

THE ENVIRONMENTAL PROTECTION EXPENDITURE AND THE CHAOTIC INFLATION GROWTH MODEL: U.S.

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Abstract. *This paper examines inflation dynamics in the United States since 2000. The basic aims of this paper are: firstly, to provide a relatively simple chaotic inflation rate growth model that is capable of generating stable equilibria, cycles, or chaos, and secondly, to analyze the inflation rate growth stability in the period 2000-2017 in the U.S. economy. This paper confirms the existence of the convergent fluctuations of the inflation rate in the U.S. economy in the observed period. The net export rate, the government environmental protection expenditure – government spending ratio, the government environmental protection expenditure rate, and the investment rate are important factors of inflation rate stability.*

Keywords: *Single equation model, Inflation, Unemployment, The environmental protection expenditure.*

JEL classification numbers: *C2, E31, J64*

Introduction

The existence of negative externalities linked with environmental challenges means that economic policy has an important role to play in environmental management. The government expenditures on environmental protection and their effective management are key aspects of a national environmental policy. Environmental protection covers the activities directly aimed at the prevention, reduction and elimination of the environment degradation.

Global output growth is estimated at about 3 percent for the third quarter of 2016. Economic activity rebounded strongly in the United States and the economy is approaching full employment. Long-term nominal and real interest rates have risen substantially in the United States. Also, U.S. fiscal policy is projected to become more expansionary, with stronger

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future demand implying more inflationary pressure. The U.S. dollar has appreciated in real effective terms by over 6 percent since August 2016. (www.imf.org)

In the short run, the fall in aggregate demand leads to falling output and price level and rising unemployment. What should policymakers do when faced with such a recession? One possibility is to take action to increase aggregate demand. An increase in government spending or an increase in money supply would increase aggregate demand. According to the Phillips curve, when aggregate demand is low, then unemployment is high and inflation is low. On the other hand, Okun's law is a relationship between changes in the unemployment rate and economic growth. In this sense, this relationship predicts that growth slowdowns coincide with rising unemployment.

Okun's (1962) paper regarding the unemployment – output relationship considers the measurement of potential output. Okun believed that the potential output should not be defined as the maximum output the economy could produce. Instead, he argued that the potential should be measured at full employment, which he characterized as the level of employment absent inflationary pressures. In accordance with Okun's Law, the rising GDP growth rates were accompanied by declining unemployment rates, while declining growth rates went hand in hand with higher unemployment rates in the United States in the period 2000-2017.

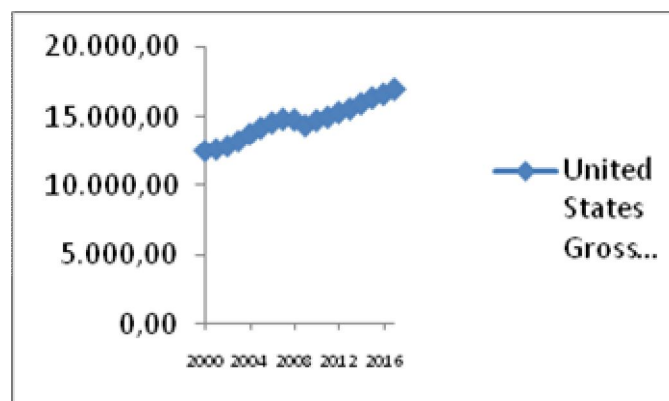


Figure 1. Gross domestic product, constant prices, national currency, billions: U.S. 2000-2017 (www.imf.org).

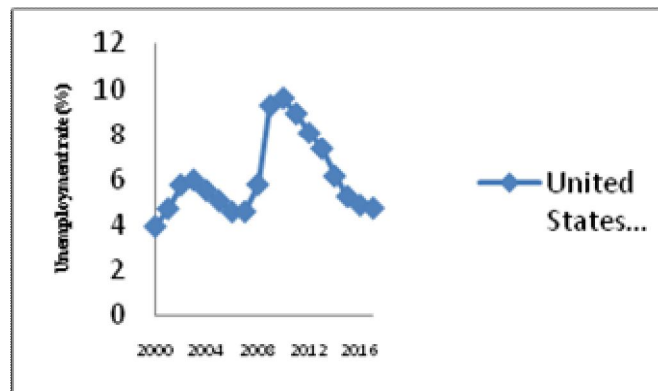


Figure 2. Unemployment rate (% of total labor force): U.S. 2000-2017 (www.imf.org).

The Phillips curve (1958) represents the negative short-run relationship between the rate of inflation and the unemployment rate. The Phillips curve shows that cyclical unemployment is related to unexpected movements in the inflation rate.

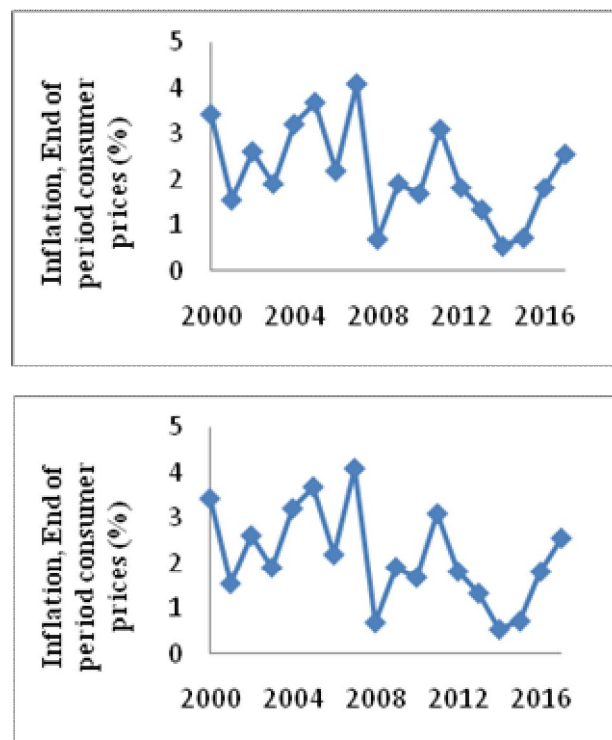


Figure 3. Inflation, end of period consumer prices (%), 2000-2017, U.S. (www.imf.org).

Chaos theory states that small changes can result in large differences. Chaotic system is unpredictable. Namely, a slight difference, in the decimal place, resulted in prediction failure. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change. Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behavior. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983, 1992, 1997), Grandmont (1985), Goodwin (1990), Medio (1993, 1996), Lorenz (1993), Jablanovic (2011, 2013, 2016), among many others.

This paper considers whether the chaotic inflation growth model can explain the recent behavior of inflation in the United States. The basic aims of this paper are: firstly, to provide a relatively simple chaotic inflation growth model that is capable of generating stable equilibrium, cycles, or chaos; and secondly, to analyze the inflation growth stability in the period 2000-2017 in the U.S. economy. This paper confirms convergent fluctuations of the inflation rate in the U.S. economy in the observed period. The net export rate, the government environmental protection expenditure – government spending ratio, the government environmental protection expenditure rate, and the investment rate are important factors of inflation rate stability.

The model

This paper focuses on the inflation rate growth stability in the U.S. economy, combining the output/unemployment relationship (Okun's Law) with the inflation/unemployment relationship (the Phillips Curve), and the Aggregate Demand curve. In this sense, the chaotic inflation growth model is presented by the following equations:

$$u_t - u^n = -\alpha (Y_t - Y^n) \quad \alpha > 0 \quad (1)$$

$$u^n = \beta u_t \quad \beta > 0 \quad (2)$$

$$Y^n = \gamma Y_t \quad \gamma > 0 \quad (3)$$

$$u_t - u^n = -\mu (\Pi_t - \Pi^e) \quad \mu > 0 \quad (4)$$

$$\Pi^e = \omega \Pi_t \quad \omega > 0 \quad (5)$$

$$Y_t = C_t + I_t + G_t + Nx_t \quad (6)$$

$$C_t = \delta Y_{t-1}^2 \quad 0 < \delta < 1 \quad (7)$$

$$I_t = \lambda Y_{t-1} \quad 0 < \lambda < 1 \quad (8)$$

$$N_{x,t} = n Y_t \quad 0 < n < 1 \quad (9)$$

$$G_{e,t} = \rho G_t \quad 0 < \rho < 1 \quad (10)$$

$$G_{e,t} = \theta Y_t \quad 0 < \theta < 1 \quad (11)$$

with Y – real output, Y^n – the potential output, I – investment, C – consumption, Nx – net exports, G – government spending, G_e – the government environmental protection expenditure Π – actual inflation, Π^e – expected inflation, u – unemployment rate, u^n – the natural rate of unemployment, α – the “Okun’s coefficient”, δ – the private consumption rate, β , μ and γ – the positive constants, n – the net export rate, λ – the investment rate, ρ – the government environmental protection expenditure – government spending ratio, θ – the government environmental protection expenditure rate.

(1) shows the Okun’ law; the negative correlation between GDP growth and unemployment has been named “Okun’s law.” The relationship between contemporaneous changes in economic growth and unemployment is often referred to as “Okun’s Law”. The parameter α is often called “Okun’s coefficient.” Okun’s relationship connected the level of unemployment to the gap between actual output (Y) and potential output (Y^n). Potential output explains how much the economy would produce “under conditions of full employment”. (2) shows that the natural rate of unemployment which is known as the non-acceleration inflation rate of unemployment (NAIRU) is proportional to the current unemployment rate; (3) shows that the potential output is proportional to the actual output; (4) the form of the short-run Phillips curve; μ is a parameter which shows the responsiveness of unemployment to inflation; (5) shows expected rate of inflation; (6) shows GDP (Y) as the sum of consumption (C), investment (I), government spending (G) and net exports; (7) In this model, the consumption function displays the quadratic relationship between

consumption (C_t) and real output of the previous period (Y_{t-1}). Real output is multiplied by the coefficient δ , “the marginal propensity to consume” (MPC). The MPC coefficient can be between zero and one. (8) shows the investment function; (9) shows the relation between net export (Nx) and real output (Y), (10) shows the relation between the government environmental protection expenditure (G_e) and government spending, G , and (11) shows the relation between the government environmental protection expenditure (G_e) and real output (Y).

Now, putting (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), and (11) together we immediately get:

$$\Pi_t = \left[\frac{\lambda}{\left(1 - n - \frac{\theta}{\rho}\right)} \right] \Pi_{t-1} - \left[\frac{\mu(1-\omega)\delta}{\alpha(\gamma-1)\left(1 - n - \frac{\theta}{\rho}\right)} \right] \Pi_{t-1}^2. \quad (12)$$

Further, it is assumed that the current value of the inflation rate is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the inflation rate growth rate depends on the actual value of the inflation rate, Π , relative to its maximal size in its time series Π^m . We introduce π as $\pi = \Pi/\Pi^m$. Thus π range between 0 and 1. Again we index π by t , i.e., write π_t to refer to the size at time steps $t = 0, 1, 2, 3, \dots$. Now the inflation growth rate is measured as

$$\pi_t = \left[\frac{\lambda}{\left(1 - n - \frac{\theta}{\rho}\right)} \right] \pi_{t-1} - \left[\frac{\mu(1-\omega)\delta}{\alpha(\gamma-1)\left(1 - n - \frac{\theta}{\rho}\right)} \right] \pi_{t-1}^2. \quad (13)$$

This model given by equation (12) is called the logistic model. For most choices of α , γ , μ , δ , λ , n , ω , θ , and ρ there is no explicit solution for (13). Namely, knowing α , ω , γ , μ , δ , λ , n , θ , and ρ and measuring π_0 would not suffice to predict π_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (13) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of π_t . This difference equation (13) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point π_0 the solution is highly sensitive to variations of the parameters $\alpha, \omega, \gamma, \mu, \delta, \lambda, n, \theta$ and ρ ; secondly, given the parameters $\alpha, \omega, \gamma, \mu, \delta, \lambda, n, \theta$, and ρ , the solution is highly sensitive to variations of the initial point π_0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

The Logistic Equation

It is possible to show that iteration process for the logistic equation (see Fig. 3.)

$$z_t = \eta z_{t-1}(1 - z_{t-1}), \quad \eta \in [0,4], \quad z_t \in [0,1] \quad (14)$$

is equivalent to the iteration of growth model (12) when we use the identification

$$z_{t-1} = \left[\frac{\mu \delta (1 - \omega)}{\alpha \lambda (\gamma - 1)} \right] \pi_{t-1} \quad \text{and} \quad \eta = \left[\frac{\lambda}{\left(1 - n - \frac{\theta}{\rho}\right)} \right]. \quad (15)$$

Using (13) and (15) we obtain:

$$\begin{aligned} z_t &= \left[\frac{\mu \delta (1 - \omega)}{\alpha \lambda (\gamma - 1)} \right] \pi_t = \\ &= \left[\frac{\mu \delta (1 - \omega)}{\alpha \lambda (\gamma - 1)} \right] \left\{ \left[\frac{\lambda}{\left(1 - n - \frac{\theta}{\rho}\right)} \right] \pi_{t-1} - \left[\frac{\mu (1 - \omega) \delta}{\alpha (\gamma - 1) \left(1 - n - \frac{\theta}{\rho}\right)} \right] \pi_{t-1}^2 \right\} = \\ &= \left[\frac{\mu \delta (1 - \omega)}{\alpha (\gamma - 1) \left(1 - n - \frac{\theta}{\rho}\right)} \right] \pi_{t-1} - \left[\frac{\mu^2 \delta^2 (1 - \omega)^2}{\alpha^2 \lambda (\gamma - 1)^2 \left(1 - n - \frac{\theta}{\rho}\right)} \right] \pi_{t-1}^2. \end{aligned}$$

On the other hand, using (13) and (14) we obtain:

$$\begin{aligned}
 z_t &= \eta z_{t-1} (1 - z_{t-1}) = \\
 &= \left[\frac{\lambda}{\left(1 - n - \frac{\theta}{\rho}\right)} \right] \left[\frac{\mu \delta (1 - \omega)}{\alpha \lambda (\gamma - 1)} \right] \pi_{t-1} \left\{ 1 - \left[\frac{\mu \delta (1 - \omega)}{\alpha \lambda (\gamma - 1)} \right] \pi_{t-1} \right\} = \\
 &= \left[\frac{\mu \delta (1 - \omega)}{\alpha (\gamma - 1) \left(1 - n - \frac{\theta}{\rho}\right)} \right] \pi_{t-1} - \left[\frac{\mu^2 \delta^2 (1 - \omega)^2}{\alpha^2 \lambda (\gamma - 1)^2 \left(1 - n - \frac{\theta}{\rho}\right)} \right] \pi_{t-1}^2.
 \end{aligned}$$

Thus we have that iterating (13) is really the same as iterating (14) using (15). It is important because the dynamic properties of the logistic equation (14) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that:

- (i) For parameter values $0 < \eta < 1$ all solutions will converge to $z = 0$;
- (ii) For $1 < \eta < 3.57$ there exist fixed points the number of which depends on η ;
- (iii) For $1 < \eta < 2$ all solutions monotonically increase to $z = (\eta - 1) / \eta$;
- (iv) For $2 < \eta < 3$ fluctuations will converge to $z = (\eta - 1) / \eta$;
- (v) For $3 < \eta < 4$ all solutions will continuously fluctuate;
- (vi) For $3.57 < \eta < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Empirical Evidence

The main aim of this paper is to analyze the inflation rate growth stability in the period 2000-2017 in the U.S. economy. In this sense, it is important to use the logistic model (16):

$$\pi_t = \eta \pi_{t-1} - \upsilon \pi_{t-1}^2 \quad (16)$$

where π – inflation rate, $\eta = \left[\frac{\lambda}{\left(1 - n - \frac{\theta}{\rho}\right)} \right]$, $\vartheta = \left[\frac{\mu \delta (1 - \omega)}{\alpha (\gamma - 1) \left(1 - n - \frac{\theta}{\rho}\right)} \right]$.

Now, the model (16) is estimated (see Table 1).

Table 1.
The estimated model (16): U.S., 2000-2017. R= 0.53765

	η	ϑ
Estimate	2.238269	1.956742
Std.Err.	0.325209	0.425380
t (15)	6.882561	4.585884
p-level	0.00000	0.00036

Source: www.imf.org

Conclusion

This paper creates the chaotic unemployment rate growth model. For most choices of α , ω , γ , μ , δ , λ , n , θ and ρ there is no explicit solution for (12). Namely, knowing α , ω , γ , μ , δ , λ , n , θ , and ρ and measuring π_0 would not suffice to predict π_t for any point in time, as was previously possible. But even slight deviations from the values of parameters: α , ω , γ , μ , δ , λ , n , θ , and ρ and initial value of inflation rate, π_0 show the difficulty of predicting a long-term inflation rate behavior.

A key hypothesis of this work is based on the idea that the coefficient

$\eta = \left[\frac{\lambda}{\left(1 - n - \frac{\theta}{\rho}\right)} \right]$ plays a crucial role in explaining the local inflation rate

growth stability, where, n – the net export rate, ρ – the government environmental protection expenditure – government spending ratio, θ – the government environmental protection expenditure rate, λ – the investment rate. An estimated value of the coefficient η (2.238269) confirms convergent fluctuations of the inflation rate in U.S. in the observed period.

REFERENCES

- [1] Benhabib, J., Day, R. H. (1981), *Rational Choice and Erratic Behaviour*, Review of Economic Studies 48: 459-471.
- [2] Benhabib, J., Day, R. H. (1982), *Characterization of Erratic Dynamics in the Overlapping Generation Model*, Journal of Economic Dynamics and Control 4: 37-55.
- [3] Day, R. H. (1982), *Complex Economic Dynamics: Obvious in History, Generic in Theory, Elusive in Data*, Journal of Applied Econometrics, Vol. 7, Issue Supplement, 1992, S9-S23.
- [4] Day, R. H. (1982), *Irregular Growth Cycles*, American Economic Review 72: 406-414.
- [5] Day, R. H. (1983), *The Emergence of Chaos from Classical Economic Growth*, Quarterly Journal of Economics 98: 200-213.
- [6] Day, R. H. (1997), *Complex Economic Dynamics, Volume I: An introduction to dynamical systems and market mechanism*, MIT Press, In: Discrete Dynamics in Nature and Society, Vol. 1, 177-178.
- [7] Goodwin, R. M. (1990), *Chaotic Economic Dynamics*, Clarendon Press, Oxford.
- [8] Grandmont, J. M. (1985), *On Endogenous Competitive Business Cycles*, Econometrica 53: 994-1045.
- [9] Jablanovic, V. (2011), *Budget Deficit and Chaotic Economic Growth Models*. Aracne editrice S.R.L., Roma.
- [10] Jablanovic, V. (2013), *Elements of Chaotic Microeconomics*, Aracne editrice S.R.L., Roma.
- [11] Jablanovic, V. (2016), *A Contribution to the Chaotic Economic Growth Theory*. Aracne editrice S.R.L., Roma.
- [12] Li, T., Yorke, J. (1975), *Period Three Implies Chaos*, American Mathematical Monthly 8: 985-992.
- [13] Lorenz, E. N. (1963), *Deterministic nonperiodic flow*, Journal of Atmospheric Sciences 20: 130-141.
- [14] Lorenz, H. W. (1993), *Nonlinear Dynamical Economics and Chaotic Motion*. 2nd edition, Springer-Verlag, Heidelberg.
- [15] May, R. M. (1976), *Mathematical Models with Very Complicated Dynamics*, Nature 261: 459-467.
- [16] Medio, A. (1993), *Chaotic Dynamics: Theory and Applications to Economics*. Cambridge University Press, Cambridge.
- [17] Medio, A. (1996), *Chaotic dynamics. Theory and applications to economics*. Cambridge University Press, In: De Economist 144 (4), 695-698.
- [18] Okun, A. M. (1962), *Potential GNP: Its Measurement and Significance*, Proceedings of the Business and Economic Statistics Section, American Statistical Association, 98-104.
- [19] Phillips, A. W. (1958), *The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957*, Economica 25: November, no.100, 283-299.