \ldots\ldots\ldots\ldots x_n$ = positive

For further analysis, let us now consider the 2 scenarios (i) n is even, and (ii) n is odd separately.

Case B Scenario (i) (n is even): In such a situation, $(-1)^n = 1$.

Thus, $x_1 x_2 \ldots\ldots\ldots\ldots x_n$ = positive.

In other words, there is even number of negative real roots of r. Since n is even, the number of real roots of r is even. So, even number of negative real roots for r would imply even number of positive real roots for r. In other words, the number of physically acceptable solutions of IRR [only positive real solutions of r are acceptable as IRR] is even. Thus, either IRR is nonexistent or there are multiple IRRs. So, existence of a unique IRR is ruled out in this case.

Case B Scenario (ii) (n is odd): In such a situation, $(-1)^n = (-) 1$. Thus, $x_1 x_2 \ldots\ldots\ldots\ldots x_n$ = negative.

In other words, there is odd number of negative real roots of r. Since n is odd, the number of real roots of r is odd. So, odd number of negative real roots for r would imply even number of positive real roots for r. In other words, the number of physically acceptable solutions of IRR [only positive real solutions of r are acceptable as IRR] is even. Thus, either IRR is nonexistent or there are multiple IRRs. So, existence of a unique IRR is ruled out in this case also.

Case B i.e. $\sum a_i > (-) 1$, therefore, completely rules out existence of a unique IRR irrespective of whether n is even or odd. Either IRR does not exist or multiple IRRs exist. Thus, $\sum a_i < (-) 1$ is an essential prerequisite for a unique IRR to exist. In other words, $\sum a_i < (-) 1$, besides being a sufficient condition for existence of IRR, is also a necessary condition for uniqueness of IRR. In case we ignore complex cases involving 3 or higher solutions, we have a unique IRR. Since practicing managers restrict their domain of consideration to such projects, they have no cause for concern while dealing with the IRR method.

For the sake of completeness of discussion, we note that $\sum a_i = (-)1$ yields a null solution for IRR.

Case B represents instances where algebraic sum of the future cash flows do not add up to even the initial loan amount disbursed. So, the net outflow (signified by the negative sign) towards repayment of a loan does not even equal the disbursed amount. Such situations are not practical and are beyond the realm of consideration of the practicing managers. These, therefore, may not bother them.

In case of project 2 (borrowing project) of example 2 above, we observe that P0 = 105 units; A1 = -250 units and A2 = 150 units. Thus, a1 = -250/105 and a2 = 150/105 and (a1 + a2) = -100/105 > (-) 1. Thus, nonexistence of IRR in this illustration is as predicted by our theory and does not come as a surprise.

Finally, we observe that while the condition for existence and uniqueness of IRR for investment projects is $\sum a_i > 1$, that for borrowing projects is $\sum a_i < (-)1$. Here we need to realize that for investment projects $a_i = A_i / P_0$ with $P_0 > 0$, where P_0 is the magnitude of the initial outflow. In other words $P_0 = (-)\ CF_0$. But for borrowing projects, $a_i = A_i / P_0$ with $P_0 > 0$, where $P_0 = CF_0$, i.e. the initial inflow. Had we followed the same nomenclature for both types of projects and defined $a_i = A_i/CF_0$, then the condition would have been identical for both types of projects viz. $\sum a_i < (-)1$. The negative sign here implies that the algebraic sum of all future cash flows will have a sign opposite to that of the initial cash flow. Also, the magnitude of the algebraic sum has to exceed the magnitude of the initial cash flow. This would ensure a positive IRR (return on investment for investment projects and cost of borrowing for borrowing projects).

Conclusions

(a) Although IRR is the most popular capital budgeting technique for the practicing managers, some issues have been raised against it by certain researchers.

(b) Some of these issues related to possible non – existence and non – uniqueness of IRR are rather serious in nature whereas certain other issues are relatively trivial.

(c) In particular, some discrepancies or anomalies have been pointed out for borrowing projects where IRR, unlike NPV, is shown to falter.

(d) Capital budgeting is a widely used technique and holds a very important position in Corporate Finance. Serious issues in this field naturally call for a proper conceptual clarity and resolution.

(e) We have observed that IRR has an even parity while NPV has an odd parity towards reversal of signs of all project related cash flows. So, some differences may surface while examining them – particularly by reversing the signs of all the project related cash flows, there by transforming an investment project to a borrowing project and vice versa. However, as explained, this does not require us to discard either IRR or NPV.

(f) As we have demonstrated, most of the discrepancies or anomalies sought to be highlighted for borrowing projects are apparent and owe their existence to a mechanical application of few criteria that do not at all represent the physical realities on the ground. They fail various tests for logical consistency and coherence. For instance, if we impose the requirement of positive NPV as a criterion of decision making for borrowing projects, it may lead to rather abnormal situations. NPV of borrowing projects can be positive only if each fresh borrowing can be made at a cost lower than the current discount rate. Positivity of NPV would require all other borrowing proposals to be discarded as non-viable. This is quite an unrealistic scenario. If only we modify the decision criteria for borrowing projects appropriately to make these compatible with the physical realities on the ground, such apparent discrepancies disappear and a harmonious coexistence of IRR and NPV becomes feasible.

(g) The general condition for existence and uniqueness of IRR for borrowing projects could be derived in a compact form. What is more, this condition has a very direct relationship with the physical requirements of the situation.

(h) For physically acceptable scenarios, serious issues such as non – existence and non – uniqueness of IRR are virtually ruled out. This explains why practicing managers find IRR so useful and attractive.

(i) Contrary to what is suggested by some researchers, IRR does not fail to deal with problems of ranking mutually exclusive projects with identical initial cash flows. As a matter of fact, it may even enjoy an edge in such situations. For projects having different scales of investment, IRR – being a ratio, should enjoy a preference just as PI does.

(j) The specific examples analyzed here have been taken from a free courseware. However, these cases clearly bring out the essential features of the relevant issues. The discussions here are completely general in nature and are capable of being readily applied to all kinds of scenarios.

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