FOUR CASES AND FOUR ALGORITHMS FOR CALCULATING ALL BASIC ELEMENTS OF THE MOBILE TRAJECTORY LAUNCHED IN THE GRAVITATIONAL VERTICAL PLANE

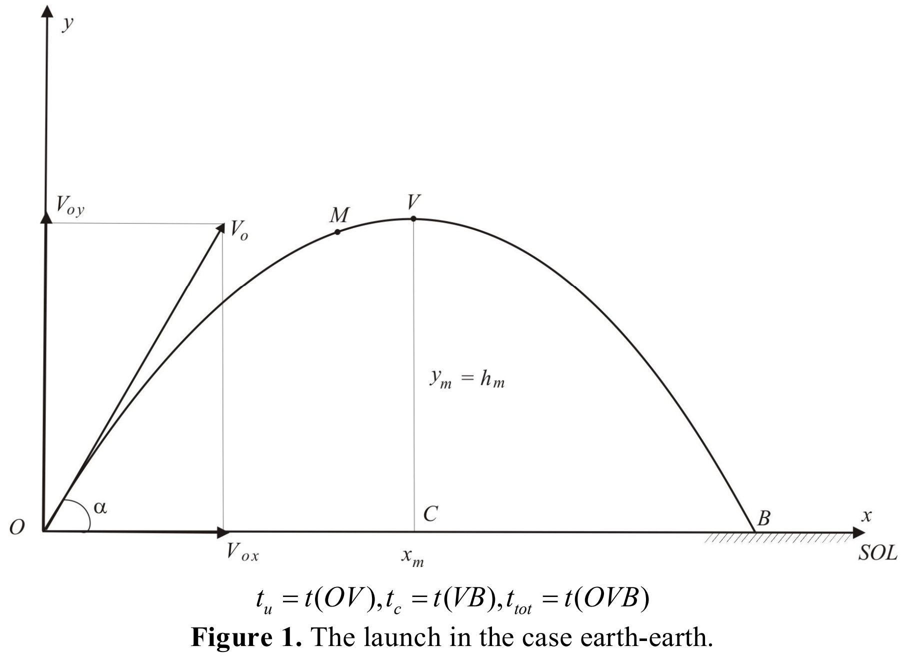
Nicolae POPOVICIU*

Abstract. *The work contains an unitary and synthetic theory for the launch of a mobile in vertical gravitational plane. Four cases with theoretic algorithms and the corresponding informatics algorithms are presented (earth – earth; tower – earth; trench = earth; earth – earth with break in two components).*

1. Problem formulation for the case earth – earth

Find **the equation** of Γ trajectory and **all the basic elements** of the mobile trace.

Solution. We study the mobile movement in many steps *Pn*.



$t_u = t(OV), t_c = t(VB), t_{tot} = t(OVB)$

Figure 1. The launch in the case earth-earth.

* Hyperion University of Bucharest

Theoretic algorithm 1 for the case earth – earth

P1. The given data are the initial speed $\overline{v_0}$ and the angle α. We decompose the initial speed

$\overline{v_0} = \overline{v_{0x}} + \overline{v_{0y}}$, where $v_{0x} = v_0 \cos\alpha$, $v_{0y} = v_0 \sin\alpha$ (1earth earth)

P2 Parametrical equations of trajectory Γ.

Γ: $x(t) = v_0 \cos\alpha\, t$, $y(t) = v_0 \sin\alpha\, t - \frac{1}{2} g t^2$, t is the time (2earth earth).

P3 The explicit equation of trajectory.

Γ: $y = -\frac{g}{2 v_0^2 \cos^2\alpha} x^2 + tg\alpha\, x$, $y = -\frac{g}{2 v_{0x}^2} x^2 + tg\alpha\, x$ (3earth earth)

P4. The highest position of trajectory is h_m. The top of Γ is $V(x_m, y_m = h_m)$.

$\Delta = tg^2\alpha$, $x_m = \frac{v_0^2}{2g}\sin 2\alpha$, $y_m = \frac{v_0^2}{2g}\sin^2\alpha$, $y_m = h_m$ (4earth earth)

P5 The speeds on axis.

$v_x(t) = v_{0x}$ $\quad v_x(t) = v_0 \cos\alpha$ =const (5earth earth)

$v_y(t) = v_{0y} - gt$ $\quad v_y(t) = v_0 \sin\alpha - gt.$

$v(t) = \sqrt{v_x^2(t) + v_y^2(t)}.$

P6 The climbing time, the descent time, the total time

$t_u = \frac{v_0 \sin\alpha}{g}$, (climbing time) (6earth earth)

$t_u = t_c$ (t_c is descent time), $t_u = t_c$, $t(OVB) = 2t_u = t_{tot} = \frac{2v_0 \sin\alpha}{g}$

P7 The range *b*.

$b = 2x_m = OB = \frac{v_0^2}{g}\sin 2\alpha$ (range) (7earth earth)

P8 The current points on the trajectory, for the total time.

$M_i(t_i))$, with $t_i \in [0, t_{tot}]$, $M_i(t_i)) = M_i(x_i, y_i)$ (8earth earth)

The tangent equation in the current point $M(t) = M(x(t), y(t))$

$$T(t):\ y=\left(tg\alpha-\frac{g\,x(t)}{v_0^2\cos^2\alpha}\right)x+tg\alpha\,x(t)-y(t)\quad \text{(tangent)}\quad \text{(9earth earth)}$$

P10 The slope of tangent.

$$m(t)=tg\alpha-\frac{g\,x(t)}{v_0^2\cos^2\alpha}\quad \text{(slope)}\qquad \text{(10earth earth)}$$

P11 The speed angle $\alpha(t)$ in the current point.

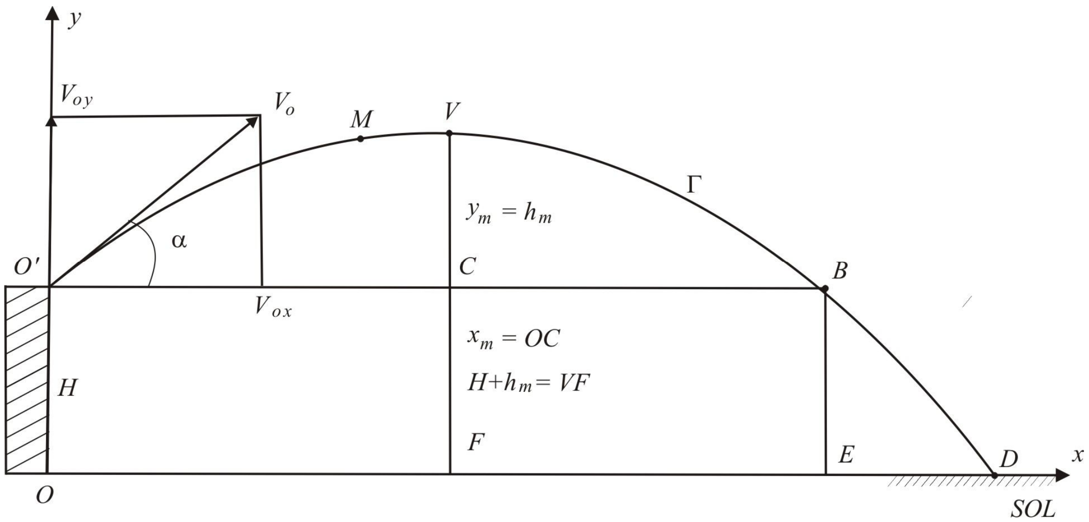
$$tg(\alpha(t))=m(t),\ \ \alpha(t)=arctg\,m(t),\ \ \alpha(t)\in[0,\pi]\qquad \text{(11earth earth)}$$

Informatics algorithm 1 for the case earth – earth
Write a computer program with all steps P1 – P11.

2. Problem formulation for the case tower – earth

Find **the equation** of Γ trajectory and **all the basic elements** of the mobile trace.

Solution. We study the mobile movement in many steps *Pn*.



$$t_u=t(O'V),\, t_c=t(VB),\, t_{SOL}=t(BD)$$
$$t_{tot}=t_u+t_c+t_{SOL},\, b=O'B,\, d=OD$$

Figure 2. The launch in the case tower-earth.

Solution. Launch movement is a compound movement that is studied in several steps. Note step *n* of the problem analysis through *Pn*.

Theoretic algorithm 2 for the case tower – earth.

P1..The given data are the initial speed $\overline{v_0}$ and the angle α. We decompose the initial speed

$\overline{v_0} = \overline{v_{0x}} + \overline{v_{0y}}$, where $v_{0x} = v_0 \cos\alpha,\ v_{0y} = v_0 \sin\alpha$ (1tower earth)

P2 Parametrical equations of trajectory Γ.

$\Gamma:\ x(t) = v_0 \cos\alpha\, t,\ y(t) = v_0 \sin\alpha\ t - \frac{1}{2} g t^2$; t is the time (2tower earth)

P3 The explicit equation of trajectory, obtained by the variable *t*.

$\Gamma:\ y = -\frac{g}{2 v_0^2 \cos^2\alpha} x^2 + tg\alpha\, x,\ y = -\frac{g}{2 v_{0x}^2} x^2 + tg\alpha\, x$ (3tower earth)

P4..The highest position of trajectory is h_m. The top of Γ is $V\left(x_m, y_m = h_m\right)$.

$\Delta = tg^2\alpha,\ x_m = \frac{v_0^2}{2g}\sin 2\alpha,\ y_m = \frac{v_0^2}{2g}\sin^2\alpha$, $y_m = h_m$ (4tower earth)

P5 The speeds on axis.

$v_x(t) = v_{0x}$ $\quad\quad$ $v_x(t) = v_0 \cos\alpha$ =constant

$v_y(t) = v_{0y} - gt$ $\quad\quad$ $v_y(t) = v_0 \sin\alpha - gt.$

$v(t) = \sqrt{v_x^2(t) + v_y^2(t)}$ (5tower earth)

P6 The climbing time, the descent time, the partial time, the total time

$t_{tot} = t(O'VBD) = t(O'VB) + t(BD),\ t_u = t_c,\ t_u = t(O'V),\ t_c = t(VB).$

$t_u = \frac{v_0 \sin\alpha}{g}$ (climbing time), $t(O'VB) = 2t_u = t_{partial} = \frac{2v_0 \sin\alpha}{g}.$

The mobile goes on the parabolic way BD under the gravitational force, namely it uses the speed component $v_y(t) = v_0 \sin\alpha - gt.$ This component has in the point *B* the value $v_y(t(O'VB)) = v_y(2t_u).$

The space on vertical is $H = BE.$ By using the space formula we obtain a second order equation in unknown *t*

$s(t) = vt + \frac{1}{2}gt^2$, $H = v_y(2t_u)t + \frac{1}{2}gt^2$, $t(BD)_1 > 0$, $t(BD)_2$

$t(BD) = t(BD)_1$, $t_{tot} = t(O'VB) + t(BD) = 2t_u + t(BD)$ (6tower earth)

P7 The range b (on the horizontal direction).

$b = 2x_m = O'B = \frac{v_0^2}{g}\sin 2\alpha$ (range) (7tower earth)

P8 The current points on the trajectory, for the total time.

$M_i(t_i))$, with $t_i \in [0, t_{tot}]$, $M_i(t_i)) = M_i(x_i, y_i)$ (8tower earth)

P9 The tangent equation in the current point $M(t) = M(x(t), y(t))$

$T(t)$: $y = \left(tg\alpha - \frac{g\,x(t)}{v_0^2 \cos^2 \alpha} \right) x + tg\alpha\, x(t) - y(t)$ (tangent) (9tower earth)

P10 The slope of tangent.

$m(t) = tg\alpha - \frac{g\,x(t)}{v_0^2 \cos^2 \alpha}$ (slope) (10tower earth)

P11 The speed angle $\alpha(t)$ in the current point.

$tg(\alpha(t)) = m(t)$, $\alpha(t) = arctg\, m(t)$, $\alpha(t) \in [0, \pi]$ (11tower earth)

P12. Compute the maximum value *VF* of the top *V*, related with the earth.

$V(x_m, VF)$, where $x_m = \frac{v_0^2}{2g}\sin 2\alpha$

$VF = H + h_m$, $h_m = y_m = \frac{v_0^2}{2g}\sin^2 \alpha$ (12tower earth)

P13 Compute the distance $d = OD$ on horizontal, from the tower base.

The mobile traverse the distance $d = OD$ with the speed $v_{0x} = v_0 \cos\alpha$, during the total time and it results $d = OD = v_{0x}\, t_{tot} = v_0 \cos\alpha\, t_{tot}$ (13tower earth).

P14 Compute the speed $v(t_{tot})$ in the point *D*. Successively we obtain

$v_{0x} = v_0 \cos\alpha$, $v_y(t_{tot}) = v_{0y} + g\, t_{tot}$, $v_y(t_{tot}) = v_0 \sin\alpha + g\, t_{tot}$

$$v(t_{tot}) = \sqrt{v_{0x}^2 + v_y^2(t_{tot})} = v(D) \quad \text{(14tower earth)}$$

P15 Compute the speed angle $\alpha(t)$ of $v(t)$ in the point D.

First on obtains the slope $m(t_{tot})$ in the point $M(t_{tot}) = D$.

Then we solve the trigonometric equation

$$tg(\alpha(t_{tot})) = m(t_{tot}),\ \ \alpha(t_{tot}) = arctg\, m(t_{tot}) = \alpha(D) \quad \text{(15tower earth)}$$

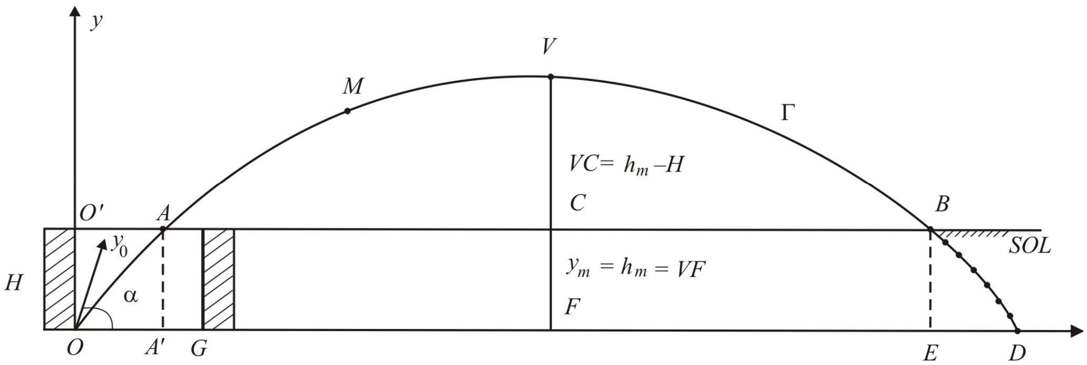
P16 Optional. We write an numerical table for the speeds and slopes in the points O', V, B, D.

Informatics algorithm 2 for the case tower – earth.
Write a computer program with all steps P1 – P16.

3. Problem formulation for the case trench – earth

Find **the equation** of Γ trajectory and **all the basic elements** of the mobile trace.

Solution. We study the mobile movement in many steps *Pn*.



$$L = OG \text{ (width)},\ L_{\min} > x_1,\ x_1 = OA',\ H = OO'$$
$$t_u = t(OV),\ t_c = t(VD),\ t_{SOL} = t(VB),\ b = OD$$
$$t_{tot} = t_u + t_{SOL},\ d = O'B = OE = x_2$$

Figure 3. The launch in the case trench-earth.

Theoretic algorithm 3 in the case trench - earth.

P1. The given data are the initial speed $\overline{v_0}$, the depth H and the angle α. We decompose the initial speed

$\overline{v_0} = \overline{v_{0x}} + \overline{v_{0y}}$, where $v_{0x} = v_0 \cos\alpha$, $v_{0y} = v_0 \sin\alpha$ (1trench earth)

P2. Parametrical equations of trajectory Γ.

$x(t) = v_{0x} t$, $y(t) = v_{0y}\, t - \frac{1}{2} g t^2$, t is the time, $\alpha = \alpha_0 = \alpha(0)$

Γ: $x(t) = v_0 \cos\alpha\, t$, $y(t) = v_0 \sin\alpha\, t - \frac{1}{2} g t^2$ (2trench earth)

P3. Eliminate time t from parametrical equations and obtain explicit equation.

$$\Gamma: \; y(t) = -\frac{g}{2 v_0^2 \cos^2\alpha} x^2(t) + tg\alpha\, x(t) \text{ or}$$

$$\Gamma: \; y = -\frac{g}{2 v_0^2 \cos^2\alpha} x^2 + tg\alpha\, x, \; y = -\frac{g}{2 v_{0x}^2} x^2 + tg\alpha\, x \qquad \text{(3trench earth)}$$

P4..The highest position of trajectory is h_m.. The top of Γ is $V\left(x_m, y_m = h_m\right)$.

$$V\left(x_m = -\frac{b}{2a}, y_m = -\frac{\Delta}{4a}\right), \; \Delta = b^2 - 4ac$$

$$\Delta = tg^2\alpha \,, \; x_m = \frac{v_0^2}{2g} \sin 2\alpha = OF, \; y_m = \frac{v_0^2}{2g} \sin^2\alpha = VF.$$

The maximum height is $h_m = VC = y_m - H$ (4trench earth)

P5. Compute the width $OG=L$ of the trench, $L = L_{\min} = = L(v_0, \alpha, H)$.

Use intersection from Γ and the line $y = H$ and solve the equation $\frac{g}{2 v_0^2 \cos^2\alpha} x^2 - tg\alpha\, x + H = 0$. The roots are $x_1 < x_2$ and we take

$OA' = O'A = x_1 = L$, where $OG \geq L$ (5trench earth)

The other root is $x_2 = O'B = OE$.

P6. Find the **fictitious range** $b = OD$ and **real range** $b' = O'B$.

We use the value x_2 from the step P5.

$$b = OD = 2x_m = \frac{v_0^2}{g} \sin 2\alpha, \; b' = O'B = x_2, \; B(x_2, H) \qquad \text{(6trench earth)}$$

P7. We compute the speeds on axis and the resultant speed in the current point.

$v_x(t) = v_{0x}$, $\qquad$ $v_x(t) = v_0 \cos\alpha$ = constant

$v_y(t) = v_{0y} - gt.$ $\qquad$ $v_y(t) = v_0 \sin\alpha - gt.$

The accelerations are $a_x(t) = 0,\ a_y(t) = -g,\ g = 9{,}8\frac{m}{s^2}$ or $g = 10\frac{m}{s^2}$.

The function $v_y(t)$ is decreasing and $v_x(t) =$ const.

The resultant speed is $v(t) = \sqrt{v_x^2(t) + v_y^2(t)}$ (7trench earth)

P8 Compute the time for the main points $O, A, V, B, D,$ denoted by $t(O) = 0,\ t(OA),\ t(AV) = t(VB),\ t(BD)$. Many successively steps are necessary.

Compute the total time for the point $B\, t_{tot} = t(OAVB) = t(OAVBD) - t(BD)$.

At the beginning we put $v_y(t) = 0$ and compute $t_u = \frac{v_0 \sin\alpha}{g} =$ $= t(OAV)$. Then $t_u = t_c$, $t_u = t(OAV),\ t_c = t(VBD),\ t(OAVBD) = 2t_u$.

Now we need the time $t(OA)$. We use the results from step P5

$$v_{0x};\ O'A = OA' = x_1 \text{ (from P5)};\ x_1 = v_{0x}\, t(OA),\ t(OA) = \frac{x_1}{v_{0x}}.$$

Consequence: $t(OA) = \frac{x_1}{v_{0x}} = t(BD)$ (due to the symmetry).

Compute the time $t(AV)$. Method 1. $t(AV) = t_u - t(OA)$ and $t(AV) = t(VB)$.

Method 2. Use v_{0x} and the space $AC = x_m - x_1$ and obtain

$x_m - x_1 = v_{0x}\, t(AV),\ t(AV) = \frac{x_m - x_1}{v_{0x}}$ (method 2 is a verification for method 1)

Compute the time $t(BD)$. Version 1. $t(BD) = t(OA) = \frac{x_1}{v_{0x}}$.

Version 2. We use the component v_{0x} and the space ED and obtain

$$ED = 2x_m - x_2 \text{ (from step P5); } ED = v_{0x}\, t(BD),\ \ t(BD) = \frac{2x_m - x_2}{v_{0x}}.$$

Now the total time is $t_{tot} = t(OAVB) = 2t_u - t(BD)$ (8trench earth)

P9. Optional. Compute the coordinates for several important points and the corresponding speeds and put the values in a table.

$M_i(t_i))$, cu $t_i \in [0, t(OAVB)]$, $t_i \in [0, t_{tot}]$, $M_i(t_i)) = M_i(x_i, y_i)$, (9trench earth)

P10. Find the explicit equation for tangent $T(t)$ in the current point $M(t)$.

$$\frac{y + y(t)}{2} = -\frac{g}{2v_0^2 \cos^2 \alpha} x\, x(t) + tg\alpha \frac{x + x(t)}{2},\ \ M(t) = M\left(x(t), y(t)\right)$$

$$T(t):\ y = \left(tg\alpha - \frac{g\, x(t)}{v_0^2 \cos^2 \alpha} \right) x + tg\alpha\, x(t) - y(t) \qquad \text{(10trench earth)}$$

The speed $v(t)$ has the same direction with tangent $T(t)$.

P11. The slope of tangent $T(t)$ is $m(t)$

$$m(t) = tg\alpha - \frac{g\, x(t)}{v_0^2 \cos^2 \alpha} \text{ (slope)} \qquad \text{(11trench earth)}$$

P12. Find the angle $\alpha(t)$ of the speed $v(t)$ with the axis Ox. Solve a trigonometric equation

$$tg(\alpha(t)) = m(t),\ \ \alpha(t) = arctg\, m(t),\ \ \alpha(t) \in [0, \pi] \qquad \text{(12trench earth)}$$

The value $\alpha(0)$ is a given initial number $\alpha(0) = \alpha$.

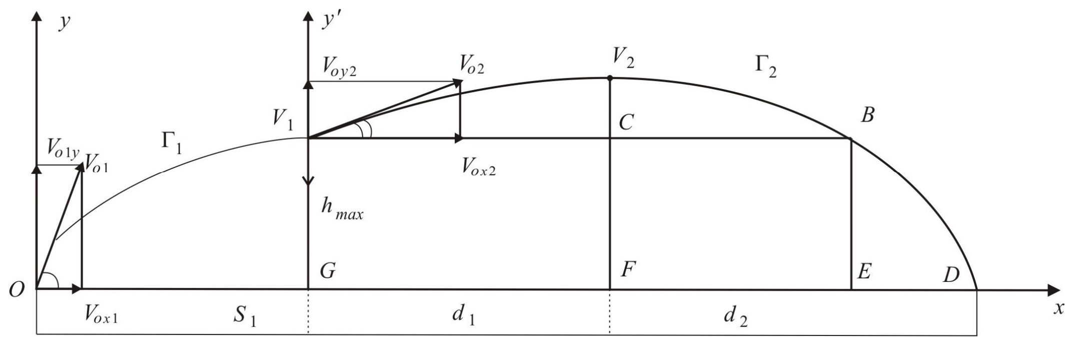
P13. Optional. Write a table containing the points, the time, the speeds and the angles.

Informatics algorithm 3 for the case trench – earth.

Write a computer program with all steps P1 – P13.

4. Problem formulation for missile launch earth – earth. Break in two components

The missile is composed of two components and the action has two stages.



$$OG = S_1 = x_{m1}, h_{\max} = y_{m1} = V_1G$$

Figure 4. The missile launches earth – earth.

Stage 1. The missile is launched from earth and at the given height $h_{\max}$ the missile is divided into two parts: C_1, C_2, where C_1 has the material mass c_1 and C_2 has c_2; c_1 and c_2 are known values; suppose $c_1 = c_2$.

Stage 2. The element C_1 is landing on the earth at the known distance S_1 and the element C_2 continue its ascendant movement. So, we have to study two trajectories: C_1 with Γ_1 and C_2 with Γ_2.

Find **the equations** of Γ_1 and Γ_2 and **all the basic elements** of the traces.

Solution. We study the mobile movement in many steps Pn.

Theoretic algorithm 4 for missile launch in the case earth – earth and a numerical application with $S_1 = 1000$ m, $h_{\max} = 20$ m.

Stage 1 (initial stage)

P1 We equivalent the given data S_1 and $h_{\max}$ with the general launch theory: $S_1 = x_{m1}$ and $h_{\max} = h_{m1} = y_{m1}$, namely the top (vertex) is $V_1(x_{m1}, y_{m1})$ of Γ_1.

Denote by O the launch point and $O_1 = V_1$ is the top point on Γ_1, where the missile is splitting in C_1, C_2.

The speed on Γ_1 is denoted $v_1(t)$, and $v_1(0) = v_{01}$.

P1.Apl. $S_1 = x_{m1}$=1000 m, $h_{\max} = h_{m1}$ =y_{m1}=20 m.

P2 By using the vertex V_1 we compute the initial speed v_{01} and the initial launch angle α_1. Successively we obtain

$\frac{y_{m1}}{x_{m1}} = \frac{1}{2} tg\alpha_1$, $\alpha_1 = arctg\frac{2y_{m1}}{x_{m1}}$ and

$v_{01}^2 = \frac{2gy_{m1}}{\sin^2\alpha}$, $v_{01}^2 = \frac{g}{2}\frac{x_{m1}^2 + 4y_{m1}^2}{y_{m1}}$, because $\sin arctgx = \frac{x}{\sqrt{1+x^2}}$, $\cos arctgx = \frac{1}{\sqrt{1+x^2}}$.

P2.Apl. $\alpha_1 = arctg\frac{2y_{m1}}{x_{m1}} = arctg\frac{2\times 20 = 40}{1000} = arctg 0.04 = 2,29^0$

$$v_{01}^2 = \frac{g}{2}\frac{x_{m1}^2 + 4y_{m1}^2}{y_{m1}} = \frac{10}{2}\frac{1000000 + 4\times 400}{20} = 250400$$

$\alpha_1 =$ $2,29^0$, $v_{01} = 500,4\,m$ (initial missile speed).

P3 Compute the projections of v_{01} on the axis, denoted v_{01x} and v_{01y}.

Method 1. $v_{01x} = v_{01}\cos\alpha_1$, $v_{01y} = v_{01}\sin\alpha_1$, $\sqrt{v_{01x}^2 + v_{01y}^2} = v_{01}$.

Method 2. Use the equation of Galileo for v_{01y}, namely $v_{01y}^2 = 2gh_{m1}$.

P3.Apl. Method 1. For v_{01x} and v_{01y} we obtain

$\cos\alpha_1 = 0,99920$, $\sin\alpha_1 = 0,03995$, $tg\alpha_1 = 0,03998$

$v_{01x} = v_{01}\cos\alpha_1 = 499,99968$ m, $v_{01y} = v_{01}\sin\alpha_1 = 19,99098$ m.

Approximation: $v_{01x} = 500$ m și $v_{01y} = 20$ m.

Verification $\sqrt{v_{01x}^2 + v_{01y}^2} = \sqrt{250400} = 500,39984 = v_{01}$. True.

Method 2. Galileo equation for v_{01y}: $v^2 = 2ad$, namely

$v_{01y}^2 = 2gh_{m1} = 400$, $v_{01y} = 20$ m.

P4..Obtain the parametric equations and explicit equation of trajectory Γ_1.

$\Gamma_1: \ x(t) = v_0 \cos\alpha t, \ \ y(t) = v_{01} \sin\alpha_1 \ t - \frac{1}{2} g t^2$

Observation. The current point on trajectory is $M_1(t) = M_1(x(t), y(t))$.

$\bar{r}_1(t) = x(t)\bar{i} + y(t)\bar{j}$ (position vector).

$\Gamma_1: \ y = -\frac{g}{2 v_{01}^2 \cos^2 \alpha_1} x^2 + tg\alpha_1 \, x, \ \ tg\alpha_1 = \frac{v_{01y}}{v_{01x}}.$

In V_1 the element C_1 falls down on earth at the distance $S_1 = x_{m1}$. Stop.

P4.Apl. Parametric equations and explicit equation of Γ_1.

$\Gamma_1: \ x(t) = v_0 \cos\alpha t, \ \ y(t) = v_{01} \sin\alpha_1 \ t - \frac{1}{2} g t^2$

$\Gamma_1: \ x(t) = 500 \, t, \ \ y(t) = 20 \, t - 5t^2$ (parametric equations of Γ_1)

$\bar{r}_1(t) = x(t)\bar{i} + y(t)\bar{j} = (500t)\bar{i} + (20t - 5t^2 \bar{j}$ (position vector).

$\Gamma_1: \ y = -\frac{g}{2 v_{01}^2 \cos^2 \alpha_1} x^2 + tg\alpha_1 \, x, \ y = -0{,}00002 x^2 + 0{,}03998 \, x.$

Approximation for Γ_1 is the line $y = 0{,}03998 \, x$.

P5.Cumpute the climbing time $t_{u1} = t(OO_1) = t(OV_1)$ on Γ_1.

$v_{01y} = v_{01} \sin\alpha_1$ (from step P3) and $v_{01y} = g \, t_{u1}, \ t_{u1} = \frac{v_{01y}}{g} s.$

Verification: $s = vt$ and $v_{01x} \, t_{u1} = S_1 = x_{m1}; \ \ V_1(x(t_{u1}), y(t_{u1}))$ from P4.

P5.Apl. Compute the rise time $t_{u1} = t(OO_1)$. This value corresponds to the time in which v_{01y} reaches the vertical value $v_{01y} = v_{01} \sin\alpha_1$ (from P3).

Such, $v_{01y} = g \, t_{u1}, \ t_{u1} = \frac{v_{01y}}{g} = \frac{20}{10} = 2s, \ \ t_{u1} = 2s.$

Verifications. $t_{u1} = 2s$: $v_{01x} = 500$ m; $S_1 = x_{m1}$=1000 m.

Formula $s = vt$ gives $v_{01x}\, t_{u1} = 500 \times 2 = 1000$ m. True.

$V_1\,(x(t_{u1} = 2), y(t_{u1} = 2))$, $x(t) = 500\, t$, $y(t) = 20\, t - 5t^2$, $V_1\,(1000, 20)$. True.

$x(t) = 500\, t$, $y(t) = 20\, t - 5t^2$, $V_1\,(1000, 20)$. Correct.

P6 Find the speed $v_1(t)$, for $t \in [0, t_{u1}]$.

Method 1. Use $\overline{r}_1(t) = x(t)\,\overline{i} + y(t)\,\overline{j}$ and by derivation on obtains

$\overline{v}_1(t) = x'(t)\,\overline{i} + y'(t)\,\overline{j}$ and $v_1(t) = \sqrt{x'^2(t) + y'^2(t)}$.

Method 2. Use the speed definition $\overline{v_1}(t) = \dfrac{\overline{r_1}(t + \Delta t) - \overline{r_1}(t)}{\Delta t}$, $\Delta t \to 0$.

Optional. Write a table for $v_1(t)$ and the projections v_{1x}, v_{1y}.

For $t > t_{u1}$, the speed of C_1 is $v_1 = 0$.

P6.Apl. $v_1(t)$ is $\overline{v}_1(t) = x'(t)\,\overline{i} + y'(t)\,\overline{j}$ and $v_1(t) = \sqrt{x'^2(t) + y'^2(t)}$

$x(t) = 500\, t$, $y(t) = 20\, t - 5t^2$, $\overline{v}_1(t) = 500\,\overline{i} + (20 - 10t)\,\overline{j}$

$v_1(t) = \sqrt{500^2 + (20 - 10t)^2}$. Construct the table for Γ_1

time t	0	1	$2 = t_{u1}$	$t + \varepsilon$
speed $v_1(t)$	500,4	500,1	500 m/s	0

In V_1 we have $v_{1x} = 500\, m/s$, $v_{1y} = 0$ and for $t > t_{u1}$, $v_1 = 0$.

Etapa 2 (main stage).

The speed of C_2 on Γ_2 is denoted $v_2(t)$, with $v_2(0) = v_{02}$.

The speed angle in V_1 is noted with α_2.

For the trajectory Γ_2 the origin of axis is $O_1 = V_1$.

In V_1 we separately study the projections of the speeds for mobiles C_1 and C_2.

For C_1 the speed projections are v_{c1x} and v_{c1y}, with $v_{c1x} = 0$, $v_{c1y} \neq 0$.

For C_2 the speed projections are v_{0c2x} and v_{0c2y}, with $v_{0c2x} \neq 0$, $v_{0c2y} \neq 0$.

We denote V_2 the top on trajectory Γ_2.

P7 Find the speed projections of C_1 in O_1.

We have $v_{c1x} = 0$ because C_1 is falling down from the heigh $h_{\max}$.

For computing v_{c1y} we use Galileo's formula (like in P3), $v_{c1y}^2 = 2g\,h_{m1}$.

P7.Apl. $v_{c1x} = 0$ and $v_{c1y}^2 = 2g\,h_{m1} = 2\times10\times20 = 400$, $v_{c1y} = 20\ m/s$.

P8 Find the falling time of C_1 on earth, in G, denoted t_{c1}.

Use the equation in *t*, having the form $h_{\max} = v_{c1y}\ t + \frac{1}{2}g\,t^2$ and take the positive root $t_1 = t_{c1}$.

P8.Apl. Find t_{c1}; $h_{\max} = v_{c1y}\ t + \frac{1}{2}g\,t^2$; $20 = 20t + 5t^2$

$t_1 = t_{c1} = 2\sqrt{2} - 2 = 0{,}828\ s$.

P9 Find the speed projections of C_2 in O_1.

We have $v_{0c2x} \neq 0$ and $v_{0c2y} \neq 0$ for Γ_2

Hypothesis: In the point O_1 we have the equal values for c_1, c_2 and the speeds $v_1(t_{u1})$, $v_{c1x} = 0$, $v_{c1y} \neq 0$, $v_{0c2x} \neq 0$, $v_{0c2y} \neq 0$.

The mass and the speed generate impulse. Apply the impulse theorem and obtain successively:

On Ox (with origin in O_1) we have

$(c_1 + c_2)v_1(t_{u1}) = c_1 v_{c1x} + c_2 v_{0c2x}$, $2c_1 v_1(t_{u1}) = c_1 \times 0 + c_1 v_{0c2x}$, $v_{0c2x} = 2v_1(t_{u1})$.

On Oy (with origin in O_1) we have

$(c_1 + c_2)v_{1y} = c_1 v_{c1y} - c_2 v_{0c2y}$, because the speeds v_{c1y} and v_{0c2y} have opposite sense.

Because $v_{1y}=0$ (from P6), we obtain $v_{c1y}=v_{0c2y}$, where v_{c1y} is known from the step P7.

P9.Apl. Find the speed projections of C_2 in O_1.

We have $v_{0c2x}\neq 0$ și $v_{0c2y}\neq 0$, and $c_1=c_2$, $v_1(t_{u1})$, $v_{c1x}=0$, $v_{c1y}=20$, $v_{0c2x}\neq 0$, $v_{0c2y}\neq 0$.

Apply the impulse theorem.

On Ox (with origin in O_1) we have $(c_1+c_2)v_1(t_{u1})=c_1v_{c1x}+c_2v_{0c2x}$, $2c_1v_1(t_{u1})=c_1\times 0+c_1v_{0c2x}$ $v_{0c2x}=2v_1(t_{u1})$.

$v_{0c2x}=2v_1(t_{u1})=2\times 500$, $v_{0c2x}=1000\, m/s$.

On Oy (with origin in O_1) we have

$(c_1+c_2)v_{1y}(t_{u1}=2)=c_1v_{c1y}-c_2v_{0c2y}$, because the speeds v_{c1y} and v_{0c2y} have opposite sense. The result is

$(c_1+c_2)\times 0=c_1v_{c1y}-c_2v_{0c2y}$, $0=c_1v_{c1y}-c_2v_{0c2y}$, $v_{c1y}=v_{0c2y}$.

But $v_{c1y}=20\ m/s$ (from P7) and $v_{c1y}=v_{0c2y}=20\, m/s$.

Hence: $v_1(t_{u1})=500$, $v_{c1x}=0$, $v_{c1y}=20$, $v_{0c2x}=1000$, $v_{0c2y}=20$ m/s.

P10 Find the launch speed and $v_2(0)=v_{02}$ and the angle α_2 on Γ_2.

$$v_{02}=\sqrt{v_{0c2x}^2+v_{0c2y}^2},\ tg\alpha_2=\frac{v_{0c2y}}{v_{0c2x}},\ \alpha_2=arctg\frac{v_{0c2y}}{v_{0c2x}}.$$

P10.Apl. $v_2(0)=v_{02}$ and α_2 on Γ_2 are

$$v_{02}=\sqrt{v_{0c2x}^2+v_{0c2y}^2},\ v_{02}=\sqrt{1000^2+20^2}=1000,2\ m/s$$

$$tg\alpha_2=\frac{v_{0c2y}}{v_{0c2x}}=\frac{20}{1000}=0,02,\ \alpha_2=arctg\frac{v_{0c2y}}{v_{0c2x}}=arctg0,02=1,145763.$$

$\sin\alpha_2=0,02$, $\cos\alpha_2=0,9998$, $\cos^2\alpha_2=0,9996$.

P11 Write the parametrical equations and the explicit equation of Γ_2.

$$\Gamma_2:\ x(t)=v_{02}\cos\alpha_2\, t,\ \ y(t)=v_{02}\sin\alpha_2\, t-\frac{1}{2}g t^2$$

$$\Gamma_2:\ \ y=-\frac{g}{2v_{02}^2\cos^2\alpha_2}x^2+tg\alpha_2\, x,\ \ y=-\frac{g}{2v_{02}^2}x^2+tg\alpha_2\, x.$$

P11.Apl. For Γ_2 we obtain

$$\Gamma_2: \ x(t) = v_{02}\cos\alpha_2\, t, \ \ y(t) = v_{02}\sin\alpha_2\ t - \frac{1}{2} g t^2$$

$$\Gamma_2: \ x(t) = 1000,2\ t, \ \ y(t) = -\frac{1}{2} g t^2$$

$$\Gamma_2: \ \ y = -\frac{g}{2 v_{02}^2 \cos^2\alpha_2} x^2 + tg\alpha_2\, x, \ tg\alpha_2 = \frac{v_{0c2y}}{v_{0c2x}}.$$

$$\Gamma_2: \ y = -\frac{10}{2\times 1000,2^2 \times 0,9998^2} x^2 + 0,02\,x, \ y = -0,000005\,x^2 + 0,02\,x,$$

with the coefficients $a = -0,000005$, $b = 0,02$, $c = 0$.

Verification. **v02=1000,2 a2=0,020002 v0c2x=1000m/s v0c2y=20 sina2=0 cosa2=1 tga2=0,02.**

P12 Find the vertex peak $V_2(x_{m2}, y_{m2})$ of Γ_2.

$$x_{m2} = -\frac{b}{2a} = O_1C, \ \ y_{m2} = -\frac{\Delta}{4a} = CV_2, \ \ a = -\frac{g}{2 v_{02}^2}, \ \ b = tg\alpha_2, \ \ c = 0.$$

Consequence. The range is $b = 2x_{m2} = O_1B = GE.$

$H_{\max} = h_{\max} + y_{m2}$, $H_{\max} = GV_1 + CV_2.$

P12.Apl. Find $V_2(x_{m2}, y_{m2})$ of Γ_2.

Γ_2 >: $y = -0,000005\,x^2 + 0,02\,x$

$$x_{m2} = -\frac{b}{2a}, \ \ y_{m2} = -\frac{\Delta}{4a}, \ \ a = -0,000005, \ \ b = 0,02, \ \ c = 0.$$

$x_{m2} = 2000\ m, \ \ y_{m2} = 20\ m.$

Consequence. $x_{m2} = 2000\ m\ = O_1C = GF = CB = FE$, $y_{m2} = 20\ m = CV_2$.

The range $b = 2x_{m2} = O_1B = GE = 4000\,m.$

$H_{\max} = h_{\max} + y_{m2} = \ 20 + 20 = 40\,m.$

P13 Find the climbing time t_{u2} on Γ_2. Denote $t_{u2} = t(O_1V_2)$.

From the condition $v_y(t) = 0$, with $v_y(t) = v_{02}\sin\alpha_2 - gt$ we obtain $t_{u2} = \dfrac{v_{02}\sin\alpha_2}{g}$ (climbing time).

Find the descending time t_{c2} from V_2 in B.

$t_{u2} = t_{c2}$, $t_{u2} = t(O_1V_2)$, $t_{c2} = t(V_2B)$, and $t(O_1V_2B) = 2t_{u2} = \dfrac{2v_{02}\sin\alpha_2}{g}$.

P13.Apl. $t_{u2} = t(O_1V_2)$, $t_{u2} = \dfrac{v_{02}\sin\alpha_2}{g}$ (descending time).

$$t_{u2} = \frac{v_{02}\sin\alpha_2}{g} = \frac{1000,2\times 0,02}{10} = \frac{20,004}{10} = 2s.$$

Hence $t_{u2} = t(O_1V_2) = t_{c2} = t(V_2B) = 2\,s$, $t(O_1B) = 2+2 = 4s$.

Verification. **v02=1000,2 a2=0,020002 v0c2x=1000m/s v0c2y=20 sina2=0 cosa2=1 tga2=0,02.**

B = *O1B*=4000, *ym*2=20, *H*max=20+20=40 m

P14 Find the partial distance ED and the total distance OD.

First we find the time $t(BD)$ on Γ_2.

$h_{\max} = \dfrac{1}{2}gt^2$, $t = t(BD) = \sqrt{\dfrac{2h_{\max}}{g}}$. Then we use the space formula $ED = v_{0c2x}\,t(BD)$, with the known speed v_{0c2x} and $OD = OG + O_1B + ED =$ $= OG + 2x_{m2} + ED.$

P14.Apl. $h_{\max} = \dfrac{1}{2}gt^2$, $20 = \dfrac{10}{2}t^2$, $t = t(BD) = 2s$. Then $S = vt$ and with $ED = v_{0c2x}\,t(BD)$, $ED = 1000\dfrac{m}{s}2s = 2000\ m$, $ED = 2000\ m$.

$$OD = OG + O_1B + ED = OG + 2x_{m2} + ED$$

$$OD = 1000 + 4000 + 2000 = 7000\ m.$$

5. **Conclusions**

The theory is presented in an unitary form. This way is the result of finding of all common ideas for the four cases.

REFERENCES

[1] Constantin Necşoiu, *Physics problems*, Third edition, Technical Publishing, Bucharest, 1971.

[2] Ioan Druica Zeletin, Armand Popescu, *Mechanical and Acoustic Problems*, Technical Publishing, Bucharest, 1974.

[3] Mihail Sandu, *Collection of physics problems*, Volume 1, Society of Physical and Chemical Sciences, Bucharest,1986.

[4] A. Hristev, V. Falie, D. Manda, *Handbook of Physics, Didactic and Pedagogical Publishing House, Bucharest,* 1990.

[5] T. I. Crețu, *Physics. Theory and problems*, Technical Publishing, Bucharest, 1991.

[6] http://en.wikipedia.org/wiki/Buoyancy

[7] http://en.wikipedia.org/wiki/Drag_(physics)

[8] http://en.wikipedia.org/wiki/Density_of_air .

[9] http://en.wikipedia.org/wiki/Properties_of_water.