ECONOPHYSICS Section

TIME, DURATION, SCALES, FORCING
AND THE RIEMANN HYPOTHESIS:
WHEN THE TIME REBORN

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Abstract. *Is human creation a free act? Mathematics leads us to assert that from the point of view of set theory this question is undecidable. To remove this indecision and act effectively, each creator must build his place of thought, namely a Topos formalizing the tree of the choices of the creator. Beyond the serendipity backed against only chance, the personal Topos is the first place of truly disruptive creativities. These ones are then the children of a forcing. Since there is a germ of mania involving duration in any creative process, it will be shown that the generic unity of the creative category is a differential monad. To account for the complexity of some systems which cannot be understood in the frame of set theory, this monad is based on the "Scale-Duration" couple itself issues on the "Space-Time" of the usual physics. The forcing creative tree then acquires a fractal metric. Surprisingly, the monadic pinning will determine the irreversibility of the creative process and the awareness of the existence of an "arrow of time". We will formalize this point of view using Riemann's zeta functions seen as an entropic foliation of the complex plane truncated by the emergence of a field of virtuality. The analysis will help to show how the will is linked with spectral, even orchestral representations of the being in the world.*

Keywords: *Category Theory. Set Theory. Forcing. Fractal and Hyperbolic Geometries. Riemann Function. a-exponential. Non-Integer Derivatives.*

1. Introduction: Homogeneity or Heterogeneity of Partitions

Time is the parametrization of change and movement. However, as anyone can now experience, for example in a car or in an airplane,

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movement is as nothing as Galileo's principle of relativity asserts. Change is never more than relative. The perception of motion requires a couple of comparable universes. Otherwise, a single universe must be divided into different commensurable sub-universes, but this partition does not fail to take time. In addition, this division can certainly be done in a homogeneous way (for instance two indistinguishable parts) but it can also be done in order to distinguish the different parts. We will then qualify it as inhomogeneous the partition to express that the division is enriched by an asymmetric. If iterations of divisions are applied and if the progression in the division scales is systematically operated in a strictly symmetrical manner, the usual arithmetic equipped with addition and multiplication is the suitable tool for formalizing the progression in the details. The operator successor brings out Newtonian time as the only parameter of a dynamics in which the homogeneous division takes on a spectral content. Indeed, the function $S(x)=log(x)$ expresses the existence of an integral surface (from the unit) whose derivative expresses *stricto sensu* the homogeneous division via the ratio $1/x$. The logarithm is then the relevant function to express the successive scales onto which the partition dynamics are projected. A normalization is possibly ensured by an appropriate calibration of the base of the logarithm. The normalization asymptotically grants a tropical geometry to the main components of the change. The dual function of the logarithm, namely the exponential $x=exp(S)$ ensures the existence of a functorial relation between the product and the categorical co-product (addition) over the set of integers. The unit of arithmetic representation is then the constant $e\in\mathbb{R}$ introducing the power of the continuous in an initially discrete problematic but which consequently becomes extensible to $\mathbb{R}$. Obviously in the case where the division is inhomogeneous, all this beautiful "orchestration" goes out of tune. At the very least, a factor capable to express the non-homogeneous character of the partition of the arithmetic unit and its parts must be used. A smart approach can then bring into play the notion of cut in the field of complex numbers to represent the a-symmetrization. The unit of the whole can then be expressed in a pendular mode by means of the function $\theta exp(2i\tau\omega\pi)$. τ is then a time constant used to normalized a « time » written through an angular variable θ via a pulsation ω. The location of the cutting phase can then be introduced using the sharing in the set of rational numbers $\theta=exp(2iq\pi$ $1>q=n/m\in\mathbb{Q}$. The cut, whose modular nature will be noted, is used to express the inhomogeneity of a foiled partition which only makes sense in relation to a referential homogeneity. The relevance of the logarithm and exponential functions disappears without the disappearance of the addition (succession) / multiplication (change of scale) which structures any need for comparison. Complexity always referees to a closure. Herein the weather is clouding over, the haze disrupts our representations and the storm is rumbling over usual physics; it is then that Time takes precedence over Space [1]. A categoric approach is then required by the heterogeneity of our "scores" in physics.

It has been suggested in previous works that any cognitive approach, any creative process requires the conception of a time extended to the complex number field $\mathbb{C}$ [2.3]. Newtonian time, that of our clocks without alarm or ringing, that of their dials tempered by homogeneous division into hours, minutes, etc must then be revisited in order to understand the temporal spring of "processes opened by located events". Their singularity leads from the universe of our static representations to the universe of dynamic correlations, characterized or not by final attractors. But how to parameterize these dynamics in a relevant way. For geometrical and arithmetic reasons, events combine together within dual ways, segregation and identification, reversibility and irreversibility, entropy and anti-entropy, flat and curved manifolds, etc. [2], because how to avoid referencing and the commensurable enumerations if not ? Their spectrum is a set of durations. The central concept here is "the notion of cutoff of the dial due to a singular event when it appears in a prior given spectrum". The evens modify this spectrum because the constraint of a heterogeneous sharing shows, after Cantor, that any constructible limit becomes evanescent, multiple, arborescent, structured around the event. The iterated heterogeneous partition structures the tree around dualities between the discrete universe of the event and the continuous universe of its space of measurement. The question that arises is that of the edges and their separability as Raymond Devos had clearly seen after Gaston Choquet. Any arithmetic on rational numbers is then naturally incomplete if the singularity is forgotten. Fortunately, the field of complexes being an algebraically closed field and the event being able to be written easily therein, any inhomogeneous partition opened by an event constructs ad infinitum a dual universe in which the phase expressing the heterogeneity imposes an asymmetry of perspectives between the left and right $P : [P_{\alpha}(s)P_{1-\alpha}(1-s)-P_{1-\alpha}(1-s)P_{\alpha}(s)]\neq 0$ with $s=\alpha\pm i\theta$ therefore something like a vague idea of an irreversible time; an idea that is all the more vague as time cannot simply formalize the relationship with the event. A question naturally emerges, however: since, in the heterogeneous case, the perspective changes according to the direction assigned to the relation with the space direction then, according to which modalities the usual Newtonian time parameter, able to be written as the absolute value of an angle of rotation $\pm i\theta$ would it remain "THE" benchmark of our relationship to change? How can such a permanence be justified other than by considerations of conscience having a priori nothing in common with an objective science taking into account the event?

We owe to the category theory built on the notions of arrow associated with object as well as to Riemann's zeta function which redesigns all the arithmetic by taking into account power laws, the formalization of the problem of inhomogeneous partition such as suggested above [2,3]. Indeed, for categorical reasons which involves in the same steps the arithmetic, the topology, the logic and the analysis, this function universally interpolates the set of dualities[1] structuring the pendular unit of the field of complex numbers heterogenized by an internal cutoff (here unique) over the modulus. Let us briefly recall why by considering the universe of natural numbers $\mathbb{N}$, on which the zeta function is built

2. Zeta function representing a Topos over $\mathbb{N}$

By authorizing the definition of exponentiation in Grothendieck's Topoi [4,5] which then appears as the locus of a virtual algebra [6], – i.e. of an algebra of systems open over the singular event –, the notion of categorical product, therefore of partition, confers a formalism on the set power operator $X \rightarrow \wp(X)$. Unlike set sub-objects, "categorical sub-objects" can be enriched for example by fibration and stacking. Heterogeneity can then be considered as a particular fibration (stacking). Let us observe, among the categorical objects are the arrows and functors themselves. They can therefore give rise to the same partitions and associated algebras (in the frame of fractions theory). In this extended context, it is by calling on a classifier, which can ultimately be based on the set of natural numbers $\mathbb{N}$, it becomes possible to label these sub-objects, while they themselves can generate new ones, these new objects having themselves to be labeled and this ad libitum in a process which can be simply recursive, repetitive. Thus each sub-object, in $\mathbb{N}$ can even be associated to set of $\mathbb{N}$ itself. The central notion here becomes that of the condition of "tree genericity". If the sub-object expresses a cardinality, the labeling in $\mathbb{N}$ brings out the quadratic self-similarity of the set of natural numbers $\mathbb{N}x\mathbb{N}=\mathbb{N}$, and after stacking the not natural but relevant application $\mathbb{N}\rightarrow 1$ allowing the

[1] More precisely it will shown that interpolation requires, through a catagorical square diagram, the couple of symmetric zeta function

pullback[2]. Thus ℕ has a dual dimension, namely 2 and not 1. The power law defining the Riemann zeta function $\zeta_\alpha(s)$ based on the set $\{1/n^\alpha\}$ obviously responds to these quadratic constraints, (in octaves) while also responding to the constraints imposed by the fractal factor "α". The central question is then that of the entanglement conveyed by $\zeta_\alpha(s)$ as well as his role of intercessor. Beyond that, while the Topos confers quasi-set properties on the objects of our categorical representations, it is by guaranteeing a formal relevance of the notions of inductive and projective limits that the zeta function ensures via the functional relation $\zeta_\alpha(s)=F|[\ \zeta_{1-\alpha}(1-s)]$ the closure of the field of complexes ℂ equipped with heterogeneity (cutoff). This closure ensures the duality of categorical limits, including asymmetric ones. In this, the zeta function carries within it classes of ambiguities, of correlations which deserve to be understood in particular with the light of the Riemann conjecture which opens explicitly on the question of the analycity opportunities of zeta. But precisely these ambiguities can be exploited exactly as they were using Galois theories. Thus, as demonstrated using the Voronin's theorem [7], it is through the complex factor «$i\theta$ of s», self-parameterizing an additional stacking on the zeta function by using recursive action, that the universality of this function makes sense as an approximation of any holomorphic function [3].

3. To a logic of creation from Topos

All creation is event and singularity. Beyond the serendipity well known to be based on chance, the logic of predicates (namely the principle of the excluded third middle, the principle of double negation, Morgan's theorem, Russell's paradox, etc.), is empirically far too elementary to suffice for a formalization of the creative instant in its context [5]. The primary logic is directly attached to Boolean algebras and to strictly nominalist binary choices (true-false, white-black, 0-1, pipe-chair, or vice versa), namely a homogeneous partition and this by means of set theory under the ZFC axioms. The avoidance of the set theory constraints, by means of Grothendieck's Topoi which are capable of enriching at will, any

[2] The induction $n\to n+1\ldots\to\infty$ leads to a step forward in the succession to be considered as nothing since the prospect of succession does not change. So the succession is not a cut in ℕ because the countable set is the categorical co-kernel of the successor kernel. It is not the same with ℝ for which the operator succession is an explicit cut each real number being able to be considered as a prime number.

[3] See C. Voronine's works : $|\zeta(s+i\theta)-\sum_{n\in\mathbb{N}}(s^n)|_{\theta\to\infty}<\varepsilon$

category, makes it possible to go beyond the nominalist symmetries which limit the elementary logic[4] to a particular form of homogeneity of our partitions. Defined in the theory of categories, the heterogeneous sharing[5] as a sub-category filtered and / or bundled as a set of sub open sets gives rise to so-called a-symmetric and "phased" logics, intuitionist, multivalued, ultimately not fuzzy but modal. The creator is then led to use the subtleties of the included third [8] to think of entangled and complex situations. Or for topological reasons[6], call for this modal logic is crucial for those who want to define a new concept of time including the quadratic referential form which we will call Noetherian energy. Indeed, to think and operate an asymmetrical partition or not requires an irreducible duration associable with a given reference, for example a clock that need energy. In practice, the request for included third middle based on event and carried for being defined up to infinite limits requires defining the concepts of "density" and "adhesion" in order to understand the status of an evanescent present seen as a particular interpolation between a spectral past and a future. multi-hypothetical[7]; a present therefore straddling the hyperbolic saddle of our will (whatever the prime ideals) and of our contingencies (there is an arithmetic constraint linking all succession (+) to a law of scale (x)). This results in a link, of which we will understand the universality further, with the zeta function seen as a mathematical bridge coupling multiple modal dualities $\exists(\mathbb{N}, s)\vdash\zeta(s)\vdash\forall(\Im, s)$[8]. Thus far from the cliché of mental chaos that is often attributed to creative minds and provided that the logic in play is enlarged within the framework of Grothendieck's Topoi endowed with relevant classifiers [8], creativity must be associated with a "spectral rationality". Therefore multi-scale, based on the definition of a duration expressed in a field of complex numbers capable of accepting the event. The irreversibility of time then emerges naturally from the a-symmetry of

4 As well as Cauchy convergence constraints associated with the locally Hausdorff topology According to this topology it is always possible to find separate neighborhoods

5 This sharing comes from a partial order relationship.

6 In practice, the intersection of two ordered sets brings into play $\mathbb{N} \cap \mathbb{N}$ which prefigures the concept of open in topology. Moreover, there is a close link between modal logic and topology as was established by McKinsey-Tarski then Jónsson-Tarski.

7 This distinction requires in fact to implement the operators "it is necessary $\Box$ » and « it is possible $\Diamond$»

8 We owe the proof of the universality of parametric interpolation methods to Alexandrer Craig in logic (1957) and to Harry R. Pitt in category theory (1983).

cuts and personal choices made by the creative spirit in a set of opportunities.

This rational approach to creativity has been approached fairly recently in a light that does not involve category theory. Thus, innovation engineering work, based on forcing theories[9] allow, according to Armand Hachuel and his collaborators [10,11], to think of procedures for transgressing mental truisms by backing creative rationality to the local extension of set theory according to Paul Cohen. The techniques initially developed to resolve Cantor's paradox according to which any countable set can contain sets of cardinality equal to or greater than zero, then bring into play generic tree-like processes to subsume the ZFC axioms [12,13]. Subtle similarities between both approaches (set or / and categorical) then challenge the authors. Do the approaches have categorical links? What role does the tree structure and its possible self-similarity play? Do the links relate only to the work of Cohen (forcing first stage 1964 [14]) or do they relate to their extensions through the works of Martin, Solovay or / and Takeuti (second stage of forcing [15-16] as commensurability)? Under the hypothesis of a link, what role could the universality of the Riemann zeta function play in the categorical formalization of forcing operations? These set questions are surprisingly open into arithmetic and algebraic problems entangled between dual categories. Let's see how.

4. Back to arithmetic

Recall that arithmetic is based first on the characteristics of the ordered set of integers $\mathbb{N}$. Addition equipped the set of a successor operator *« s »* (($\mathbb{N}$,+) is an additive monoid characterized by a total order) while the multiplication equips $\mathbb{N}$ of sharing operator (divisibility) namely a scaling. It is through the procedures of partitions, of scaling, and the extension of the rings using complex fields, that the constraints of self-similarity will be played out. This self-similarity interests us as a framework for simplifying a monadic approach to time. The natural characteristics in $\mathbb{N}$ are projected onto the set of so-called prime numbers by means of the fundamental theorem of arithmetic. The prime numbers equip $\mathbb{N}$ with a partial order of

[9] There is a link between forcing and the principle of genericity. More precisely, forcing can be associated with the principle of enumeration of sets of sets of sub-objects *ad libitum*. A typical example of forcing is the design of the imaginary number $(-1)^{1/2}$ and the creation of the field of complex numbers by stacking. In this corp the Riemann zeta function allows to reconstruct the continuum from the discrete.

divisibility seen as a categorical arrow. Divisibility allows us to build from $\mathbb{N}$, the set $\mathbb{Q}$ of rational numbers after possibly passing through $\mathbb{Z}$ (commutative ring). The set $\mathbb{Q}$ is characterized by its dense character but not connected in the set of real numbers $\mathbb{R}$[10]; however, their vicinities do not have any extrema. Except therefore to express the sets using filters and categorical ultrafilters, the theory of networks and the Heyting algebra (generalization of algebras using GCD and LCM), in other words, an algebra on the correlations (arrows) between ideal subcategories, cannot be applied ex nihilo to rational data. In practice, however, the extension can fortunately operate by building inclusive structures by progressive saturation according to a recursive game and therefore by enriching step by step, the structures starting from subcategories by means of internal dualities such as those using quadratic self-similarity of natural numbers $\mathbb{N}\text{x}\mathbb{N}=\mathbb{N}+\mathbb{N}=\mathbb{N}$[11]. This relation is likely to be itself projected over the set of prime numbers $\mathfrak{I}$. Thus, the divisibility problem can be considered by using the sets of cardinal n, $k \in \mathbb{N}$ capable of fiberizing $\mathbb{N}$ itself as mentioned above with regard to the techniques of *ad libitum* classifications. Self-similarity, which is necessarily based on a differential monad (see appendix 1, figure 4) then naturally opens the door to the use of Heyting algebras and Killing forms on filtered subcategories and stacked upper categories. These last operations introduce arithmetic, algebraic and topological intricacies between scales. Ultimately, whatever the complexity of the entanglements, the monad contains all of the arithmetic correlations between the initial (limit) seed and final (colimit) attractor, and the self-similarity is one particular expression.

This "differential" approach in a total and partial order in the scales opens very opportunely both on the properties of the zeta function $\zeta_\alpha(s)$ and on the cyclotomy operators [17,19][12] that is to say on the set of rational orbits adapted to the topology of openings of type $\text{O}=\mathbb{N}\cap\mathbb{N}$ locally quadratic and globally fractional to the order α, that is to say both orders of fractionality of the interpolation saddle local and analytically condensed in the zeta function. Although bent by the Cantorian metric (α) naturally incomplete, the support manifold of the dynamics is, according to a certain fold of the saddle, a surface (namely a smooth fractal). The examination of the constructive method makes it possible to understand why and how the

[10] We don't take into account here the algebraic numbers.

[11] in categorical terms, this constitutes a space of Chu, the duality in question falling within what is commonly referred to as the duality of Isbell [5].

[12] Note the role of phase locking between systems already pointed out by Michel Planat [17-19]. The importance of this role will be seen in the remainder of the analysis.

extension of $\mathbb{N}$, to $\mathbb{Z}$ and $\mathbb{Q}$, then $\mathbb{C}$ (carefully avoiding $\mathbb{R}$[13]) allows to keep the quadratic self-similarity properties of $\mathbb{N}$ (in fact its hyperbolic representation) while extending the recursion ensuring self-similarity via a cut in the complex number field $\mathbb{C}$. Now the Riemann zeta function $\zeta_\alpha(s)$ carries within it this double constructability[14]. More precisely, it has been shown that, upstream of any considerations of the quadratic characteristic of $\mathbb{N}$, the zeta function bases its construction on a basis which is none other than the fractional dynamics called by us [3, 20, 21] α-exponential[15]. The fibration (stacking) via $\mathbb{N}$ of this spectral base indeed leads to the construction of the zeta function in particular by highlighting the monadic (initial object) / Integral (final object) and elliptical (double periodicity identity and scaling) status of the needs for defining the durations base of spectra. According to standard physics this ellipticity is too quickly summarized under the misleading name of "Space-Time" reducing the problem of double dualities in the order of the scales of categorically indexable sub-objects to a core of duality reduced to a unity by the notion of velocity (see Figure 2).

5. Zeta Function. Non-Integer Operators. Phase locking

By attempting to answer the questions opened by the preceding affirmations this note sheds a new light on the question of the ambiguous status of the Newtonian time continuum but also on “arrow of thermal time” proposed in the Alain Connes et al. works [20-23]. The analysis is simplified here under the assumption of an self-similar genericity (the tree structure generating subcategories is supposed Fractal [24]) and of a recursion of the differential operators involved in the order of the scales (Annexe 1 Figure 4), in other words, in category theory, of their monadic character. Under this assumption bridges are created between topology,

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[13] The issue at stake is the continuous representation of the whole $\mathbb{N}$, a problem that calls into question the continuity of the successor operator. The absence of a cut (commutativity of successors) which is not conceivable in $\mathbb{R}$ can be in $\mathbb{C}$.

[14] These operations intersect the determinisms of a quadratic norm and a phase specific to complex numbers. We find here the concept of Fractal-Lisses involving isometries. In the simplest case, the phase angle which defines the hyperbolic curvature associated with the fractality is given by: $\varphi = \pm(1-\alpha)\pi/2$.

[15] We call α-dynamics and more precisely α-exponential and open dynamics whose transfer function is given by the spectral equation $Z(\omega)\sim 1/[1+(1/(i\omega\tau)^{\alpha}]$. See in this regard the work published for 40 years by the authors.

and analysis using fractional differential operators [25]. It is recalled that the use of non-integer order operators was originated (Annex 1-Figure 4) in the absence of relevance of standard derivation for all the dynamics performing in fractal geometries [25-29]. This mismatch is linked to the fact that if "any differentiable function is continuous" it is established that "any continuous function is not necessarily differentiable" which very naturally leads to the design of operators adapted to self-similarity and capable of expressing discrete differential behavior[16] by deploying a bundle structure of the tangent cone. As shown by experimental dynamic analyzes in self similar geometries[17], non-integer operators are perfectly suited to the problems linking the local and the global, including if expressed through the zeta function [4]. This link is, in both cases, achieved by means of a phase locking exerted by a non-integer metric as well as in the *L* functions[18]. This characteristic which resonates implicitly with the relation $\mathbb{N}x\mathbb{N} = \mathbb{N}$ explains that the zeta function $\zeta_\alpha(s)$ can be built by using a stacking [30-32] under conditions specified elsewhere [4,33,34] from α-exponential. This type of dynamics can be associated with fractal geometries with metrics $d=1/\alpha$. The authors can then associate the open character of the dynamics on such geometries (divergent series of subsets) with the internal and limits arithmetic correlations of the zeta function. Among these correlations we will note the structure of the local saddle, associated to a hyperbolic topology, folded and a-symmetrized by the double quadratic metric on $\mathbb{N}$ and fractional by means of "α". As such, the zeta function is the expression of a functorial and topological bridge (the saddle) between the total order (that of arithmetic succession) entangled with the partial order associated with changes of non-integer metric when the scaling proceeds. The internal duality of the topology makes it possible not only to distinguish but also to add to the representation (i) the closed

[16] The authors relied from the outset on the work of Gustave Choquet, work which found successors with the study of the role of convergence in topology and the role of adhesion, among others by Szymon Doleki and Frédéric Mynard

[17] … from which it follows that the zeta function is autosimilar.

[18] It will be observed that, as it was underlined by Jocelyn Sabatier and co authors, the physical meaning of these operators can appear very questionable according to a standard vision of physics. The authors confirm this critique within the framework of set theory under ZFC axioms. The relevance of the critical remarks, however, does not call into question the heuristic effectiveness of the use of fractional operators, nor the rigor of their design within the framework of category theory (Monads, Filters, Adjunctions, Kan extension, etc.)

dynamics associated with filling fractal geometries (example Peano curves $d = 2$ in the plane; quantum case with thermal uncertainty due the quadratic metric of standards) and (ii) the open dynamics associated with the co-space associated to any fractal geometries. The use of the co-dimension originates in the need of completion [35] (example Sierpinsky curves $d<2$ in the plane. Experimental example: the electrochemical dynamics of electrodes of batteries; the mechanics of certain heterogeneous materials). The difficulty then arises from the definition of the categorical adjunction between the co-product and the product when two different kinds of scaling entangle a total order and a partial order. In this regard, we recall that the zeta function derives from two definitions arithmetically attached to the field of integers $\mathbb{N}$, either as a sum or co-product and induction $\sum(\mathbb{N},s)$ or as a product $\prod(\Im,s)$ with $\zeta_\alpha(s)$ as intercessor; then, we can write: $\sum_{n\in\mathbb{N}}(1/n)s=\zeta\alpha(s)=\prod_{pi}\in\Im(1/[1+(1/p_i)s]$ with $p_i\in\Im$ the set of prime numbers. Here we find an analog of the Graig theorem (logic) and Pitt theorem (category) in which equality would be seen as an adjunction in $\mathbb{C}$ due to the role of $s = \alpha + i\theta$. The double role of divisibility arithmetic appears very clearly here in the conjugation of the current countable and discrete $\aleph_0$(n) with ($n\pi\in\mathbb{N}$) category allied to a virtual continuous $\aleph_1$(s) as analytical limit (with the role of $\alpha\log(n)$, π and the Neper constant « e »), but it also appears in a tree structure as a cascade of sums in the third middle[19]. It is in practice the dual adjunction based on product / co-product // completeness / incompleteness // continuous / discrete // projection / induction // injection / surjection, taking into account the singularity of the event "α", that formalizes categorically the universality of the zeta function as intercessor between sum (co-product) and categorical product itself dual. The definitions of this zeta function therefore implicitly fold within the dual covering, with quadratic metrics on the one hand, self-similar, formalized by the factor "α" on the other hand, i.e. two distinct classes of adjunct self-similarity, of which we can conjecture that the first is probably referential since the second must be completed for being closed and not the the first. We come, as the work already published [4] shows, to an extended intercession of $\zeta_\alpha(s)$ concerning any parameterized predicate: $\exists(\mathbb{N},s)\vdash\ \zeta_\alpha(s)\ \vdash\forall(\Im,1-s)$, very subtly taking into account the algebraic closure of the complex plane

[19] …as for example the product $(1-p^{-s})^{-1}=1+p^{-s}+p^{-2s}+...$ leads to an expression reduced to a sum of products of highly entangled prime numbers.

a-symmetrized by the event « α » and open over a tangent cone and determined by the co-metric $(1-\alpha)$.

Given the above, the asymmetry of metric of the local saddle, based on co-dimensions, can be took off; this is precisely the constraint forced by the Riemann hypothesis. The constraint is reduced to a phase locking which is no longer general $½ < \alpha < 1$ but specific $\alpha=1/2$ (namely being quadratic the metrics in play simply express that $(\mathbb{N}x\mathbb{N})^2=\mathbb{N}$ [20]). This relation gives a flaky Riemannian metric to the saddle points. If the symmetry breaking homogenizes the metric, it does not for all that remove the arithmetic heterogeneity, because it plays in the background of the addition with a total order based on the co-product upon $\mathbb{N}$ but also of the partial order of $\Im$ resulting from the divisibility in scale on the ideal basis of prime numbers. The local saddle remains representative of the product / co-product adjunction. Obviously at this stage the question arises of the countable character or not of the component produced during the definition of zeta, therefore in particular of its non-trivial zeros of zeta function. We will come back later to that aspect.

Reduced to a question of a particular metric d=2 [4], the singularity of a phase factor $\alpha = 1/2$ becomes perfectly relevant to account for the Newtonian time (since the ½-exponential spectral function basing the zeta fibration has an inverse Fourier transform). Conversely, this result suggests a questioning of the concept of absolute time in the general case $½ < \alpha < 1$. Such a questioning goes well beyond the mere irreversibility of the Connes thermal time[21] [20-23] whose irreversibility complies the Riemann hypothesis and therefore depends on chance, that is to say from two ½-exponentials in adjunction. According to above approach thermal time then comes from an uncertainty over the metric d=2. From that point let us observe that if chance is self-similar and gives rise to an arrow of time, we emphasize that any self-similar dynamic does not necessarily arise from chance but from the dynamics of partitions, most often heterogeneous, physically expressed by the balance between dissipation and utilization of energy flows.

[20] The Riemann Hypothesis indeed suggests in this context a phase locking similar to that which a white noise ($d= 1/\alpha$ with $\alpha=2$) would impose on experimental data, hence the role of chance and probabilities for example in the access to quantum data the zeta function establishing the appropriate bridge between the discrete universe and the continuous universe.

[21] Subject to the existence of thermodynamic state functions, temperature is the extensive quantity associated with entropy.

The analysis of the structure of the local saddle point of the hyperbolic geometries associated with these partitions, leads to think the status of the time as a functor (Figure 3) for instance formalized by means of the function $\mathrm{Exp}(\alpha\pm i\theta)$ with $½<\alpha<1$ using the parameter to run the succession $\pm\theta\in\mathbb{N}, \mathbb{Q}, \mathbb{R}$, taking into account the quadratic self-similarity of any manipulation of integers, that can then be assimilated to the concept of durations. Associated with the fibration (stacking) of self-similar dynamic functions of the α-exponential type, the stacking parameter « ±iθ » giving birth to zeta Riemann function à $\zeta_\alpha(s)\neq 0$, expresses a kind of integration; this last taking into account a phase at the origin, thus plays with clear asymmetry of the complex plane created by the event « α ».

The main point then as to the dual unity of the self-similar process (terminal object analogous to the final object; identity of limits and co-limits, etc.) implies the closure of the analysis by design of a functor of "set derivation » giving abundle content to the tangent cone[22]. The derived functor makes it possible to close the monadic point of view by using the transformation $s\longrightarrow(1-s)$ as shown in Figure 4 of Annex 1, based on the fact that any integer is necessarily the product of two of them $n=(\omega\tau)$, themselves able to be divided into multiple ways on the basis of prime numbers. The defining the tangent cone of any spectral α-exponential dynamic gives rise a co-limit spectral distribution, named by analogy (1–α)-exponential. We will observe in figures 3 and 4 that this last it then closes over the unity metric, the cantor set of non-integer metric "α" by closing the α-exponential dynamics on a pure exponential which then appears to be the merest standardization of dual issues. The functor naturally opens the way to the constructability of the dual Riemann function, leading the birth of $\zeta_{1-\alpha}(1-s)\neq 0$ according to an inverse stacking using $\mp\theta\in\mathbb{N},\mathbb{Q},\mathbb{R}$. The asymmetry of the complex plane is, among other approach, expressed via the functional $\mathrm{F}{:}\zeta_\alpha(s)=\mathrm{F}[\zeta_{1-\alpha}(1-s)]$. This relation then acquires the content of an Isbell duality [5]. This explains the incompleteness of the embedding of the folded topological saddle in the space of octaves mentioned above[23]. This relation is incidentally associable with

[22] It takes the place of the notion of velocity for the derivable continuous function.

[23] The Riemann hypothesis treatment must take into account the following: [(1– α)–exp] $\Rightarrow \zeta_{(1/2)+\varepsilon}(1-s)$ while [α-exp] $\Rightarrow \zeta_{(1/2)-\varepsilon}(s)$; both size of the complex plan cutoff have not the same categorical content because the inversion of theta, is not associated to the inversion of the complex plane axis. This peculiarity is not expressed in the context of quantum mechanics through a dualities involved by the only conjugation of complex variables $s^* = (1-s)$.

Euler's Totient function which then qualifies in dual ways the set obstructions to inside correlations.

6. Kan extension. Symmetry breaking. Commutativity

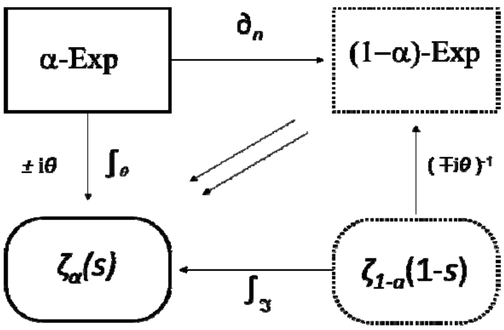

Figure 1. Schematic view of the Kan expansion and the square of interpolation.

Starting from the above, we now have an integration functor and a α-exponential set derivation functor, both pointing respectively in the general case to $\zeta(s)\neq 0$ and to (1-α)-exponential itself linked to $\zeta(1-s)\neq 0$ [4]. The factor "α", involving an asymmetry of the complex plane is clearly brought into play. However if, as assumed, time can be formalized by the factor of stacking $\pm i\theta \in \mathbb{N}, \mathbb{Q}, \mathbb{R}$, we do not yet see emerging the slightest irreversibility of "time" here. Let us therefore continue the categorical analysis by examining the role of "α" on the commutativity of the integration functor along the stacking « $\int_\theta$ » with respect to set derivation « ∂_n » namely the formal expression: $[\int_\theta,\partial_n - \partial_n,\int_\theta]$.

By backing reasoning to the central role played by adjunction and square categories and by using the naturally dual functor called Extension Kan, – operator pointing to the problem of functorial commutativity –, we can prove that the Riemann hypothesis $\zeta_{1/2}(s)=0$ expresses nothing else than the commutativity of the derivative functor (partition and local fibration of an integer via prime numbers) and the stacking functor considered as an integration of α-fractional dynamics (here approach via the factors $exp(\pm 2i\pi\theta log(n))$). In other words $\zeta_{1/2}(s)=0$ leads to an identification of the right and left Kan expansion with both functors [4,32,34]. This indifference explains why the non-trivial zeros come into play as an ideal set of projection (x) / induction (+) of the zeta function. The temporal content of commutativity is then based on a nominalist equivalence between $\pm i\theta \sim \mp i\theta$ relation expressing the indifference of the representation to the orientation of the complex plane.

Conversely, if the Kan expansion at left is different from the Kan expansion at right, the commutator is different from zero $[\int_\theta, \partial_n - \partial_n, \int_\theta] \neq 0$ (and the square adjunction diagram taking into account $(1-\alpha)$ -tangent cone does not commute). The constructability of $\zeta_\alpha(s) \neq 0$ and of $\zeta_{1-\alpha}(1\text{-}s) \neq 0$ implies a taking into account of the dissymmetry induced by the cutoff of the complex plane and then $\pm i\theta \neq \mp i\theta$. The sign loses its nominalist status because the orientation of the complex plane cannot be reduced to an inversion of the sign of rotation of θ. Via the stacking this relation subsumes the functional relation F: $\zeta_\alpha(s) = F\,[\zeta_{1-\alpha}(1\text{-}s)]$ from the a-exponential and its tangent cone via the $(1-\alpha)$-exponential. The irreversibility of time then emerges formally from a problem of the derivation and integration commutativity involving either zeta as a sum or zeta as a product of prime numbers in a linear structure of attributes: $\sum_{\aleph 0}(\mathbb{N}, s) \vdash \zeta(s) \vdash \prod_{\aleph 1}(\Im, 1-s), \quad n)$; this zeta intercession emphases the difference of cardinality between both external terms as an interpretation of the irreversibility of the operation of succession $\sigma \rightarrow \sigma + 1$ herein for scaling[24]. Conversely the Riemann hypothesis for $\zeta(s)$=0, implicitly expressed by the relation $\mathbb{N}\text{x}\mathbb{N}=\mathbb{N}$. While one raison of irreversibility is probably the fact that set based on primes mostly possesses the power of continuous $\aleph_1$ in the critical domain, Riemann hypothesis could involved a countable character $\aleph_0$ of the set of subsets of tangent sheaves if α=1/2. The Riemann hypothesis therefore appears to be a form of parametric completion admittedly associated with a suitable symmetry breaking of the complex plane, but in addition involving that the dynamic orbits associated with the zeta product writing, must then be dense, separable and countable in a 2D embedding space (smooth 2D fractals). All this implies that, except for the very particular case in accordance with the Riemann hypothesis, the time operator «θ», not be an inverse of the spectral parameter « n » or, expressed in current tongue: *except to be immersed in a totally deterministic world, the action carried out from a representation of the world posed a priori is, in no way equivalent to a will to act in search of this same representation in the absence of a priori.*

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24 This irreversible succession takes into account a cutoff analogous to the passage from one prime number to the next. This succession is in no way similar to a succession in $\mathbb{N}$, known not to be similar to a succession in $\mathbb{R}$ (each point of $\mathbb{R}$ is analogous to a prime number $\Im$). The resulting orientation, associated with $\aleph_1$ leads to consider the co-limit of the cone of progression in the scale equivalent to a real number.

The impossibility of parameterizing the two approaches by means of the same universal and absolute Newtonian time is partly due that, except in special cases α =1/2, the change cannot be reduced to a countable set of separable orbits (Hausdorff space). The entanglement of correlations coming from divisibility on the primes set, the is, moreover, the reason why[25], Newtonian time appears as a very poor concept; a concept badly adapted to the constraints of renormalization [37]. However, as Poincaré noted already in his scientific couriers, there is a quirk in quantum physics, namely that the revolution which was initiated by Planck in 1905 with the discretization of the energy levels of the black body, was associated, it seems without any questioning, at a Newtonian absolute time thus giving pride of place to the continuous, for emerging in fine with the functions of probability. This oddity whose origin can be understood through above categorical analysis (Riemann Hypothesis), opens a perspective upon an universe infinitely more intricated than Quantum Mechanics in which the concept of time would be banned. As the second author notes in his lectures on quantum computation, understanding the above information requires a panoramic and transdisciplinary vision of the arithmetic, topology, measure theory etc, within the extended framework of category theory and not in the sole framework of sets theory under ZFC axioms. The theory of sets appears too poor to analyse the whole implications of the Riemann hypothesis.

7. Open questions for students in Quantum Computing (ESIEA Lectures)

The links between the quantum structure of matter and the zeta function are rather well established in a classical way [38,39]. it is the same when regularization techniques is required [40]. However, the links between irreversible thermodynamics of complex systems and Riemann conjecture remains to be established and understood. According with above arguing, the function zeta as interpolation functors infers a continuous approach of the set of integers while the breaking of symmetry seems to cancel this hypothec to return to a duality over a couple of countable set; but along what kind of ways? Let us observe that if this switch is confirmed, the Riemann hypothesis can only be an additional

[25] … although at that time now old without anticipating the above links with the Riemann zeta function

axiom with respect to the set of ZFC axioms. This observation suggests that the conjecture is not provable from the use of the only set axioms ZFC. More precisely, the symmetry breaking would be a particular formulation of Martin's axiom playing with the smallest uncountable ordinal ω_1 According to Solovay and Martin $(2^{\aleph_0}\simeq\aleph_0)$ as limit and co-limit this cardinal can take either the power of the continuous or the power of the countable (Yoneda lemma). In the last case, by, using $\mathbb{N}$, it opens the ability to index each element of subsets in distinct ideal classes within any kind of set. $\mathbb{N}$ then takes the structure of a Russian doll named matryoshka. In accordance with this last hypothesis, because $\mathbb{N}$ can be seen as co-nucleus of the operator of succession and due to the quadratic self-similarity $\mathbb{N}\mathrm{x}\mathbb{N} = \mathbb{N}$, this operator must be reversible (a couple of data turns like one). Therefore, it becomes natural to associate with quantum mechanics an absolute and reversible Newtonian time as the inverse of a Fourier quadratic form thus giving credit to velocity concept. At this stage, the Riemann hypothesis is clearly related to forcing techniques and their tree structures (any couple of events have a common supremum) seen as genericity conditions for finding distinguished states (for zeta, the isolated non-trivial zeros associated with the set of prime numbers). This hypothesis which explicitly states that the substructures can be indexed by means of a classifier thus appears to be only an axiom in ZFC[26] like the choice axiom.

So we have to go back to the basics, here to the theory of ordinals and cardinals to understand the universal status of the zeta function (and therefore also of *L* functions) in both mathematics and physics. Three entities must be considered: (i) first of all aleph zero $\aleph_0$, since all reasoning in spectral space is mainly based on the set of natural numbers; moreover, by construction (ii) the zeta function brings into play the power of the continuum, aleph one, $\aleph_1$ by means of the parameter "s", a complex number. This imperative does not change the issue of cardinality of the categoric objects considered and finally there is a third entity, (iii) named omega 1 (ω_1), that is to say the smallest no countable ordinal. This ordinal corresponds to the number of possible orbits in $\mathbb{N}$, a number which

[26] Forcing techniques do not find their origin only in the works of Cohen (1964) but also in those of "Martin-Solovay" (see Internal Cohen extension Ann. Maths Logic 1970) Nevertheless the axiom of Martin aiming at the character countable of the set of subsets, true if the continuum assumption is true, is independent of the assumption proved by Paul Cohen, because it could be assumed to be true even if the continuum assumption was false.

involves the operators of divisibility, fibration (stacking) and ultimately cyclotomic properties. The consequences of coupling are then the following: (i) either omega one is equal to aleph one, and we are dealing with the continuum hypothesis, or else (ii) both properties are different, the all subsets of zeta are countable and, in this case, the ZFC theory is insufficient to deal with the Riemann hypothesis. In this second case, of the topology is a pre-topology[27], the only solution to force indexing is to introduce an additional axiom. This is when Martin's work comes into play. His axiom of internal extension aims to extend a classical theorem, the Rasiowa-Sikorski theorem associated with the theory of Boolean algebras[28]. However, by carefully re-reading the statement of this result, it appears obvious that Martin's axiom is a reformulation of the Riemann hypothesis, which itself is only a disguised formulation (of a categorical nature) of the fundamental theorem. arithmetic, namely that every integer admits a unique decomposition into the product of powers of integer prime numbers. The links between the zeta function, the set of none trivial zeros, the set of prime numbers and the mathematical structure of quantum mechanics [38,39] then appear to be fully natural[29].

8. When Time Reborn!

Based on the above approaches, it becomes possible to give heuristic content to the mathematical abstractions regarded. Theoretical analysis leads quite naturally to a radical conclusion: Newtonian time is not a generic notion. It is at most a benchmark parameter. However, at this step, its status of limit related to categoric filters and ultra-filters themselves associated with tree structures (here constrained by self-similarity) is far from clear. To understand

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[27] More precisely, Vietori-type pre-topology which plays a central role in partitions and filters.

[28] Let $(P, \leq)$ be a poset and $p \in P$. If D is a countable family of dense subsets of P then there exists a D-generic filters F in P such that $p \in F$.

[29] The fact that zeta relates directly to the technique of forcing is extremely interesting for understanding quantum formalism. The Japanese mathematician Gaisi Takeuti established around 1980 a formalism for quantum mechanics precisely based on forcing; this work was extended more recently by another Japanese mathematician Ozawa. By formally establishing that the Riemann hypothesis is an axiom, the approach developed opens up useful new perspectives in quantum computing. What indeed does Martin-Riemann's axiom say very concretely in quantum terms: it says that a neutron is not an electron.

its content, it must first be stated that the notion of duration seems, on the contrary, a generic and universal notion associated with the notions of frequency and pulsation, notions that unfold in fractal geometry from a monadic category based on a bi-division of the ℕ-set (Annex 1 Figure 4). The concept of frequency $\nu \in \mathbb{N}$ (i.e. of standard and a-dimensioned pulsations: $\nu = wt$) is used to arithmetically and continuously express these notions in the form of a pendular oscillation $exp(iwt)$ written in the complex plan. Further, expected quadratic self-similarity $\mathbb{N}x\mathbb{N}=\mathbb{N}$ justifying the limitation of the analyzes within the framework of only one octave[30] itself associable with the interval [0,1], the concept of duration is at least intrinsically dual (for example product of a pulsation by an inverse quantity commonly called "time constant" or relaxation time). In practice, time is called the inverse of a pulsation, but then it remains to categorically define the rules of divisibility allowing this concept to be reached via the concept of frequency, when the partition is non-linear (eventually fractal). The advantage of the pendular expression is that the arithmetic character in no way excludes a continuity, including nonlinear. Linearization of this movement is useful but not necessary. Subject to the imperatives of divisibility / inversion and in accordance with the fundamental theorem of arithmetic, any dynamic expressed in the space of frequencies[31] necessarily contains internal correlations associated with its distribution over the set of prime numbers. A partial order is superimposed on a total order destroying (except in special cases if d=2) any temporal succession order (linear or along scaling). For example, some primes numbers are common to spectrum very distant components, in the order of scales. Further, the Goldbach hypothesis, which is shown to be associated with the understanding of the Riemann hypothesis, arithmetically poses the problem of the ideal partition of heterogeneous divisions (two different parts could be associated with two prime numbers). Much more subtle than the only correlations to the octave $\mathbb{N}x\mathbb{N}=\mathbb{N}=\mathbb{N} + \mathbb{N}$, and since any integer dynamic quantity can be extended by the means an open vicinity, these arithmetic correlations based on $\Im$, therefore on an ideal sharing, can among others, be associated with sets of pre-sheaves (structuring the local tangent cone). This results in symmetries, couplings and therefore ambiguities that must be able to express similar

[30] The frequency n and its octaves has the same decomposition on the basis of prime numbers.

[31] Spectral space here centered on the universal transfer function of the α-exponential type, does not have any inverse Fourier transformation, but, in close connection with the Riemann function, it nonetheless has a relevant parameterization in the field of the complexes which should be linked to the concept of time.

functions, but much more general than Standard Entropy (in particular overcoming its elementary logarithmic expression). The entanglement of arithmetic couplings on α-exponential type dynamics and the constructibility of the zeta function by stacking of dynamics along $\pm i\theta$, leads to credit the zeta function $\zeta(\alpha+i\theta)$ of a role quite close to entropy while meeting the constraints of asymmetric partitions mentioned in the introduction. Moreover, given the existence of the functional relation which locks in a double way the universality of zeta, this function implicitly interpolates correlations between the discrete universe of durations $n=\omega\tau$ and the continuous universe of temporalities $r=(\omega\tau)^{\alpha}$. Starting from the observation that the current moment must also be considered as an interpolation between past and future, it of course comes to mind that the interpolation provided by zeta probably constitutes the missing link between the notion of temporality or even of causality and the status of any contingency opened by the multiplicity of partitioning along a tree of choices.

Considering the above, and even if temporal succession does not mean causality, any causality can only be founded on correlations coupling two universes. Both universes are here dynamic categories relating to metrics, therefore of distinct non-linear tempi [17-19,41]. Among the couplings that interest us we note those indicated in figure 2. We owe to Voronin, the proof of the universality of zeta function $\{1/n^s\}$ looked as a stacking asymptotic approximation of any complex variable function based on $\{s^n\}$. Let us observe that, provided that one of the dynamics is expressed by means of a complex function such as for example $\sum f(s^n)|_{n \in \mathbb{N}}$, it is established that the stacking of the zeta function such as $\sum f(n^s)|_{n \in \mathbb{N}}$ along the covering $s(\theta) \rightarrow s(\theta)\pm i\theta$ leads to a change of scale of analysis asymptotically allowing the proved approximation [42]. In a way zeta is the "putative derivative" of any analytical function[32]. Note however that the metric considered in integration is not similar, but complementary to the fractal metric in play during the derivation (due to the non-integer metric determined by the factor "α" which introduces the Cantorian

[32] Recall that the Voronin theorem expresses the ability to approximate any function of complex variables. To do this, it operates by means of a fibration by doubling the use of the imaginary component. Each integer value of the zeta function can be associated with the entire set $\mathbb{N}$, which amounts to a fibration according to this component; in practice, it is an asymptotic change of scale backed by recursion [42]. Thus the object of the function is progressively bundled by bringing into play a quadratic autosimilarity $\mathbb{N} \times \mathbb{N} = \mathbb{N}$ that is to say the role of the octaves equivalent to a projection on the segment [0,1].

co-dimension "1–α"). The integration/derivation switch is generally different from zero, a situation which is often expressed in physics by pointing from a standard point of view, asensitivity to initial conditions[33]. We can perceive through this last phrase how constraint the current for mind the paradigm of derivable trajectories is.

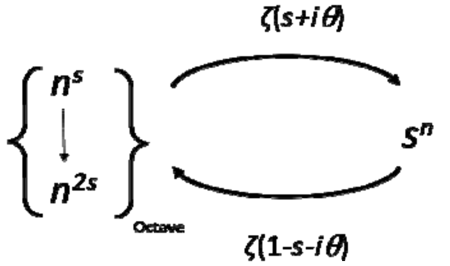

Figure 2. Holomorphic functions approximation scheme according to Voronin.

At this step, let us observe that Bagchi [42], having left scientific research just after the publication of his founding lemma about the self-similarity of the zeta function, he, in no way, has took the time for distinguishing the self-similarities (i) in accordance with Riemann's hypothesis (Filling fractal), of the more general self-similarity (uncomplete fractal) (ii) which interests us in this note: 1> α > 1/2, implying the bringing into play of the Cantorian co-dimension 0<(1–α)< 1/2. It will be observed that the arithmetic structure of internal correlations shared between the a-exponential functions and the zeta functions use jointly two metrics on ℕ: the quadratic metric and the fractal metric of order "α" (and 1–α). The first is compatible with Newtonian time while the second, for lack of commensurable measurement for both categories combining two metrics and immersions so different, sets aside the question of temporality. Observe that, as shown in Figure 3, in the absence of "analycity", the probability of finding commensurability if $\alpha \neq 1/2$ is at first glance much too

[33] As such, everyone can easily understand the condition of validity of the Voronin theorem, namely the absence of zeros in the domain of definition of the approximate holomorphic function. This condition is however lifted with Bagchi's lemma, if the approximate function is the zeta function itself. The non-trivial zeros are isolated and they can respond in the critical domain, to the quadratic self-similarity metric d = 2 associated with the relation ℕ x ℕ = ℕ also valid in the vicinity of the same zeros, hence the asymptotic validity of Riemann hypothesis with α=1/2. The internal correlations concerning the α-exponential serving as a basis for the constructability fibration (stacking) of the zeta function are based on diffusion laws $r = (\omega\,\tau)^{1/2}$ where the emerging constant is of dimension D = [L2t], the notion of Newtonian time retains its validity via the inverse Fourier transformation.

weak[34,35] for designing a simple and universal methodology; at least too weak in mind to justify the importance of the role of zeta in physics and mathematics. Martin's axiom however opportunely suggests that universality could originate from the adjunction, under forcing, of the quadratic metric (implying at boundary a countable continuum) and the fractional metric (countable, discrete but incomplete) conjunction implying infinity like it is shown by the Voronin theorem, under Bagchi's lemma (self-similarity) and schematically in the diagram 3. This conjunction matches particularly well with Rovelli's assertion that "time does not exist and only relative changes have a physical meaning" therefore here the need to use a categoric adjunction adapted to the duality emphasizing the relativity. The analysis of zeta for a quadratic metric (Riemann hypothesis) then naturally suggests to consider this metric as a reference or like a standard (Figure 3 right). In the frame of this this benchmark, Newtonian time then takes the meaning of an inverse of Fourier variable, seen like the oscillation of a pendulum. However, the quadratic character makes it possible to analyze this same oscillation within the framework of a hyperbolic suites of partitions characterized by negative curvature controlled by the factor « α=1/2 ». It is the unique representation likely to facilitate the construction of a functor between two hyperbolic dynamics having two different metrics. Thus, the second system (the one on the left) can itself be represented by a pseudo-pendulum; "pseudo" because, characterized by an internal cutoff of the complex plane emphasizing the α-exponential dynamics as a partial circular arc. « Pseudo » because a simple completion is allowed by the introduction of its set complement namely the $(1-\alpha)$-exponential.

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[34] Obviously universality is acquired if the zeta function is equipped with the phase imposed by the Riemann hypothesis, i.e. $1/2=\alpha=1/d$. Indeed, as shown in Figure 3, the significant angle is then $2(\pi/2)$ parameter causing the asymptotic emergence (i) a dense orbit basis on rational numbers and (ii) a von Neumann algebra on the transitions between orbits therefore quantum like behavior. The ultimate basis for such behavior is once again the quadratic self-similarity relationship. $(\mathbb{N}\text{x}\mathbb{N})^2= \mathbb{N}^2 = \mathbb{N}$.

[35] Although treated discreetly in practice in the rational set, the extension of the notion of time equipped with the cardinality of the continuum does not seem to pose any insurmountable problem if the Riemann condition is met. It is also necessary to introduce self-similarity in the first place within the framework of sets which can be divisible in a countable way. This is the role of Martin's axiom and of the radical assertion: the Riemann hypothesis is only an axiom in the frame ZFC set theory. Thus it can not be proved in this framework. Subject to the addition of this axiom, the relevance of a Newtonian time deriving from the countable as well as the validity of Noether's hypothesis regarding the invariance of physical laws by translation in Space-Time thus defined are warranted.

Pendulum, because it emerges from the completion a characteristic exponential time-relaxation. Both components required by the completion are related by the set derivation / partition functor. Obviously this last set cannot be assimilated to that of a pendulum and even less able to give rise to the concept of temporality, nevertheless the dual approach allows the design of an application between the reference dynamics (1/2-exponential) and the fractional dynamics (α-exponential) and only that matters, as it is shown in figure 3. This functor is based on an enumeration and on an indexation of analogous intervals, for instance in terms of hyperbolic distances in $\mathbb{N}$ but taking into account "α", notwithstanding the continuous parameterization constraints, by only using a forcing in the field of countable subsets. Any pseudo-pendulum associated with an arc having a spectral representation (namely a hyperbolic representation of the dynamics controlled by "α") can then be formally tuned to a standard pendulum for any spectral state of the dynamics. $\pm i\theta = ir\pi$. Behind the forcing is obviously hidden Martin's axiom which makes all the filters involved in the mapping of both dynamics countable. The hyperbolic distance on both spectra provides the link. Despite the abstract nature of its theoretical content, the solution that is obvious (see figure 3), involves quite simply in forcing the commensurability of the truncated spectrum even if this periodicity is here open by a tangent cone of metric "$1-\alpha$" locally as well as at infinity. The forcing then appears to be identical to the bending of an arc forced to adopt a quadratic "Scale-Duration" relationship, this curvature not being objective but only categorically adjunct in the space of frequencies. This application of Occam's razor to the internal locking of a dual "System-Benchmark" is an operation that can always be carried out. It amounts to creating an adjunction between two hyperbolic dynamics by using the set of integer numbers as standard and their primes as ideal projectors, namely $H_{1/2} \doteqdot H_\alpha$ Referred to the timescale, this adjunction leads to considering a boundary singularity which obviously involves the open status of ω_1.

Limiting the analysis of the frequencies to the set of integers $\mathbb{N}$, the adjunction would be based on a relation ostensibly written: $\mathbb{N} = \mathbb{N}^{2\alpha} \mathrm{x}\ \mathbb{N}^{2(1-\alpha)}$ which one can hypothesis as misleading in the ZFC set framework; However, it could be significant, in the categorical domain, here spectral. Subject to Martin's axiom and in the frame of Grothendieck Topos, this equation could possess a meaning, at least heuristic. The introduction of "$1 > \alpha > 1/2$" is indeed associated with a symmetry breaking of the complex plane involving a perfectly significant duality, as shown in figure 3 which, at first glance, in no way excludes the concept of mean. This concept plays on the fact that number two is the only known even prime number, therefore the heterogeneity on partitioning can be considered as an adjunct stacking upon homogeneous partition. The category H_α turns out to be enriched category compared to $H_{1/2}$. This enrichment can be shown in particular through the functional relationship characterizing zeta. While the formulation of the above analysis in ZFC seems conceptually weak, the heuristic formulation $\mathbb{N} = \mathbb{N}\ \mathbb{N}^{\alpha} x \mathbb{N}\ \mathbb{N}^{(1-\alpha)}$ strengthened via the categorical adjunction $H_\alpha \doteqdot \zeta_\alpha(s)$ is no less profound because this formulation leads to the role performed by $\zeta_{1-\alpha}(1-s)$ zeta extension attached to the closure of the body of complexes (and the closure of the dynamics over a pure exponential relaxation). The differential constructability of zeta by means of $\pm i\theta \neq \mp i\theta$ then immediately discloses the irreversibility of time taken as the analog of the variable $\pm\theta$ if the a-dynamic is referred to the standard time of our clocks. Time arises as an integration functor. Due to the non linear structure of the fractal universe, the set to "music" of any standard dynamics in self similar environment, leads the time of our clocks to take timbre linked, including with the external observer via the co-fractal.

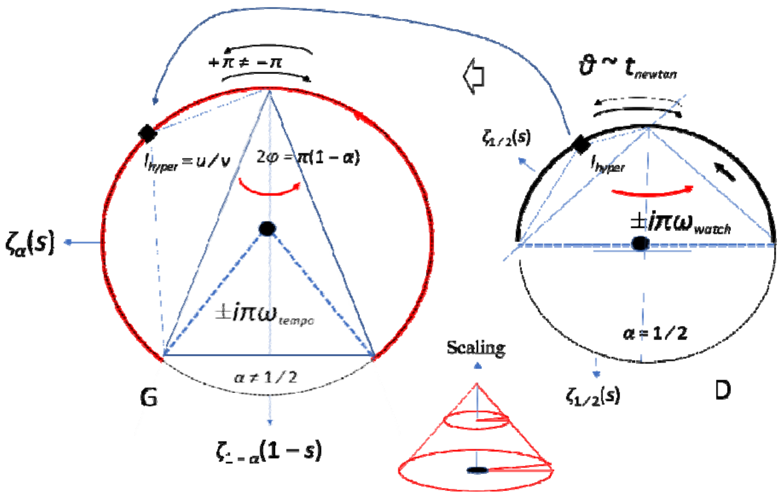

Figure 3. Construction showing the resonance of a pendulum (D) and an self-similar system (G). On the right, the reference clock if $\pm t_{Newton} \in [1, \infty]$ and the distance $l \in [0, \infty)$ are at will Euclidean or hyperbolic according to the point of view (determinist or stochastic) if $\pm\theta(modp) \in [0,1]$ since the distance is given by $l = u_{/\,0,\,1}/v_{/\,1,\,0}$. The commensurability of the clock and the dynamic system (D) shown on the left (α-exponential) is universal, provided the comparison is considered within the framework of hyperbolic geometry. The cutoff in the complex plane (figure on the left) invalidates the relevance of Newtonian time, which forced for referencing, manifests its inadequacy by bringing out an irreversibility (an arrow of time) as a singularity at boundary. This singularity, which is based on the presence of a non-integer metric of geometry, must clearly be distinguished from an arrow of thermal time as described by Alain Connes. Take care to the dual Euclidean and Hyperbolic competing points of view of the spectral analysis of the pendulum (both local and global).

The "timbre" attached to the system, corresponds to the catenation of the arithmetic correlations, written using the set of sheaves, attached to the metric of the co-system (1–α) for each frequency of the spectrum. We see here both the importance of self-similarity but also its dynamical universality formalized through the zeta function when it is matched with integral of a α-dynamic. Indeed, when two systems are in interactions, it is their "Scale-Duration" structures which must be tuned within a spectral data, and not a couple of "space-time" presuppositions. The hypothesis of the stochasticity forced by the temporal expression (for example via non-integer derivation operators $\partial^{1-\alpha}/\partial t^{1-\alpha}$) and the one of a spectral hyperbolicity required the by scaling, involve the irreversibility of time from which energy losses emerge in physics. Conversely, the irreversibility of time induced by dissipative processes confers a fractality on the resulting geometries of any physical activity. The zeta function being the integral expression of this conjunction, the local / global algebraic closure will be then based on the functional relation characterizing the dualities that found zeta. The component $\zeta_{1-\alpha}(1-s)$. Partly got off the “current” constraints, the dynamical data are then also associated with a cone of “potentialities”. The creator therefore evolves in a bi-hyperbolic local/ global universe (quadratic for temporalized chance and fractal in its potentialities) depending on the flows of exchanges which animate him and not on putative static identity of his being. The forcing, based on standard time opens into his potentialities. The local topology of this creative universe is a bi-covered saddle by the quadratic self-similarities of quantification, and by the self-similarity of the forcing tree. His "potential" is open by its being as events. These events play with the "timbres" of his personal scores. As a third middle, the zeta coupling between chance, necessity and potentialities then structures the being in front the set of his choices of living.

To summarize, standard time appears as a modal operator which matches the existence of a terminal/initial object in the categorical sense reduced to a monad. This monad serves as a “differential” basis for stacking structures (Annex 1, figure 4) giving rise to entangled connections. These structures naturally model processes playing with multiple scales, ultimately self-similar for reasons of universality. Since the monadic character leads one to think of spectral analysis as much more fundamental than temporal analysis, duration takes precedence over the singular instant. The time appears as a mere avatar, obviously required but misleading if considered as absolute in the analysis of complex systems. Nevertheless, according to above approach the "universal and absolute

Newtonian time" can be associated with the quadratic zeta function, namely the one in accordance with the Riemann hypothesis. This hypothesis, which supports the idea that any dynamic phenomenon can be projected onto a basis formed by the non-trivial zeros of the zeta function, reduces all of physics to a countable game of chance and its dual, absolute determinism. Quantum mechanics fits into this last scheme. It implicitly assumes a choice of location that locks on "points-particles" the paradigmatic objects handled. This paradigm, mainly based on chance, restricts the scope of the above categorical approach over a too much closed complexity for being general. The fibered structures which carry the constraints of entanglement within them must be designed with their bases far from zeros, for playing on the asymptotic universality of $\zeta_{\alpha}(s)$ and $\zeta_{1-\alpha}(1-s)$ whatever « α » in the critical domain. The dynamics in play relate to multiple duration, frequencies and timbres based, after integration in fractal geometry, locked together via a complex variable ($|.|$, Arg), made up of a first real quantity of quadratic content associated with an energy, and of a second entity based on a cutoff of the complex plane (phase angle) determining both an entropic component (dissipation) and an anti-entropic component, ie the creative part of the change [3]. The heuristic power of this monadic kernel is due to the characteristics of the zeta function strongly related to this approach. Ultimately, the spectral analysis which gives credit to the notion of duration, gives also credit here to the notion of Topos as the place of existence of an universal exponential, a classifier based on integers, and on relevance of categorical limits/co-limits duality, based on filters. This approach gives a categorical content to the scientific analysis of complex systems which cannot generally be able to reduce to a point analysis. Such enhanced universe are too rich to take place in a ZFC theory; the same for zeta function and the related concepts rising from it.

9. Conclusion

Above analysis leads us to consider the extension of the notion of "Space-Time" to the hyperbolic concept of "Scales-Durations". The tempi related to the spectral analysis are then associated with the notion of Duration (Annex 1 Figure 4) in a framework of a non-linear relationship with the Scales. These scales relate hyperbolic distances, globally depicted on each point of the dynamics, with the local dual-frequencies. This conceptual shift is a pivot for understanding the grounds of irreversible thermodynamics, irreversibility being herein associated with scaling.

The shift from the usual entropy function to the Riemann zeta function (power laws and parametrization via a cutoff in the complex plan if self-similarity), expresses the obstruction to the isomorphism between Time (expression of the co-product in $\mathbb{R}$ via the successor operator) and Duration (as product in $\mathbb{N}$ $\Im$ or $\mathbb{Q}$) here in the fractal context. Unlike standard time which expresses a linear relationship to space, duration expresses a monadic and non-linear relationship to scales, a relationship able to take into account a non-integer metric (Annex 1, figure 4). If self-similarity is involved one cutoff in the complex plane is the simplest expression of the open issues about let say the irreversibility. The Voronin theorem naturally opens the way (figure 3) to the use of the Riemann zeta function to express the foliation of the complex plane equipped with the self-similarity cutoff. This foliation implicitly introduces a quadratic fractal benchmark, that is to say a relation to a chaotic pendulum. It formalizes any change parameterized by a complex number taking into account the phase at the cutoff. The universality of zeta is due to the fact that this function interpolates any existential quantifier able to calibrate Duration (over $\mathbb{N}$), and any conjunctural and conjectural quantifier hidden in the the dynamics sub-structures opened by partition in accordance with local ideal projection (over $\Im$). This projection relates to the divisibility of a set in sets of ideal subsets, easily conceivable on $\mathbb{N}$ using here the set of prime numbers as classifying factors. The fractal structures give an illustration of the set of cycles on which the dynamics could be featured.

In his lectures on the status of time, Etienne Klein [43] usually asks the audience the following questions relating to the existence of an arrow of time: "In which time frame should we register this arrow? In what kind of file should the time, be able to write its own variation? "Such questions obviously exclude a priori the following answer which became obvious here:" very simply, with respect to itself in the body of complex numbers here seen as a stacking field of dualities ". It is this response to which led quite naturally, albeit implicitly, (i) on the one hand Rovelli's remark according to which any change can only be perceived as relative [41], (ii) on the other hand the constraints of immersion of the time concept in the field of complexes numbers under a condition of self-similarity, that is to say under the constraints of hyperbolic geometries, finally (iii) the categorical analysis of time / duration notion reduced to a monad (Appendix 1, Figure 4).

The double immersion in the field of complexes (duality Figure 2) using a categorical adjunction, jointly introduces (i) a reference to a normative quadratic self-similarity of arithmetic nature such as

$\mathbb{N}x\mathbb{N} = \mathbb{N}+\mathbb{N}=\mathbb{N}$, by giving a role to the zeta function projected onto its zeros, – namely a role to a stochastic temporality emerging from an inverse Fourier transformation of standard diffusion type –. However, the folding of this "surface" also introduces a temporalized adjunction constrained by an additional self-similarity, different from chance. The need for forcing the covering expresses an obstruction to perfect matching of a folding with a self-similar drum skin, damped and timbred differently from the usual white noise. The forcing is authorized by the closure of the current dynamic complex field, onto set derivative pseudo-dynamics of "the coherent possible". While using the Martin's axiom (the set of the whole subsets remains countable) the matching between forcing and tuning along scales appear to be the implicit pivot to time reversibility. In this framework, Martin axiom and Riemann hypothesis have a same content. Beyond the Martin axiom, forcing must be dualized to build a square diagram involving the functorial relationship characterizing the dissymmetry of zeta. The adjunction in the covering as a will to act and to benchmark dynamics using a standard pendulum, gives substance to an irreversible progression in scaling,

Thus, an arrow of time, originated in the folded hyperbolic geometry. Because he counts what he thinks "man never bathes in the same river". The final object of any scaling is the a dual object of the categorical of a monadic unfolding. The related co-object is the virtual, potential, even teleological component of a change quantified by an always external temporality. The use of this externality when a the complex system is considered leads to emergence of an arrow of time. It comes from the breaking of symmetry of the complex plane when a cutoff is performed (it is unique if self-similarity characterizes the dynamic process). This cutoff constraint a duality and open to the adjunction between both parts of the complex area. In the same vein, since consciousness arises from a body, objective and current action, therefore from a spectrum of biological rhythms we can easily understand that the spectral identity of an individual might be the site of an obstruction to the reversibility of times (internal or external) seen herein as the absence of inverse of Fourier; impossibility of locking of the whole rhythms. Identity then appears as a Grothendieck Topos based on a monadic spectrum characterized by an identity functor based on chance and a multiplication functor based on scaling.

In this context, Newtonian time (therefore Noetherian speed and energy) used by Quantum Mechanics which so intrigued Poincaré, is due to the filling character of Cantor-Peano metrics closed on the non-trivial zeros of the zeta function[36]. This formalization is referential because it is objectified by the pendulum seen, not as a deterministic object, but as a hyperbolic structure linked to chance if the symmetries of the complex plane are considered, hence the role of probability and the principle of uncertainty, the whose physical dimension the one of a diffusive action. In this context, and contrary to the initial opinion of the authors, the idea of thermal time [20-23], appears to be the most upstream concept of an arrow of time, because the least subject to the will or human action. We can easily see that it is associated with the uncertainty principle and therefore with the Planck action $\hbar$ and with the maxwell constant k.

In accordance with Rovelli's approach time does not exist and the spectral identity of a pendulum required for benchmarking any change. The sciences use a single pendulum to ensure the objectivity dynamic analysis. In practice, the obstruction to the matching and to the frequency locking of two pendulums can be read as a truncation of the complex space of duality. Analysis suggests that this truncation can be expressed by the means of the Riemann zeta function. Common language expresses it by the distinction between Experimentation (Freedom-Will aiming the getting of information, data emerging as an anti-entropy factor associated with $\zeta_{1-\alpha}(1-s)$) and experimentation (usual action characterized by the use of energy and a limited yield giving rise to an entropic component represented by $\zeta_{\alpha}(s)$). Creative action enrolls this duality in the personal life of the creator with a bonus given to his will. The creator accepts and alone ensures the entropic cost of his action. When the creator is an ideologue whose will is characterized by a belief in a determinism doped by a hubris, he quite naturally makes by society assume the entropic cost of his dreamlike capacities; war and destruction take shape at the end of the path that will be finaly taken, because both chance ($d = 2$) and determinism ($d = 1$) call for similar temporality resulting from the relation $\mathbb{N} \times \mathbb{N} = \mathbb{N} + \mathbb{N} = \mathbb{N}$.

The referential role of chance and determinism in the consciousness of change, parameterized or not by our clocks, is obviously not without epistemological consequences. The analysis (at least that which represents a fractal environment, asymptotically associable with zeta functions) leads to part our immersion in the real world into two classes of representations:

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[36] The peano curve is 1/2-hölderian. The factor « 1/2 » is optimal, namely the Hölder constant α expresses a surjection from [0, 1] over $[0, 1]^2$ must be always lower than 1/2 . The value 1/2 matches exactly the Riemann Hypothesis which must take into account the asymmetries involved by the cutoff.

(i) a physics of countable states handling, in line with Democritus thinking, points, particles, states analog the zeros of a dynamic function such as the zeta function. Quantum Mechanics is the paradoxical archetype of this paradigm since this dualist mechanics is satisfied with absolute continuous Newtonian time for stating the change.

(ii) a science of relations based on open and adherents sets, a science of entanglements of pre-sheaves; a "science of local incompleteness"; a science without any prior definition of scientific objects reducible to categorical points; a science where points, objects, can emerge, if so, as asymptotic final attractors. The arithmetic and algebraic problematic of standard physics then extends to categorical issues relating to measurements, and adhesions most often non local and inseparable [44,45]. This science brings into play either sheaves of interactions (thermodynamics of irreversible processes, time crystals, etc.), or even any kinds of object that cannot be reduced to such points (network science, sociology, life science, etc.) [37]

Considering that the sciences for complexity are not usually considered as sciences, we have nominated "Sci**a**nces" with "**a**", both sets of above rationality; the first approach (i) backing on the theory of sets under ZFC axioms; the second approach (ii) backing on the theory of categories as able to formalize actions and operate forcing, but also to report on geometrically irreversible time [46]. All the creative processes involve forcing. Nevertheless, we show above that either this forcing manages predefined states and then only chance is involved in the creative process. the creator then plays with the serendipity by using of a Newtonian time to progress randomly from state of consciousness to state of consciousness. There is no transgression of arithmetic order relationships and creation is reduced to innovation. However, there are other kinds of creative processes in which the states of consciousness to be considered are relations between relations, namely functors. There is no predefined state the result of any action may not be an object but a field of intricated relationships without obvious order and at most partial order. Due to the lack of order there is then no transgression possible and creation then plays with breaks in symmetries to build spaces of representations and creations made up of singularities (formalized by cutoffs within complex plane which can involve or not, a self-similarity in scaling)… The analysis

[37] So comparable to stable states of a physics characterized by a discretization of measurable quantities, and a suitable algebra (von Neumann algebra) such as those of Quantum Mechanics.

then reveals a field of rational thought, based on modal logics, that is still largely unexplored. The analysis of the mathematical foundations of this field of research will give rise to a detailed mathematical work being written [47].

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In accordance with the diagram in figure 3, the self-similarity expressed by the cutoff and phase angle imposes at left (G) the non-integer fractal metric while the forcing, from a reference clock given by the diagram at right (D) forces the tuning of the quadratic dual frequencies (D) onto fractional dual frequencies (G). We suggest here that the duality formalizes the procedure of construction of a fractal structure (Measure of scale versus Number of parts). The standard frequencies based on chance traverses the body $\mathbb{Q}$ by filtering the scales of space, while, according to the metric, the tempo adjusts the scaling by determining the number of parts. Duality simply expresses the correlations of autosimilar connections in the order of scales. However, there is immediately a problem of standardization aimed at avoiding divergence (renormalization). This is the aim of the fractality relation expressed in the complex plane by means of a Cantorian dimension "α" explicitly referring to the phase angle of fractional self-similarity: $\eta \times N^{\alpha} = exp(+\, i\alpha\pi) \times exp(-\, i\alpha\pi)$.

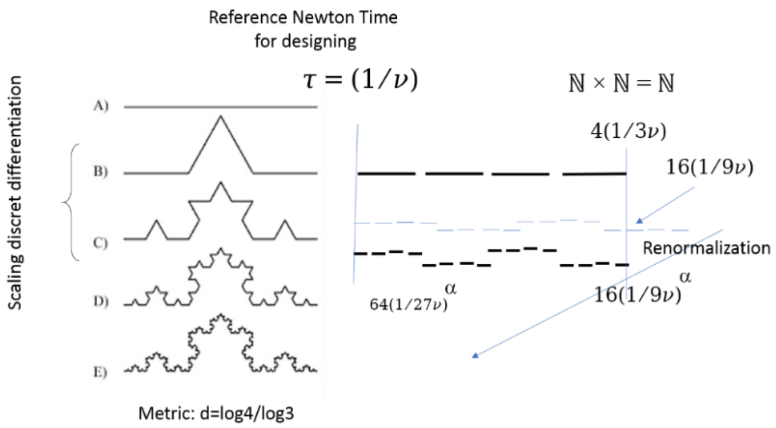

Figure 4. Monadic structure of the "Scale-Duration" relationship taking into account the self-similarity. The embedding of the structure in the complex plane by using the co-dimension (1-α) gives all its quantitative meaning to the dual object: "Actual-Virtual". Any dynamic system open in fractal structure can then be completed by a system of "virtuality" seen as a co-system with respect to the current action. It spreads this one upon many of them also coherent with the current action.

Such structures form the basis of a covering of the complex plane – equipped with a cut – which formalizes the Riemann zeta function and closes the functional relation characteristic of symmetric functions.

Annex 2

By basing the reasoning on the spectral characteristics of the function *exp*[(a ± *imπ*)log(n)], constituting set of orbits or of open system, and establishing the links between *m*, and *n* within the framework of the fundamental theorem of arithmetic[38] then by functionally widening the concept of □-exponential [3] by fibration (stacking) [3,24,25] we show that the zeta function is constructible. The links between phase trapping, the non-integer metric of the manifold associated to the fractal dynamics and the functional and symmetry properties of zeta function have been the subject of many works already published. By differentiating the Hausdorff topology (quadratic space-time) and non Hausdorff topologies (fractal Scale-Duration), it is then possible to revisit the questions relating to the links between reversible and irreversible time by associating them with the intrinsic duality of the zeta function (symmetric or not) seen as an expression of an functorial adjunction[39]. Indeed according to its value the phase factor "a" imposes or not, an asymmetry of the relation of order not over $\mathbb{N}$ seen as total ordering but on the referent quadratic product $\mathbb{N}\text{x}\mathbb{N} = \mathbb{N}$ pointing jointly on the self-similarity and on an order partial linked to a binarity spreading the decomposition upon the set of prime numbers. Thus, two distinct axes of self-similarity can be identified that work together, one being standard (d = 2) and the other conjunctural (d = 1/α). Therefore, time can be expressed by means of an angle of rotation □ in the body $\mathbb{C}$ without any cutoff [1, i] which would correspond to a pure determinism or referred to [+ iπ/4, − iπ/4] allowing to point (Figure 3) the quadratic self-similarity $\mathbb{N}\text{x}\mathbb{N} = \mathbb{N}$ therefore depending on chance namely via $\theta_r = \pm n\pi/4$ within Monte Carlo approaches. This stochastic approach is then ready to respond to Martin's axiom, when designing the perception functor either for motion or change: $\zeta_{1/2}(s) \rightarrow \zeta_\alpha(s)$ et $\zeta_{-1/2}(s) \rightarrow \zeta_{1-\alpha}(s)$. It is then interesting to consider the topological aspect of the problem. The dynamics in accordance with the Riemann hypothesis is in fact associated with a donut-type topology (with a dual tangent torus). On the other hand, in the general case although the topology remains of the donut type there is an asymmetry at the edge determined by the factor "α" (Double secant torus) which introduces an asymmetry of time arrow due to the direction of

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[38] If $\theta_r \in \mathbb{R}$, the set of real numbers $\mathbb{R}$ any integer can be expressed as products of integer powers, therefore also according to $\mathbb{N}$, of prime numbers. We see here the recursive looping in power, ultimately a source of fractionality.

[39] See above Kan extension.

rotation on the junction. Here we find Gromov's idea of the impossibility of reducing a binary choice to the pure point inside a Topos. The concept of point is no more relevant for heterogeneous cutoffs.

The reversible Newtonian time and the proper time of the recursive process are not conflicting because they correspond to an adjunction for seizing any change. The double duality disclosed above is necessary to understand irreversibility as scaling in the only possible way: in a relative manner. Behind this relativity hides in part the functional relation characterizing the zeta function, a relation which constitutes a first approach to the analytical ambiguities of which every creative mind obviously plays. That this involvement of the functional relation is based on arithmetic is far from obvious, as according to Anaximander, could yet be less more obvious the inverse, namely the fact that the parting mind [18, 26] at the source of arithmetic, could also be the origine of the (subjective) ordering of time.