THE CHAOTIC GROWTH MODEL AND COVID-19

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Abstract. *This paper considers the COVID-19 epidemic within the framework of the chaotic growth model. The basic aims of this paper are: firstly, to create a relatively simple chaotic growth model for the spread of COVID-19 that is capable of generating stable equilibria, cycles, or chaos, and secondly, to analyze the COVID-19 epidemic growth stability in the period March 6, 2020 – February 22, 2021 in the Republic of Serbia. This paper confirms the existence of the stable growth of COVID-19 epidemic in the Republic of Serbia.*

Keywords: *Logistic Map, Growth, Stability, Chaos, COVID-19.*

1. Introduction

In the Republic of Serbia, the first case of COVID-19 was reported on 6 March 2020, and the outbreak is still ongoing. At the moment, the epidemiological situation is stable. However, the SARS-CoV-2 virus is still circulating in the territory of the Republic of Serbia.

This paper uses the elements of chaos theory. Namely, chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2013, 2016), among many others.

Jones A. & N. Strigul (2020) show that the spread of COVID-19 exhibits the major characteristics of chaotic systems, namely, determinism, high sensitivity, large number of equilibria, and unpredictability. It is shown that spread of COVID-19 is likely a chaotic system. They cannot assume that spread of COVID-19 will follow a logistic path in any one country and they cannot predict the behavior of a particular logistic curve.

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Pelinovsky E., Kurkin, A., Kurkina, O., Kokoulina, M., & A. Epifanova (2020) use the generalized logistic equation to interpret the COVID-19 epidemic data in several countries: Austria, Switzerland, the Netherlands, Italy, Turkey and South Korea. The growth rate, the expected number of infected people, and the exponent indexes are the model coefficients in the generalized logistic equation. They conclude that the dependence of the number of the infected people on time is well described on average by the logistic curve.

The basic aims of this paper are: firstly, to create a relatively simple chaotic growth model for the spread of COVID-19 that is capable of generating stable equilibria, cycles, or chaos, and secondly, to analyze the COVID-19 epidemic growth stability in the period March 6, 2020 – February 22, 2021 in the Republic of Serbia

2. The chaotic growth model and the spread of Covid-19

The first aim of this paper is to create a relatively simple chaotic growth model for the spread of COVID-19 that is capable of generating stable equilibria, cycles, or chaos. It is important to set up a model for the spread of COVID-19.

The logistic map is used to interpret the COVID-19 epidemic data.

Let x be the fraction of a population which is infected with a disease (COVID-19). Namely, **x=** number of active cases / total population of the country.

Then the percentage of uninfected persons is $y = 1-x$ (by using percentages, the total has to add up to one). Then $1-x$ is the fraction of the population which is uninfected.

The COVID-19 is only transmitted when infected people come in contact with healthy persons. In this sense, interactions between healthy individuals do not contribute to the spread of the COVID-19, neither do contacts of infected people.

If the interactions within the country are more or less random, than the COVID-19 increases at a rate of proportional to the product

$$x\,y = x\,(1-x). \tag{1}$$

Now, the difference equation is obtained, i.e.,

$$\Delta x = \alpha x\,(1-x) \tag{2}$$

where α captures the rate of transmission of the COVID-19 in the population

Further, the model coefficients becomes: the rate of transmission of the COVID-19 in the population (α); the rate of the recovery of the infectious population (β), and the COVID-19 mortality rate **(**γ).

More precisely,

$$\text{Ä}\, x = \text{á}\, x\,(1 - x) - \text{â}\, x - \text{ã}\, x \qquad (3)$$

or

$$x_{t+1} = (1 + \text{á} - \text{â} - \text{ã})x_t - \text{á}\, x_t^2 \qquad (4)$$

where α captures the rate of transmission of the COVID-19 in the population and β reflects the rate of the recovery of the infectious population, and γ reflects the COVID-19 mortality rate.

This model given by equation (4) is called the logistic model. For most choices of á, â, and ã, there is no explicit solution for (4). Namely, knowing á, â, and ã, and measuring x_0 would not suffice to predict x_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

3. The logistic map – the famous equation in chaos theory

The logistic map is a one-dimensional discrete-time map that is describd by

$$Z_{t+1} = \text{ð}\, z_t\,(1 - z_t), \quad \text{ð} \in [0\,,4], \quad z_t \in [0,1]. \qquad (5)$$

This map was popularized by the biologist Robert May (1976). He was interested in fluctuations of insect populations. The logistic map uses a nonlinear difference equation. The discretized form of the logistic equation displays much more complex behavior than its continuous counterpart.

It is possible to show that iteration process for the logistic map (5) is equivalent to the iteration of growth model (4) when we use the identification

$$z_t = \left[\frac{\alpha}{(1+\alpha-\beta-\gamma)}\right] x_t \quad \text{and} \quad \text{ð} = (1 + \text{á} - \text{â} - \text{ã}). \qquad (6)$$

Using (4) and (6) we obtain:

$$z_{t+1} = \left[\frac{\alpha}{(1+\alpha-\beta-\gamma)}\right] x_{t+1} = \left[\frac{\alpha}{(1+\alpha-\beta-\gamma)}\right]\left[(1+\alpha-\beta-\gamma)\, x_t - \alpha\, x_t^{2}\right] =$$

$$= \alpha\, x_t - \left[\frac{\alpha^2}{(1+\alpha-\beta-\gamma)}\right] x_t^{2}.$$

$$z_t = \left[\frac{\alpha}{(1+\alpha-\beta-\gamma)}\right] x_t \quad \text{and} \quad ð = (1+ á–â –ã).$$

On the other hand, using (5) and (6) we obtain:

$$z_{t+1} = ð\, z_t\,(1 - z_t) =$$

$$= (1+\alpha-\beta-\gamma)\left[\frac{\alpha}{(1+\alpha-\beta-\gamma)}\right] x_t \left\{1 - \left[\frac{\alpha}{(1+\alpha-\beta-\gamma)}\right] x_t\right\} =$$

$$= \alpha\, x_t - \left[\frac{\alpha}{(1+\alpha-\beta-\gamma)}\right] x_t^2.$$

The dynamic properties of the logistic map (4) have been widely analyzed (Li & Yorke, 1975; May, 1976).

It is obtained that:

- For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- (ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;
- (iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$;
- (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- (v) For $3 < \pi < 4$ all solutions will continously fluctuate;
- (vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

4. Empirical Evidence

The second aim of this paper is to analyze the COVID-19 epidemic growth stability in the Republic of Serbia. Input data are collected by Institute of Public Health of Serbia "Dr Milan Jovanovic Batut" (https://covid19.data.gov.rs/?locale=en). The available dates range from the 6th of March of 2020 to the 22nd of February of 2021 (see Fig. 1-2). The Office for IT and eGovernment made available statistics to public on the number of people in self isolation, infected by COVID-19, tested, hospitalised and died on the territory of the Republic of Serbia.

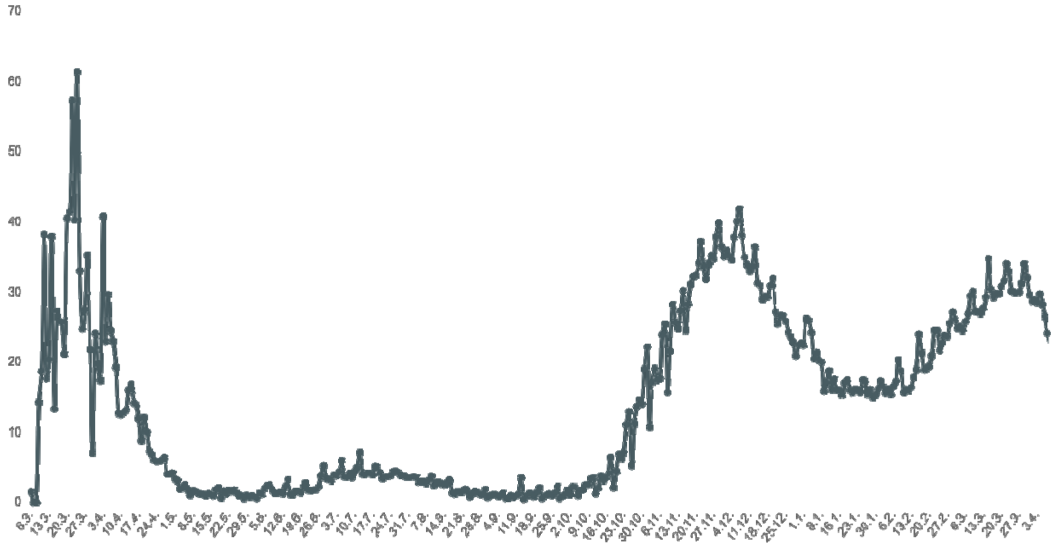

Figure 1. Daily % infected/tested . Input data are taken from covid19.rs.

Source: Institute of Public Health of Serbia "Dr Milan Jovanovic Batut".
(https://covid19.data.gov.rs/?locale=en)

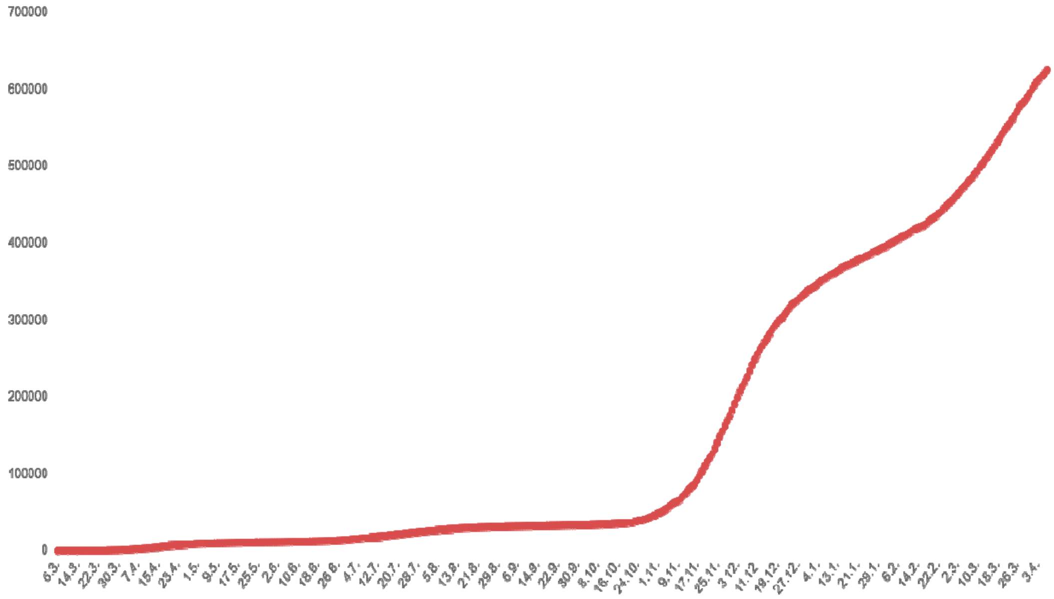

Figure 2. Total cases, Input data are taken from covid19.rs.

Source: Institute of Public Health of Serbia "Dr Milan Jovanovic Batut". https://covid19.data.gov.rs/?locale=en

The Office for IT and eGovernment made available statistics to public on the number of people in self isolation, infected by COVID-19, tested, hospitalised and died on the territory of the Republic of Serbia.

It is important to find x, as the fraction of a population which is infected with a disease (COVID-19). Namely, **x** = number of active cases / total population of the country. The estimated number of population in the Republic of Serbia in 2020 is 6 926 705.

In this sense, it is important to use the logistic model (7):

$$x_{t+1} = \text{ð}\, x_t - \text{á}\, x_t^2 \quad (7)$$

where x – the fraction of a population which is infected with a disease (COVID-19), ð =(1+á–â–ã), á – the rate of transmission of the COVID-19 in the population; β – the rate of the recovery of the infectious population , and ã – the COVID-19mortality rate.

Now, the model (7) is estimated (see Table 1).

Table 1. *The estimated model (7):6.3.2020-22.2.2021.*

	R=0.99978 Variance explained: 99.956% N=355		
		ð	á
Republic of Serbia	**Estimate**	1.0428	0.664326
	Std.Err.	0.0044	0.083966
	t(353)	236.8381	7.911855
	p-level	0.00000	0.00000

Source: *https://covid19.data.gov.rs/?locale=en*

5. Conclusion

This paper considers the COVID-19 epidemic within the framework of chaotic growth model. For most choices of á, â, and ã there is no explicit solution for the growth model (7). Namely, knowing á, â, and ã, and measuring x_0 would not suffice to predict x_t for any point in time, as was previously possible.

A key hypothesis of this work is based on the idea that the coefficient ð = (1+á – â – ã) plays a crucial role in explaining the local growth stability of of COVID-19 epidemic, where, á – the rate of transmission of the COVID-19 in the population; β – the rate of the recovery of the infectious population, and ã – the COVID-19mortality rate.

An estimated value of the coefficient ð (1.0428) confirms stable growth of COVID-19 epidemic in the Republic of Serbia in the observed period.

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