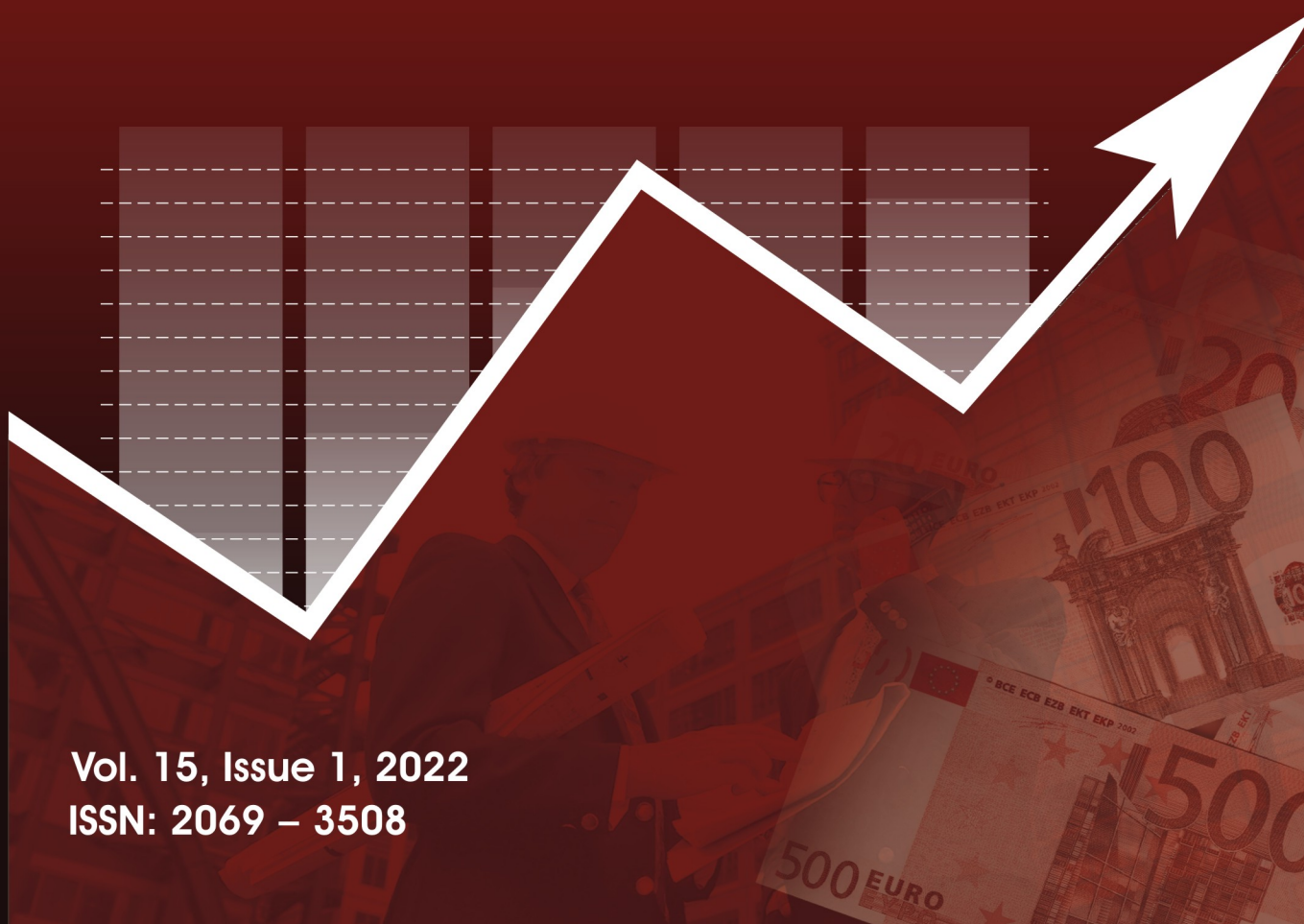




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CONTENTS

Volume 15, Issue 1, 2022

ECONOPHYSICS Section

Alain Le MÉHAUTÉ, Philippe RIOT, Alina GAVRILUT, Dmitrii Tayurskii, <i>Quid of the status of the time for creative management? A theoretical approach through zeta Riemann function.</i>	7
Vesna JABLANOVIC, <i>Natural Gas Prices and the Chaotic Growth Model</i>	49
Udayan Kumar BASU, <i>A Consistent and Comprehensive Analysis for Accretion and Dilution of EPS during M&A processes based on Rigorous Mathematical Analysis</i>	57

NEW ECONOMY Section

Andrei DOSPINESCU, <i>The Economic Impact of Covid-19 on the Low-Technology Manufacturing Industry in Romania. A Comparative Analysis</i>	79
Liviu ANDREI, Dalina ANDREI, <i>Foreign Direct Investments (Fdi) in the Central and Eastern Europe (Cee) Region</i>	91
Sidhartha Sankar LAHA, <i>India's Industrial Growth Scenario During Covid-19 Pandemic – A Time Series Analysis</i>	101

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ECONOPHYSICS Section

QUID OF THE STATUS OF THE TIME FOR CREATIVE MANAGEMENT? A THEORETICAL APPROACH THROUGH ZETA RIEMANN FUNCTION

Alain Le MÉHAUTÉ*, Philippe RIOT**, Alina GAVRILUT***, Dmitrii TAYURSKII****

Abstract: *Still young, the Professor J. Tenreiro Machado, passed away in Porto October 6, 2021. His deep scientific work had mainly concerned by the nexus between entropy, arrow of time and fractional operators. Our own works confirm the importance of the selinks by introducing how they operate through zeta Riemann function and foliation of dynamical fractional sites. Analysis of fractional Fourier spectra highlights the deficiencies of the usual notion of time and the need for its replacement by sheaves parametrization above dual dynamical Grothendieck's topos. This conclusion is based on the partition of this topos into a couple of tensorial monads whose Cantor's metrics are « $\alpha < 1$ » and « $1 - \alpha < 1$ » (non-integer dimension and co-dimension of fractal geometries folding "spectral geodesics"). The vicinity analysis of each possible state leads a pointed torus topology associated to dynamical multi-connectivity. The authors analyze the existence of the stacking of the universal spectra as origin of Riemann zeta function. This stacking accounts for arithmetic scaling that, depending on α value can play or not a role analogous to a Newtonian time. The authors show that, with the exception of cases $\alpha = 1$ and $\alpha = 1/2$ (Riemann hypothesis), the stacking parameter used to calibrate the proper time of the system, is different of the zeta complex parameter and then equipped by an "arrow of time" related to a constraint of order at infinity.*

Keywords: *fractional calculus; fractional derivative; Riemann function; entropy; category.*

1. Introduction

This note is a scientific variation which originates from the remark of Henri Poincaré, which I discussed with my friend J. Tenreiro Machado,

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especially during our last meeting in Kazan 2018. Henri Poincaré was dubious about the fact that quantum mechanics uses absolute Newton's time to develop essentially its discrete concepts. It is obvious that the laws of quantum mechanics do not change by modifying the origin of the Space-Time, thus in accordance with Noether's principles, the role of the symmetry of time and the role of energy as the fundamental quadratic invariant was never contested. However, the relevance of Poincaré's remark has never been challenged. The spectral discretization of the problems; the existence of constants such as that of Planck or that of Boltzmann; the role of probability measures on complex field; the problems of entanglements or questions of finitary constraint, exhibit still open epistemological issues. In particular, despite the paradigms at stake, energy can in no way be an invariant of systems equipped with a memory, as fractional systems are. Open questions are for instance the following: what kind of invariant can replace the energy when fractional complex systems behaves for example recursively up to infinity? By what mechanisms can complex systems lead to "universal singularities" as introduced by Daniel Sibony [1] ?

The purpose of the current synthesis is focused on a critical approach of temporality seen as an inverse Fourier transform of frequencies spectra. In discrete complex systems, the cardinal concept of entropy appears as having precedence over the notion energy. To reach this conclusion, the authors will study the fractional dynamics. By proving to be dual and bringing multi-connectivity properties, these dynamics will question the link between the non-integer metric and the topology of the systems and incidentally its entropy. In doing so, the analysis will give a particular perspective to Carlo Rovelli's point of view [2] according to which a change *per se* does not exist anymore than the notion of absolute time, these concepts only making sense technically and relatively to each other. This thermodynamical generalization of relativity will be examined here by using self-similarity, namely a non-integer metric, to shift the differential approach to a monadic folding one. The central question will then be reduced to the conception of a non-standard canonical and dual temporality bringing us closer to the perception of the consciousness with a time which flows irreversibly like "*another kind of time*" [1] in tensorial links with the Newton one.

At least since Galileo, the practice wants that a motion (if the physical extensity is space (L)) and a variation (if the extensity at stake is other than space although often proportional to L) are represented by a dynamic process using a single automorphism parameter called the *time*. To do so

scientific methods use a quantity called velocity (Lt^{-1}). The mathematical content of this notion for any continuous function is a limit named "derivative". Aside, it is showed that if the physical processes are independent of the time origin, then the dynamic processes are settled by an invariant called Energy, namely a quadratic form of a certain velocity. All formal mechanics (Hamiltonian, Lagrangian in particular) are based on Noether's principles. Implicitly this suggests that, even discrete, the processes can be fitted onto continuous and differentiable templates. But what about when they are not differentiable, as in fractional dynamics?¹. Fortunately, around 1850 after Kummer and Dedekind studies Emmie Noether has developed the notion of ideal opening the way to the notion of *filter* and of *ultrafilters* later conceived by H. Cartan in 1938 to remove many limitations about systems characterized by partial orders and differentiability issues. These concepts will be used below at least implicitly, in a frame of a categorical approach of time irreversibility.

2. Energy, entropy and time symmetries

Even now some scientists still situate thermodynamic formalism at the epistemological opposite of the mechanical formalism. Respectively linked to an (operative) ingenium *versus* a (contemplative) cosmology the philosophical approach seems obviously different. Nevertheless, this initial opposition between both sciences partly vanished with the design of statistical thermodynamics and then of statistical mechanics. However, as Cédric Villani's works point out, the constraint related to the hypothesis of continuous distributions mainly differentiable for any macroscopic behavior while the microscopic behavior is discrete (hypothesis of hard or soft balls collisions), involves huge mathematical problems that are still unresolved [3]. These difficulties are mainly due to the writing of problems within the framework of set theory under the ZFC axioms. For addressing thermodynamics of irreversible open systems likely to be non-additive, and self-similar (or monadics), we suggest here to oppose to the Maxwellian vision of time, a categorical approach, using *dynamical sites based on heaves above topos*. Such systems should highlight the fact that if energy

¹ Contrary to the PDE approach, it will be observed that non-integer derivation operators, being operators with memory, the assign values other than zero to the derivative of the constants. The arbitrary calibration of these properties (derived from Caputo and others) certainly makes it possible to deal with practical problems of control of fractional systems but does not directly address the epistemological consequences of kind of characteristic, which we propose to face here.

stays an invariant of the physics, it is most likely a dual quantity each component having no direct relation with energy but mainly with entropy. Dual on the one hand because any “*quantity*” is always based on the product of an Extensive quantity by an Intensive quality [4] and dual on the other hand because the energy involved in a process must be written as the sum (coproduct) of an efficient quantity (free energy) and a heat loss associated with an irreversibility of the process (entropy). This loss expresses the resistance to a change and is expressed by the product of a temperature (intensity as cardinal) and an ordinal function of a combinatorial nature called entropy (here a flow of extensities). Any finite physical process is characterized by a thermal effect (irreversibility) and the associated loss of efficiency increases with the power involved (energy in stake per unit of time). For some processes the inversion of the flow of extensities makes it possible to find the initial state (example charge of a battery or loop of catalysis) but this always singular operation is difficult and expensive because during any natural transformation (in the physical and not categorical sense) the entropy naturally increases. Entropy is the only oriented function in the physics of change. This function seems oriented as an arithmetic count can be, for example in the monoid $(\mathbb{N}, +)$: $n \rightarrow n + 1$ for instance represented by an oriented straight line. Far from the Newtonian time of the standard mechanics, entropy is too naturally assimilated to the irreversible time flowing over a geodesic likely to be reduced to this monoidal straight line.

The ambiguity concerning the reversibility of "mechanical time" *versus* the irreversibility of "thermodynamic time" is weaved by the following fact: since any change can only be featured with respect to an temporality external to the system *per se* (the beating of 'a pendulum, the vibration of a molecule, the electronic transition of an atom) the hypothesis according to which time itself is subject to change requires that the temporal frame of reference be defined in relation to which this change must be considered. The only possible solution in the state of knowledge may seem highly paradoxical: "*any arrow of time must be evaluated in relation to time itself*". This statement is inconceivable without a duplication of time to set an internal standard of the dual. Therefore, if time is counting down, this phenomenon can only be related to a duplication of the set $\mathbb{N}$ itself. Would this property not be linked to the quadratic self-similarity relationship that equips the set $\mathbb{N}$ namely $\mathbb{N} = \mathbb{N} \times \mathbb{N}$ and then the irreversibility perceived would not be related to self-similar metrics, our way of counting and measuring? How would this be thinkable without calling for the use of a recursion operator as it appears, for example, to

design monadic categories [4]? Evoking the “arrow of time” unrelated to a process involving “self-referencing” seems at this stage to have no heuristic future. Even without support of the self-similarity, evoking the irreversibility of time *per se* seems to be a metaphysical way of twisting a problem that remains, at this step, conceptually vague. However, in accordance to the fact that a pendulum always merges in a linear way space and time, it is possible to leave the metaphysical field by pressing the concept of time on a non-integer space metric. Beyond the idea according to which time is only defined by means of a pendulum imposing a "granularity" to measures and moreover the use of complex field, the technique defining the device for measuring time, underwrites that, like in Foucault experiment, irreversibility (angle shift) or reversibility (no shift) is related to the referential considered [3]: respectively the whole mass of the universe (inertial Galilean) or only our planet (not inertial). A correlation is therefore established between a measure of a change in a given unit of time without absolute reference and a variation after counting of numerous oscillations. These correlations will almost surely be explained by a non-linear relationship between granular time and extend measure brought at stake by any kind of energy transfer generating entropy; among these extensities requiring a measure, the space which is at the foundation of any metric. This hypothesis leads to assign a granular character to the flow of physical time and with its projection over ideal basis, thus its spectral character. Beyond, finitary rings properties and self-similarity metrics will naturally introduce a monadic generalization of ring properties. The spectral point of view amounts to giving pre-eminence the distributions over functions and duration over momenta. The test functions always required for measuring, namely the distribution theory, will also be used to unfold the ambiguities and reveal hidden symmetries. The size of the clock dial, like the length of the pendulum then associate the temporal issues previously defined with spatial quantities. *Space-Time* as the only dual entity, then enters in the cognitive game development, except that the non-linear character of the coupling introduced for instance by the recursiveness playing between cardinal and ordinal data, cannot be satisfied with the simplification given by a simple dial (complex plane) or a simple wire or rod for pendulum. The dial, like a camembert, must be cut and breach at least into a couple of pieces for dual coupling to appear and cause the emergence of morphisms, namely correlations, symmetries and then geometrical entropy. These statement raises immediately, among other things, the question of the divisibility and the morphism of dials, as scanning approach of the durations and memories. A number "*a*" is

divisible by a number " b " if there is a number " k " such that: $a=k.b$. Divisibility therefore uses the monoid $(\mathbf{N},x)$. The existence of a cut of the plane then requires the use of a complex field formalism involving the reference $I=(-1)^{1/2}$. This statement may seem paradoxical since unlike usual time, the field of complex numbers does not offer a natural order, but, because any change in a system may be referred to an external clock, the binarity of complex plane is nonetheless a powerful structure that deserves of being explored for understanding the deepness of the time concept.

3. Statistics and/or fuzzy dance?

Usually the temporality is represented by a standard oriented continuous real line and thus we can obviously associate a flow of a discrete module of time to the arithmetic successor operator in the monoid $(\mathbf{N},+)$ namely $n \rightarrow n+1$. But, in what context relating *continuous* to *discrete* is the unit of time really comparable to this too obvious unfolding operator? Zeta function takes place exactly at this gloomy node as a $\mathbf{N}$ -test of continuity within discrete sets. The major property of $\mathbf{N}$ is to be a set equipped with a good order which allows to have a principle of induction up to countable infinity. This principle is equivalent to the statement that every subset, namely all parts of $P(\mathbf{N})$, have a smallest element. From a categorical point of view, this means that we have the notion of lower limit but not natural upper limit. The asymmetry pointed out is here strictly axiomatic and linked to the successor operator in $\mathbf{N}$. The indifferent nature of the orientation of time, that is a characteristic of the mechanical laws cannot then simply be associated with accounting oriented by the succession operator. It can only be so if the physical laws are indifferent (or not) with regard to the "sign" of the extensity in stake when counting beat of time (for example symmetry of space for the clockwise or anti-clockwise or asymmetry of the electric charge, for example for the positron which "goes back in time" in Dirac models etc.). In practice, if the mathematician affirms as an axiom the existence of the successor operator, the physicist can in no way be satisfied with a transposition of the orientation of counting in order to affirm that irreversibility is present. The succession in the monoid $(\mathbf{N},+)$ is therefore a naive point of view that cannot be transposed without care to the definition of the notion of physical time. However, given the above, what about the use of its dual multiplicative monoid $(\mathbf{N}, x)$ in arithmetic? Could it not have much greater advantages by taking into account the set of rational numbers $\mathbf{Q}$ involving an internal

order much more subtle than that of $\mathbf{N}$? Would the scaling associated to division not capable to model the concept of formal irreversible time? The possible choice of the set $\mathbf{Q}$ may seem paradoxical because it operates in the absence of a lower limit of the subsets. It could however be relevant since on the one hand this absence is likely to generate a *boundary uncertainty* (uncertainty on initial conditions then naturally opposite to Noether's principle) and that on the other hand, due to intensive possible meaning, $\mathbf{Q}$, resulting from $(\mathbf{N}, x)$ can naturally take into account the notion of *duration*, the notion of *moment* being then reserved $\mathbf{Q}$ et $n \in \mathbf{N}$. The interest of this inversion of the natural order of $\mathbf{N}$ reinforces the Carlo Rovelli point of view who asserts, after a detailed examination of the problems of mechanics, that a universal absolute time does not exist and that the examination of relative changes, *namely a categorical adjunction*, is sufficient to represent any change. This epistemological position is implicitly based on the notion of *functor* and *Natural Transform*, always coupled subsets being concerned by *recursive morphisms* [5,6]. However, this principle based on relativity for any changing implies that any point of view on evolutions be at least dual, equipped of a quantifier, and therefore inscribed within the framework of a *categorical ordering* which questions the identity of the categories and the logics in stake [7-9]. One of the components of the duality associated can be for example a clock providing a standard beat. Nevertheless, any dual point of view using a norm, can be reversed, for example the same pendulum becoming object of interrogations since any measure requires vice-versa the presence of a co-data. We will see from the very particular case of the inversion of the monoid $(\mathbf{N}, +)$ as a normalized straight line, the potential relevance of the approach backed by the monoid $(\mathbf{N}, x)$ as normalized geodesic. Before that, let us observe, following Maxwell and Boltzmann works, that the link between the mechanical sciences and the thermodynamic sciences are mainly based on statistics, or even on a theory of strictly additive and normed measures namely very special and among the simplest measure theory based implicitly on Logarithmic function (see at the opposite even in mechanics the information sciences, as shown by Taganov and Babenko [10]) Considering the behavior of ideal gases, Boltzmann was led to construct a $-H$ function expressing the statistical homogenization of the distribution of velocities of the molecules of a gas, initially in an inhomogeneous state. The natural evolution of this H -function (called relaxation) is similar, up to the sign, to that of the entropy function S based on logarithm. However, the irreversible orientation of this type of relaxation is based on perfect reversibility of the local elastic shocks (as

reversible as the shock between balls on a pool table). The reversibility of the “atomic” shock (local) versus the irreversibility of the set of shocks (global) at long time then appears a paradox always open [11,12]. This leads to the impossibility of reversing the temporal order of the macroscopic process, which then appears as linked to a change of scales of analysis (local *versus* global). As shown using Maxwell's demon "thought experiment", this link seems to be assignable to an accumulation of irreducible not countable uncertainties, in fact related to the definition of the subsets, in the set of information at disposal. Therefore, information has to be considered statistically. The oriented H-function imposes a point of view originating in the details followed by integration (i.e., a bottom-up series). However, in light of what category theory [6] teach us about counter-intuitive and dual ways of thinking, fractal geometries, (namely the whole is in the part; the whole is more than the sum of the details; capacitive measures can be non-additive; completion is not necessarily acquired and requires the duality of an embedding space etc. (11)) would it not be possible to geometrically circumvent standard set statistics, – seen through measure theory (additive and normed measures) –, for switching the analysis according to a categorical and more precisely monadic and recursive approaches ignoring the usual differential way of thinking? From then on, Boltzmann's approach could be generalized through new non-integer space-time, the scale changes then leading to consider non-additive measures, whose normalization and closure would constitute degrees of freedom. The scaling then would be seen as a sequence of filters (converging series of gauges and templates) in geometries which for simplicity could be self-similar? The transposition from the differential point of view to the monadic point of view may appear clearly disruptive. Far beyond the trivial intuition linked to a simple succession, it highlights, a strong relationship with scaling hidden in its projective arithmetic. Thus, the ontological question of a dual temporality should be then open [13].

As Prigogine stated [14], not without having been roundly and paradoxically criticized by experts in formal quantum mechanics, irreversibility of processes is obviously not related to our measurements; they are intrinsically irreversible and the “presence” of external observers and therefore of measurement uncertainties is, in no way, the cause of an irreversibility that would orient or order our consciousness of time. If irreversibility is linked to statistics operating on locally reversible processes, irreversibility cannot be linked to local measurements, but to the taking into account of "global characteristics" associated with scale changes (namely the set of subsets) that can go beyond the usual statistical

framework. As it is involved by the spectral series, any physical change infers, a passage from the local to the global, and therefore a role of the metric (monadic aspect replacing the one parameter self-morphism) and of the topology (through vicinity). We find here a well-known cosmological distinction between the notion of curvature or parallel transportation (via the measurement of metric herein linked to the local mass density) and the notion of topology namely vicinity and connexity associated with principles of least actions. By taking into consideration the asymmetrical union of a meta-categorical nature $((A \cup B) \cup (A \cap B))$ which underlies all self-similarity and dualizes the operation of succession, intuition then discreetly distances itself from the notion of arithmetic succession, to take into account a monadic kernel $(T^2 \sim T)$ and a deployment through a Keisli algebra. Although still very weakly, one perceives here the emergence of an epistemological turning point supported by a recursion and fractal geometry both suggesting a progression that is no longer linear but looped and spiral, the "proper time" of the dynamic system being associated with a succession but in the order of scale therefore related to $(\mathbb{N}, x)$ and no longer calibrated according ergodically to "a standard distance" namely to $(\mathbb{N}, +)$ [14].

Importance of iterated operators, as a monadic generalization of successor operator in a spiral ring is not a detail in Ilia Prigogine's works [15] who explicitly links entropy to a composite recursive process illustrated by means of the transformation of the baker. This transformation associating 2D (composition) and 3D (associativity) -*folding* 3D, *spreading* 2D, *folding* 3D, *spreading* 2D, *etc.*-, leads very subtly to distinguish stages of dual effectiveness leading to two neighbor points at a step $2n$, find themselves very distant (or very close) at step $2(n + \omega\tau)$, in accordance or not with the Liapounov divergence laws [14]. It is established that the baker transformation is strictly irreversible; there is indeed no reverse process allowing to find the initial state because it is not only a question of considering two points² which diverge but of taking into account all the possible subsets of points of the baker dough seen as the continuity of an infinity number of subsets. The problem of irreversibility is well and truly here a topological problem of distance, connexity and vicinity of subset [11,12,14].

² The problem raised here is of a topological nature because it is questions of vicinities that are at stake. A physical object cannot be thought of in itself, it must be thought of in its categorical context by addressing open questions (Yoneda's lemma). But changing vicinity in a given topology amounts to change scale therefore to consider filters (H. Cartan), the entanglement of the latter giving rise to grids (G. Choquet).

Associate time with the variable $m = \omega\tau$ as it would seem natural does not hold a *disputatio*, because the fact of admitting that time constant τ of the composite operation of folding (2D) and spreading (3D), is unique and irreducible and therefore represents a "statutory and external arrow of time", leads to a tautology and thus proves nothing with respect to the "internal field of velocities" of the dough. By choosing an arbitrary standard length, each clock time can then be associated with a distribution of times characteristics erecting a local sheaf-times. Notwithstanding the definition of a repetition time " τ " and that the occurrence " 2τ " is the same transform in the frame of monadic properties, the baker's transformation can be associated with an increase in entropy, -i.e. an increase in the internal disorder of the dough and entanglement of the set of points initially neighbors-, only by taking into account the crisscross of sheaves (partly based on the recursive algebra involved by the monadic structure). The resulting ambiguities (Galois denomination) in terms of internal past-present and future, is associated with the development of a multi-connectivity which designs the final topology of very complex definition of what vicinity means. The recursion of the dynamics then involves long range correlations of the time-sheaf (spectral memory and prophecy) with the most often emergence of scaling properties optimizing the density of entropy-flow within the dough.

Arrived at this stage of our questioning the heuristic progress of our arguing seems very small because if the entropy is an extensive function clearly defined and oriented at the combinatorial level with as an emblematic example the Shannon formulation $S = - \sum [p_i \log(p_i)]$ at the opposite, the being of time in its consciousness complexity related to sheaves of relationships remains at this step a categorical open question, as well its links with the determinism of the successor and divisor operators respectively in additive and multiplicative monoids. The relationship between "time-sheaves" and causality is equally fully open. However, it is at this stage, by critically examining the paths previously sketched out, that we can make the hoped-for conceptual leap. To do so, we propose to revisit three questions:

- The mathematical meaning of the logarithm function and the implications of its extensive use in physics.
- The use of Fourier (or Laplace) transforms as time transforms and scaling measurement operator and its generalization.
- The relation between self-similarity and renormalization (the recursive geometries), associated with the spectral concept of time.

4. Logarithm & Exponential functions versus asymmetrical bi-partitions

Any sassy student can only be surprised at the extensive use of the logarithm and the exponential function in science and in engineering. In the framework of category theory, this couple of functions expresses a duality: namely the duality of the product (projection) and the co-product (sum). It is clear that their unbridled use first and rightly points to arithmetic constraints much more than physical ones. Any physical measurement passes implicitly or explicitly through the use of the monoids $(\mathbf{N}, +)$ and $(\mathbf{N}, \times)$, and thus through their links, or even for scaling reasons through the use of ring with modules and field. Obviously, any measurement requires the algebraic links between addition (subtraction/0) and $\mathbf{Z}$ field and multiplication (division/1) and $\mathbf{Q}$ field. Any computation calls out the serial character of composition (2D) while associativity plays with diagram division (3D). In this matter, the logarithm (log) function has a very special status. Indeed, we call logarithm the area $A(x)$ ($A = -\log(x)$) that below the hyperbole $f(x) = 1/x$, is located between the abscises 1 and $1/x$, namely a kind of integration which is expressed otherwise by writing $dA/dx = f(x) = 1/x$, or after permutation of the variables $x \sim \exp[A(x)]$. Therefore, the *logarithm* function qualifies numerically the homogeneous set of subsets of sizes larger than $1/x^3$. *The log function is thus explicitly attached to a problem of bounded and linearized scaling (by using a sum) from an initial state to a minimum size of resolution stated by $1/x$.* In this perspective, writing informational entropy like a sum of logarithms ($S = -\sum [p_i \log(p_i)]$) is nothing but a tautology: its orientation during the deployment of natural relaxation processes designates a change of scale for which it is not equivalent to go in the direction of small scales towards local detail (top down) or to go in the direction of the large scales according to a synthetic approach namely the “coarse graining” (bottom-up). This logarithmic approach is nothing else than the simplest “Matryoshka approach”. There is a stacking order of the Russian dolls although they are all strictly identical except the linear scale in between. The freedom to compute either using sums (of amalgamated details) or using product and projection over an “ideals” basis as Noether suggested, is expressed in the duality log/exp, *however this duality conceals a constraint of homogeneity and implicit simple kind of memory, most often*

³ If x is a prime number, then the generalized sum is related to the concept of filter and more over ultrafilter.

passed over in physics lectures. The derivative $1/x$ supposes a homogeneous division of the whole set X . We take an object X and cut it into x objects that are totally *indistinguishable* from each other. The set theory under ZFC axioms is then tacitly involved in every homogeneous point of view, therefore also the standard Peano-arithmetic. Each part is an atomic point and behaves like a wheel among others identical [11].

The critical analysis of this homogeneity can leave the standard approach precisely at this step. Conversely to a homogeneous fragmentation leading to indiscernibility of parts, a non-homogeneous splitting, assigning different relative “weight supports” to a generic inhomogeneous bi-partitio⁴, namely a *distinguo* between α and $1-\alpha$, immediately invalidates the use of standard set theory because Cantor sets are then involved through iteration and hence a thoughtless use of the logarithm function becomes cognitively risky; likewise the use of the standard entropy which is nothing else than a simplest functional variation about homogeneity and indistinguishability of sets in ZFC theory. With the introduction of the parameter α which is a metric one, the meaning of the variable x changes from that of a x -partition (cardinal) to that of a scaling variable (ordinal). We will see later how the logarithm function nevertheless regains some of its relevance in the particular case of dual power laws, but let us first examine what are the consequences of this metrical *distinguo* on the exponential function.

To connect the following reasoning to the above and give a metrical meaning at α , let us observe that the evoked inhomogeneity is, in subsets point of view, naturally associated with the notion of identity, iteration and recursion. Assumption of inhomogeneous partition based on α and $1-\alpha$, immediately brings to mind the construction of a Cantor set, therefore the notions of non-integer dimension and fractal co-dimensions. Indeed, by inversion of standard fractal Mandelbrot formula, the fractional metric formula for Cantor sets is such that $1 \sim n \cdot \eta^\alpha$ if n is the number of pieces of

⁴ The parameter α quantifies a relative weight of the inhomogeneous partition. This is an analytic division operation and therefore a basic engineering operation. We will draw from it a notion of cantor dimension, obviously not to be confused with multiple other definitions of the dimension which, must be stable in a given context, for example metric (distance), topological (with the notion of connectedness), algebraic in the sense usual or homological etc..

the hollow X-puzzle and $\eta=1/k$ the average scale of each pieces 5, in other words: $\alpha \log(\eta) + \log(n) = 0$. Iteration when associated with a generic in homogeneity leads at limit to a fractal structure [11]. It erects a set of weight 1 over a set of zero measure, immersed in a set of measure 1 with a zero weight. In physics the weight is given through a dynamic of exchange: TEISI non completed model [16,17]. Above two dual sets have respectively for fractional dimensions precisely the values α and $1-\alpha$. α is the dimension of the cantor set and $1-\alpha$ is the dual co-dimension of void and then the dynamics can be expressed using fractional differential operators [16-18]. The critical case is obviously $\alpha=1/2$ since the fractional co-dimension is then strictly equal to the dimension and the immersion becomes a self-immersion (Peano curves with $d=1/\alpha$). Using simple duality, let us observe that we can give to all this quadratic problematic a content of Peano metric and no longer of Cantor one; indeed: $1=\eta^2 x n^7$. Therefore, $n^2+m^2=k^2$ namely $1/\eta_n+1/\eta_m=1/\eta_k$ fulfills Pythagorean relationship. The “external” world, namely the co-universe of immersion, disappears from our intuition because it is certainly a-causal but similar to the causal one. Let us observe that there is then an obvious link between the Pythagorean relation and this fractional metric since the respect of the quadratic relation for a sum imposes the usual diffusive correlation characteristic $D_{corr}=L^2 t^{-1}$. The “time” variable t is here nothing else than the inverse of a frequency p of weighted correlations, namely a scale distribution which can then be thought of in the field of complex numbers through a spectral approach of fractal dynamics and Cantorian representation [16,17].

Still using the concept of duality, the problem opened by the inhomogeneous α and $1-\alpha$ bi-partition and then iterations, must also be expressed via the exponential function but now in an infinitely more subtle way than for the linear logarithmic form $\alpha \log(x)$ and $(1-\alpha) \log(1-x)$. It is

⁵ Let us observe that for $\alpha = 1/2$ we will affirm that the self-similarity is quadratic because this metric is related to the 1D Peano curves which have the property of covering a (2D) plane fulfilling in particular the complex plane.

⁶ It will be observed that the base of the logarithm becomes one more degree of freedom for our representations, that this degree also opens up a problem of change of scale arising the tropical geometry which is therefore asymptotically linked to our problem.

⁷ Let us observe i that in the general case of inhomogeneity of the recursive division the questions of non-additivity of magnitudes and non-linearities of distributions arise exactly in the terms in which they arise in fractal geometry and in the theory of capacitive Choquet measures.

no longer a question of starting from the piece of the puzzle to arrive at a "hollowed out set", but of starting from the "hollowed out universe" to express the possible distribution of the sizes of the pieces fitting together to represent the set at all scales of the puzzle. In practice, it is a matter of looking for a canonical basis for the distribution of relevant templates allowing the puzzle to be projected onto *ideal orthogonal subsets*⁸. With declination from the homogeneous case $x = \exp(-kA)$ which is, at this step of generalization, our only tool of thought, we must imagine all the conceivable possible divisions arithmetically consistent at the various scales with the recursion of the bi-partition α and $1-\alpha$. For this, we need to go through a spectral analysis. This analysis is very simple if homogeneity is acquired. Indeed, the scaling relation, here linear, is the following $1/\tau = n \cdot \eta$ because by writing $t/\tau \sim A$, then the Fourier Transform [TF(n)] is a TF of an exponential which must be written like a transfer impedance $Z \sim 1/(1+p)$, with only one $p = i\omega\tau$ as $TL(t/\tau)$. Passing to the Fourier Transform (TF) the passage to the field of complexes is reduced to an unfolding of scales along the complex axe " i " with $p = i\omega\tau$ with a only norm τ . There is certainly an infinity of workable partitions but all of these refer to a single normalization of the exponential transfer function: $x = \exp(-t/\tau)$. Although elementary, this analysis of the linear link between scaling factor through $A = -\log n$ and *time* makes it possible to bring out new trails which, to our knowledge, have not been explored by any other authors, at least so simply:

- the first is that even the traditional approach to time is in agreement with a time t no longer considered as a natural and intuitive notion of shift factor, but as a subsidiary notion as an inverse spectral transformation, $TL^{-1} (TF^{-1})$ of the variable of Laplace p (respectively of Fourier). The spectral statement acquires a physical meaning with the consideration of a scaling (as produced by p (if derivative) or as a divisor $1/p$ (if integrator)). The central notion of the dynamic would therefore no longer be t but p . As soon as a scaling becomes nodal, we have to think in a spectral way, therefore in the fonctorial space of recursive sequences, in magnitude (spreading) and in series (backing). The "*spectral structure*" then replaces the "*space-time*" which we use out of habit to build our "*objective and holistic*" representations of

⁸ Implicitly we reformulate here mathematical questions known under the qualifier of cyclotomics

unity via like in puzzle, scaling for change. Virtually, both FT and TL explore the different scales of the structure. It is through the p -scaling (also scanning) that the usual concept of temporality is conveyed in and through scientific experiments or even made aware at the human being through his sensation. The complete makes sense and this sense is not only holistic here spatially but also spectrally. The ontological identity of the structure only takes a meaning dynamically, namely categorically [13]. Therefore, surprisingly, there can be no identity without “temporality”; thus both without scaling and history!

- In the case of a homogeneous partitions this identity signature is written in the space of frequencies by means of a singular TF(n): $Z \sim 1/(1+i\omega\tau)$: The Fourier transform of the exponential. The complex field, implying a duality of conjugated variables, is then naturally summoned by the analysis. The spectral signature is based on the inversion of a purely imaginary straight monoidal line ($\mathbf{N}, +$) positioned at the abscise "unit: 1" with respect to a reference point of the complex field in stake (Fig. 1). A homogeneous division (here a $1/n$ partition) of the line is nevertheless involved up to infinity. If this inversion is taken with regard to a point A outside the line it brings a compactification, namely a convergence of series bringing infinity within reach. In this case, what is the meaning of the constant "1" of the formulation? The operational calculation clearly tells us that it is the normalized entropy per unit of time and it is associated with the dissipative dynamics represented by the real part R of the exponential relaxation $x \sim \exp(-t/\tau)$ with $\tau = 1/RC$ product of a Resistance R and a Capacitance C , i.e. geometrically the parameters of definition of the diameter of the semicircle representing the inversion $1/n$ in the complex plane (distance AO). In addition, the possible adjustment of the base of the logarithm introducing asymptotically *Tropical Geometry*, the universality of the precedence of the spectral analysis over the temporal analysis in all cases of scaling, including non-homogeneous divisions, becomes obvious in spite of its not very intuitive character due to the holistic approach⁹.
- The spectral exponential response has the paradoxical advantage of its ambiguity. Indeed, the spectral representation in the complex plane is

⁹ This approach can be heard all the more because it can easily show any non-linearities by using the notion of "tone" as in music.

nothing but a geodesic semicircle. From a *cardinal* point of view transformation in $(\mathbf{N}, +): n \rightarrow 1$ matches a “coarse graining” process. In the case of a homogeneous partition, the analysis of the semicircle offers the opportunity for a dual interpretation, one (i) is linear and obvious ($n \rightarrow n + 1$) the other (ii) is hyperbolic ($\eta \rightarrow \eta + 1$) it is a geodesic functor in a Poincaré models; both are related. A point of the circle is then the formal expression of the perception of the unity vector +1 at the distance n from the origin O, such as an observer could see this unit from the singular point A outer the straight line. The measures are not only reversed but distorted by a parallax (hyperbolic curvature) involved by the cardinal distance n .

- Although the homogeneity assumption hides it, the ambiguity goes far beyond what has just been pointed out. The parametrization constraints rely on an additional duality. There is not only a singular point of view in the problematic but two singular points of view completing each other because by symmetry the two ends of the $\frac{1}{2}$ circle, A and O can be exchanged. In practice there is not one straight line but two straight lines then involved. They can be parameterized in inverse ways if the orientation of the complex plane is fixed. An increase on the right leads to a decrease on the left, a situation which excludes that irreversible time can be associated with the sole representation above. The distance between both possible roots for the referential straight lines is nothing else that the flow of entropy associated to the exponential, namely the irreversibility of time related to the scaling process. Of course, having already studied No ether's works, we expected this observation! In this framework the time is only related to the relaxation process; irreversibility is not intrinsic but related to the scaling associated to the process itself; it is expressed above through the inverse Fourier transform.

We propose to shake up these paradigms by asserting that classes of transfer functions exist which, although *not giving rise to inverse Fourier transforms*, thus to the emergence of a unit of Newtonian time. Nevertheless these classes are physically relevant and equipped with an effective universality for designing a new class of temporality, *another time* [1]. In the framework of Grothendieck's topos, if subtly stacked, these spectra can contribute to introduce a degree of freedom overcoming the absence of inverse Fourier transform then giving birth to new type of proper time (that we call *musical time*). To achieve this program we have altered, falsified and

generalized the notion of arithmetic site as described by Connes and Consani [19] by shifting pure and very abstract mathematical concepts into handy and straightforward engineering notions. The idea is to equip the standard spectral exponential with a topos structure by dividing the basic spectrum to give rise to a bi-category made up of non-finite but complementary hyperbolic geodesics ensuring a physical closure. The set-forcing joins here renormalization techniques. Thereafter a couple of clever unfolding of this topos can give rise to the dual Riemann's zeta functions (with complementary s and $1-s$ exponents) in such a way that these functions can become functorial intercessors both between causal and a-causal “geodesics” and between counting sets and continuous sets. Then our approach makes it possible to design in a categorical framework the question of *the arrow of time* seen as a matching between classes of recursive inverse automorphisms within a bi-scaling closed point of view. This approach requires the understanding of a new concept: the concept of α -exponential.

5. Spectral Extension of exponential and α -exponential

Above survey is not a simple reminding of knowledge but the conceptual requisite concerning all fractional dynamics based on universal $Z_\alpha \sim 1/[1+(i\omega\tau)^\alpha]$, transfer function. We know its dynamical foundation in terms of fractal geometry [17]; for example, $\alpha=1/d$ where d is the non-integer dimension of the immersive geometry. It is used in the definition of the dual closed cantor set with characterized by an α -metric. Traditional apriori concerning PDEs leads quite naturally to think that the universality of this type of dynamics, giving a special place for memory (via convolution) and context (via test functions over the geometry) is questionable¹⁰. Indeed, the fractional derivative, -and more generally the multiplicity of fractional operators challenging together [18]-, does not simply assign the value zero to any constant, namely to initial conditions. Except for particular metrics this property generates a sensitivity to these conditions. Noether's axioms are then no more relevant leading for example the invalidation of the invariance of energy.

Let us focus first on an emblematic particular case: the derivation of order $1/2$, the least of fractional operators applications to dispute because it can be linked with spectral exponential without major breaking of symmetries of the complex plane.

¹⁰ Concerning the controversy about fractional operators we recommend the very interesting works of Jocelyn Sabatier and al. (For example *Fractional order derivatives defined by continuous kernel Are they really too restrictive?* Fractal and fractional 4(3) 40-45, 2020)

5.1. $\frac{1}{2}$ -exponential, chance and diffusion

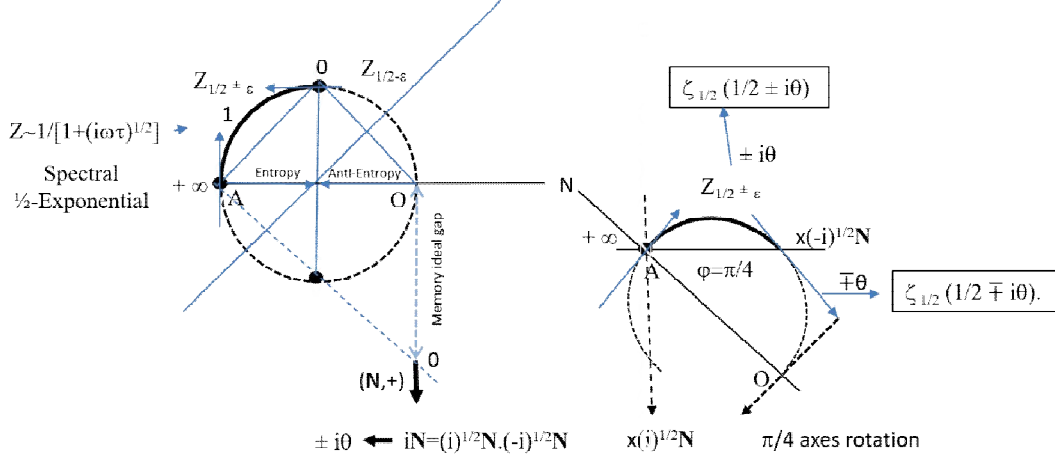


Figure 1. Euclidean (left) (and hyperbolic (Right) representations of Grothendieck dynamic topos based on $\frac{1}{2}$ exponential transfer function, namely transfer of extensive entities across a complete 2D fractal Peano interface [16-17]. The spectrum is obtained via an inversion with respect to A, of the monoid $(\mathbf{N}, +)$, equipped with a memory-gap at its origin. The figure is characterized by an entropy balanced by ananti-entropy (non-causal component) via a vertical symmetry axis. The foliation (fibering) parametrized by θ gives rise to zeta Riemann function. θ plays the role and is analog of the newton time.

It is very easy to skip from the expression of a standard spectral exponential, $Z_\alpha \sim 1/[1+(i\omega\tau)]$, to the expression of a compactified chance and (finitary) diffusion. It suffices to consider the transfer function $Z_{1/2} \sim 1/[1+(i\omega\tau)^{1/2}]$, namely again, a transformation of a straight line associated to a monoid $(\mathbf{N}, +)$. However, as shown in figure 1 (right), the line of parametrization must now be tilted at an angle $\pi/4$ with respect to the axes of the complex plane representation. This figure highlights, (diagram at right), the reference system specific to fractional dynamics given by above equation (hyperbolic) *versus*, (diagram at left), the same transfer function but represented in a reference system (Euclidean) basing usually the standard exponential. These couple of representations seems mathematically equivalent¹¹ but they do not hold the same physical meaning. In the “hyperbolic” representation (right) the $\pi/4$ -tilt of the straight line, that can be symmetrized with respect to spectral frequency limits. This representation is obviously fractionally intrinsic. In “Euclidean” representation (left), though

¹¹ It is associated to the homographic transformation of the axes of the complex plane

parameterized by a hyperbolic distance, the spectrum, is now represented by the $\frac{1}{2}$ -arc of a semi-circle, it is then explicitly referred to the whole semi-circle which corresponds to an inversion of the usual complexline $\mathbf{N}(0,+\infty)$ along $1+i$. This representation is fractionally extrinsic. The major characteristic of this representation of α -exponential (if $\alpha=1/2$) is the following: in external reference (Euclidean left), the ordinate of the origin of the complex plane (the A center of inversion) is partly disconnected from the root of the usual parametrization of the spectral exponential. The representation highlights the presence of a *gap of parametrization*. Conversely in the internal reference frame (Hyperbolic), the gap disappears and it is replaced by a simple tilt with the slope $\pi/4$ at the zero spectral limit (right). Therefore, the non-nil information contained in the gap unveiled in Euclidean representation (left) is compressed via the tilt of the line supporting the parametrization and the punctuation of the spectral fractional representation, is not reduced to a nil value (as “no being”) but is a physical emptiness full of hyperbolic non-standard correlations related to the slope of this line. The gap of parametrization can obviously be used as a specific field of inversion with respect to its own root for the monoid $(\mathbf{N},+)$ over all the field $(0,+\infty)$. The resulting duality is none other than a specific expression of the quadratic self-similarity affecting the set of integers $\mathbf{N}=\mathbf{N}\times\mathbf{N}$. The meaning of this “zero gap” will appear below in relation with the fractional universality of zeta Riemann function in relation with the mathematical status of the equivalence between both representations.

The item to emphasize at this step of arguing is the following: this specific transfer function has an inverse Fourier transform TF^{-1} which brings into stake both the power law $1/(\pi t)^{1/2}$, as in the processes of infinite diffusion, but also the error function $Erf(t)$ by relating the non-finitary scattering process (namely the structure of the gap at root) to closed probability rules with Gaussian distribution of uncertainties. Due to the symmetries involved, the dynamics with parameter $\alpha=1/2$ can then be expressed through a temporal representation using standard Newton time. The emerging time from TF^{-1} is ontologically analogous to that implemented for a standard exponential relaxation. The spectrum from which it results may be amalgamated with an “anti-spectrum” (linked to the “zero-gap” mentioned above) in order to rebuilt a complete exponential giving rise to adual Grothendieck’s topos. This spectral topos is endowed with a singularity in its middle. The associated topology is that of a pointed torus whose singular angle at infinity ϕ is such as the punctuation is given by $2\phi=\pi/2$, corresponding of a permutation of axis of the $\mathbf{C}$ -plane (the centering of this topos on the complex plane leads to a significant angle at infinity given by $4\phi=\pi$, confirming the categorical role of the amalgamated semi-circle). We called $\frac{1}{2}$ -exponential the spectrum

$Z_{1/2} \sim 1/[1+(i\omega\tau)^{1/2}]$, as well as the Newtonian dynamics which only here, can be associated with it¹².

5.2. Generalization at α -exponentials with $1 > \alpha > 1/2$

Although being an obvious generalization of above approach the spectral analysis expressed by the transfer function $Z_\alpha \sim 1/[1+(i\omega\tau)^\alpha]$, with $1 > \alpha > 1/2$, appears highly complex to interpret physically. At the end of the seventies, for engineering reasons, the TEISI model has had as aim to master this complexity [16]. Just like the simple power law $1/(\omega\tau)^\alpha$ subject to the convergence constraint $R(\alpha-1) > 0$, universal spectra experimentally relevant (transfer function known as Cole and Cole) otherwise characterized by $(\alpha-1) < 0$, do not have any inverse Fourier transform¹³. The notion of “standard temporality” (*the idea of an automorphism over the dynamic characterized by only on e parameter*) disappears [20]. The representation of the spectrum remains related to the Peano arithmetical rules, for example via the sharing of the discretization of the half-line $m = \omega\tau$, namely via $m \in \mathbf{N} = \mathbf{N} \times \mathbf{N}$. This last constraint preserves the role for the usual symmetrical bi-partition in the representation (namely the role of the phase angle $\pi/4$) but this role is now different of the metrical constraints involved by the power law m^α and the dissymmetry: α versus $1-\alpha$. This dual characteristic within the generalization gives to the α -exponential a heuristic indisputable efficiency reinforced by the numerous of experimental data as well in mechanics as in electrodynamics and even in thermodynamics [17]. Unfortunately it is balanced by an increasing level of the complexity for physical understanding. Nevertheless the topology remains that of a torus pointed by an angle $2\phi = (1-\alpha)\pi/2$. We named α -exponential the dynamics associated with such spectra even if Newton time loses its standard meaning and Noether’s principles are no more fulfilled. Without any reference to geometrical scales at this step, the

¹² It will be observed that the $1/2$ -exponential defined above conceals the capacity to transform the operator of succession upon $\mathbf{N}$ into a continuous operator. As noted elsewhere by authors $\mathbf{R}$ and even $\mathbf{R}^n$ do not provide a continuous model of the succession. To be able to do this, it is necessary to use the complex plane, therefore the stacking of $\mathbf{N}$ using $\mathbf{N}$ as fiber. A clever stacking of the $1/2$ -exponential leading the birth of zeta function appears as the expression of this possibility.

¹³ The best proof of the absence of absolute time, and the observation of a requirement for renormalization to bound the dynamic according to the flows of extensities considered. The unit of relevant “time” is a value that becomes a function of this flow in the self-similar context (See J. power Sources 26, 179-185, 1989.)

parameter α expresses nothing other than the inhomogeneous partition mentioned in the introduction. In a context that became "fractal" through recursive partition-iteration, the use of "the Newton time" seen as the ability to express the indexation of the recursive process, -valid in the case in Peano self-similar structure ($1/2$ -exponential)-, is not authorized. We cannot write any standard differential equation but only formal *fractional differential equations* without any local meaning [16-18] associated with many questions about boundary conditions. Nevertheless "harmonic analysis and transfer function keeps physical meaning in the frame of distribution theory the basis of the TEISI model [16,17]. In particular fractal capacitor behaves like a Choquet non additive capacity. Although the representation of the spectrum is nothing but an arc of a circle, therefore similar to the previous one with $\alpha=1/2$ (figure 2) the asymmetry of the couple of dual geodesical arcs Z_α et $Z_{1-\alpha}$ causes the impossibility of constructing a unique norm over the $\mathbf{N}$ set valid for the metrical duality, in other words for inversion of the real axis which backs the entropic factor. At least a couple of different norms exists depending on whether the circle is considered either clockwise « α » or anti-clockwise « $1-\alpha$ ». So we discover then surprisingly, that the physical norm to be given to $\mathbf{N}$ depends on the orientation of the complex plane. We have to switch back the reasoning to understand the strange paradox according to which we are no longer dealing with a pure standard dynamic but with a moving entanglement between experimental non additive measures and the context of these measures. This scientific approach gives a surprisingly engineering content to Schopenhauer idea expressed in: *Die Welt als Wille und Vorstellung*.

The only relevant relations in this generic bi-partitions featuring a heterogeneous context is given by the α -exponential spectrum and *Scale-Frequency* relations [16]. Both variables are connected through the non-integer dimension $d=1/\alpha$ in accordance with the simplest Mandelbrot's formula η^d . $N \sim 1$. Beyond, the writing of $\mathbf{N}$ as a component in the field of pure complex numbers (TEISI 1978) makes it possible to express without ambiguity the decoupling between the harmonic content of α -process (extensive variable requiring counting and using a unit) and the of Hausdorff weight measure (related to dynamical test function over the fractal geometry therefore an intensive ball of measure requiring another norm). While considering via the product (here expressing an open and unbounded new phase space), the convolution of a physical process with

the geometry (monadic unfolding)¹⁴, and its norms must obviously be related via the ontology of the experimental object. For a given α -metric, this complex relation expresses the correlations between a process and a fractal context. If we consider that the magnitude of a given spectral component is a quantity attached to the field $\mathbf{N}$ (discretely normed with an external clock), the α -spectral analysis takes into account an n -indexed set of durations and no longer infinitely precise moments like in differential approach. The discretization becomes intrinsic with respect to the dual analysis and the link to the continuum is, at this step, ignored and entrusted to categorical limit and colimit of the topos only. Accordingly, although without any usual temporality, this approach requires the introduction of an external clock for defining the arithmetic unit. Indeed, by taking up the conclusions of Carlo Rovelli [2], according to which physics is not interested in objects but only in their relations, -assertion which implicitly promotes the categorical approach-, any change is thus only relative; any experience even in absence of "temporality » requires the availability of a reference clock, namely an annexed stable *Space-Time* namely: "*any arc of circle requires a circle arc as support*"; an exponential geodesic is required to define a singularity there; a straight line or a plane are required to probe a curvature etc. Annexed pendulum serves only as a metronome (by its escapement mechanism the reference clock is first a metronome and measures a duration before assigning, thanks to its dial, a time to it). The pendulum ensures that any measure of change in any set of systems can be referred to him, thus it becomes a *categorical assistant* in the frame of a reality which must only be handle through a categorical *adjunction* [4-8]. Then, even under the feature of "non-exponential", any hyperbolic system needs an exponential reference and in addition an external method of compactification to analyzed unbounded process to acquire experimental meaning. The clock no longer defines ordinally the moment n but its cardinality $(n, <)$ that why any topos requires the definition of a *classifier* [7,8]. Order classification paves the way for the use of rational set numbers $\mathbf{Q}$. At this stage, we can suspect that the continuous is not very far from our issues¹⁵. Indeed: (i) α has at least any value between 1 and $1/2$. By inversion

¹⁴ Symmetrization is associated with quadratic self-similarity of natural number $\mathbf{N} = \mathbf{N} \times \mathbf{N}$.

¹⁵ The skip from $\mathbf{Q}$ to $\mathbf{R}$ corresponds to a completion operation for which there are two approaches either in terms of Cauchy sequences or in terms of cuts (Dedekind) in $\mathbf{Q}$. Categorically this double formalism is based on the notions of Cartan filters and ultrafilters. The information conveyed by the zeta function is precisely the existence of these couple of transformations reduced to the scaling properties of $\mathbf{N}$ seen as a set not above the set of prime

The authors have shown that the zeta function expresses an interpolation between the discrete set of ordinal counting (entropic) and the continuous set of cardinal measuring (geodesic); this observation is only a natural extension of the theorems of Craig (in logic of predicate) and Pitt and Beck-Chevalley (Category Theory) in the field of experimental constraints. It should be noted that the links between the Riemann zeta function and fractional operators are currently under questions [24,25] but that none of these works are based on category theory even if J. Tenreiro. Machado and Y. Luchko confirmed the link with scaling properties [25] nodal in the TEISI approach extended buy a zeta covering [21-23].

The strangest observation about the dynamic meaning of dual arcs (completion of non-integer geodesics) as dual spectra of fractional dynamics, is the fact that to understand the content of the universal transfer function $Z_\alpha \sim 1/[1+(i\omega\tau)^\alpha]$, it is necessary to implement the α -exponential phase angle $\varphi=(1-\alpha)\pi/2$ that is to say by an easy change of standard axis of the complex plane, to refer the analysis as for stochastic processes (figure 1 right)¹⁶. The new expression of the phase angle is $\psi=\pm(1-2\alpha)\pi/4$. It becomes symmetrical with respect to anew complex plane orientation and therefore the Grothendieck's topos (requiring finitary limits and cardinal classifier) built on dual "geodesics" using α and $1-\alpha$ becomes implicitly referred to the analytical content of the Riemann zeta function $\zeta(s)$ with $s=\alpha+i\omega\tau$, namely its set of non-trivial zeros if Riemann hypothesis is true. The analytical features of this stochastic reference is based on arithmetic quadratic scaling of the product $\mathbf{N}=\mathbf{N}\times\mathbf{N}$, controlling through $\alpha=1/2$ this set of non-trivial zeros [22, 25-28]. The consequences of the symmetry breaking with regard to stochasticity, namely $\alpha\rightleftharpoons 1-\alpha$ becoming $\alpha+1/2\rightleftharpoons 1/2-\alpha$ is epistemologically radical because with the non-symmetrical structure of the Grothendieck's topos being based on the adjunction $Z_\alpha\rightleftharpoons Z_{1-\alpha}$, the α -exponential dynamics matches in the complex plane a projection of the functional zeta equality $\zeta(s) = F[\zeta(1-s)]$. Due to the θ -covering required for zeta building (complex dimension, scaling and staking are related together), and far from the dynamical morphisms, this functional equation appears as the heart of the

¹⁶ The use of spectral dimensions which gives a referential role to the $1/2$ -exponential is due to the fact that this function is the natural receptacle of quadratic self-similarity $\mathbf{N}\times\mathbf{N}=\mathbf{N}$. We can measure its importance through the symmetrization of the α -exp via the $1/2$ exp. For lack of unfolded singularity on the 1-exp this symmetrization must obligatorily pass through the $1/2$ exp, hence a double game of self-similarity that can only be analyzed through Kan's extension. The stochastic aspect can therefore never be evacuated de l'analyse fractionnaire.

Scale-Frequency entanglement. Memory and prophecies could then be the fruits of this entanglement [1].

For giving additional precisions, let us recall that the universal hyperbolic Cole and Cole transfer geodesic namely $Z_\alpha \sim 1/[1+(i\omega\tau)^\alpha]$, is associated since 1978, with the convolution of a locally linear process (Differential Equation of order one, i.e. a local exponential) and an interface for exchange of extensity equipped with structure a fractal dimension d such that $\alpha=(1/d)$ relation is valid only in the simplest case of *Scale-Frequency* coupling but other coupling canal so be considered [29]. Matching with a infinite numbers of possible norms and test functions, the unfolding of dynamic of any irreversible process related to exchanges of extensities (of electrons, ions, momentum, entropy, energy, etc) within a self-similar geometrical context, must be seen as a scan with regard to the scaling. It is expressed via spectra. Initially for experimental reasons, the TEISI, was focused on fractal *Scale-Frequency* coupling law which determines not only an *entropy factor* via the real component of Z_α but also an *anti-entropy factor* via $Z_{1-\alpha}$. This anti-entropic term is associated with internal arithmetic correlations, namely a memory, able to be matched with an virtual inversion of the flow of stochastic time (identical to Newton time) as seen by mean of the phase angle ψ . When the *Scale-Frequency* coupling can be at least partly split, this anti-entropy which is the foundation of any creative processes, appears as the completion to exponential standard¹⁷. This reference is here due to the local law of exchange that if it can be chosen arbitrarily is often referred to first order processes (Newton time). Therefore, for a given fractal dimension and a given entropic dissipation, the anti-entropy magnitude can always be mastered. This characteristic offers the opportunity for managing the creative process involved by any dissipative process subject to a “wasting time”.

6. Quid about “time-gap”: toward a wasted time?

The linear interpretation of the irreversibility of time is usually based on an analogy with the coproduct categorical operator defined from the monoidal straight line $(\mathbf{N},+)$ but spectral approach of the dynamics the much greater importance of the inversion namely the roles of $(\mathbf{N},x)$ and of A . The relation between both monoid appears the issue to be considered, the ordinal scale being the new temporal beat, now equipped with a phase (gap located at the

¹⁷ In fact we come at the next stage with the introduction by two distinct pathways of the zeta function building.

origin of the straight line). This presentation arises the issue concerning the status and characteristics of infinities: what kind of infinity is concerned and thus why kind of zeros? Due to the difference between set and the Cantor set of subsets namely diagonal proof of the existence of different level of infinities, above analysis leads to distinguish countable infinite $\infty_N(Z_\alpha)$ from continuous infinite $\infty_c(Z_{1-\alpha})$. This distinction is valid even for $\alpha=1/2$ and its symmetrical adjunction based on $\mathbf{N}:\mathbf{N}=\mathbf{N}\times\mathbf{N}$, namely the content of the octaves. It rises the general question of the structure of the singularity at left and at right. The structure of the dynamical site over the dual and spectral structure of a Grothendieck topos appears as the heart of this issue.

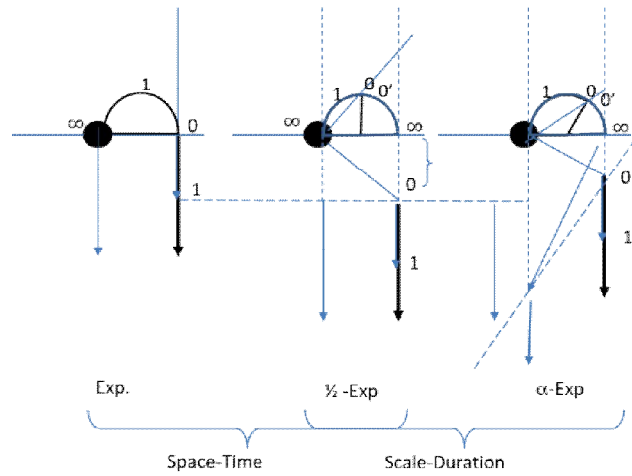


Figure 3. Three possible diagrams of inversion of the semi-infinite straight line with respect to a point external to this semi straight line according to the position of this external point and the value of the norm of the operation of succession in the set of natural numbers $\mathbf{N}$.

Why to think of the symmetrization using spectral dimension as the most useful breakthrough for representing the dual Grothendieck's topos ${}^{1/2}Z_1.Z_\alpha \rightleftharpoons Z_{1-\alpha}$? The standard exponential transfer function Z_1 points very precisely, hyperbolic distances $u/v \sim 1/(v/u)$ on the semicircle via the symmetry/inversion of the frequencies $[1, \infty] \rightarrow [1, 0]$. However, the linearity $1 \sim \eta.(i\omega\tau)$ hides the interest of this hyperbolic symmetry that can be also pointed out two time in the topos merging Z_α and $Z_{1-\alpha}$. The insights required for understanding the issue are, up to any isomorphism, based on the positioning of the outer reversal point A and O in the complex plane with regard to the referencing line supporting the both monoid $(\mathbf{N}, +)$ and the determination of the unit required for the successor operators over both transfer functions. From that three observations must be done:

First observation: the transfer function of the exponential is a whole semicircle, parameterized by $[1, \infty_{\mathbf{N}}]$ therefore directly related to usual arithmetic in $\mathbf{N}$: $[n \rightarrow n + 1] \rightarrow [n \rightarrow n \pm f_{\eta}(\pi)]$: i.e. exactly the operation that hides behind the measurement balls required for Fourier or Laplace transformations implementation (Figure 1). Due to the normalization of the lines to be reversed (unit of succession), it is also symmetrized to the standard unity value and $u/v \rightleftharpoons v/u$ the second part being parametrized over $[0,1]$. The unit becomes inseparable from the $1/2$ entropy associated to the exponential relaxation (Transfer resistance) expressed mathematically through the Euclidean point/line minimal distance. This means that the flow of entropy is minimal in an irreversible exponential process.

Second observation, bounded stochastic transfer function $Z_{\pm 1/2} \sim \pm 1/[1 + (i\omega\tau)^{1/2}]$ associated to diffusion processes, is represented by a couple of half arcs of a circle. The tmonoid $(\mathbf{N}, +)$ axis "temporal straight line" here reversed is characterized by a gap at the origin. The $+1/2$ -exp spectrum can be completed with a similar arc backed to a vertical symmetry in the complex plane. This symmetrization occurs with an inversion of the orientation of the real axe and a permutation of the imaginary axis and the monoid $(\mathbf{N}, +)$ axis. The symmetrical group in the complex plane not only has a reference $(1, i)$ but also another one obtained by a $\pm \pi/4$ -rotation of the complex plane axes $(\pm i^{1/2}, \mp i^{1/2})$. The phase angle defines the phase locking of the global process expressed by the fractional derivation operator $\alpha=1/2$. Above observations allows the design of a dynamical "topos" merging causal (positive time) and non-causal (negative time) processes. The symmetry of this topos confirms the relevance of writing the Laplacian operator using a product of $1/2$ -operators with $(d^2/dx^2 - Dd/dt) = (d/dx - D^{1/2} d^{1/2}/dt^{1/2})(d/dx + D^{1/2} d^{1/2}/dt^{1/2})$ respectively causal (entropic) and non-causal (anti entropic) product. Here anti entropy is only due to uncertainty related to the freedom of referring the octave $(\mathbf{N}=\mathbf{N} \times \mathbf{N})$. The limit and co limit are always countable. Entropy and anti-entropy links are kept through all symmetries. Due to the symmetries highlighted above, the sign of the time variable is irrelevant if the physical process is expressed in its topos, namely here with referential of energy¹⁸.

Third observation, the physical meaning of non-causal component may be generalized. The universal transfer function $Z_{\alpha} \sim 1/[1 + (i\omega\tau)^{\alpha}]$ also occurs itself as an arc of a circle of exponential support. However, in this

¹⁸ If the gap is associated with an inversion of $\mathbf{N}$ (the field itself associated with time) then the link between the flow of time and entropy (here stochastic) is constrained by the same dynamic norm.

case a part of the previously mentioned symmetries revealed for the $1/2$ -exp disappear and the symmetrical non-causal completion is now ensured through the complementary spectrum $Z_{1-\alpha} \sim 1/[1+(i\omega_c\tau)^{1-\alpha}]$ up to any isomorphism. In this case, the temporal backing of both components ensured by the physical meaning of Laplacian operator, if $\alpha=1/2$, disappears to reappear rethought and transformed by the categorical approach of the set derivation due to the bipartition $n=\omega\tau$. The role of the Riemann zeta function and the Kan transformation then emerge of the matching of the couple of sets $\mathbf{N}$ in the bipartition $\mathbf{N} \times \mathbf{N}$. Despite obvious differences, above analysis highlights a remarkable categorical filiation between standard exponentiation, bounded stochasticity ($1/2$ -exp) and fractality (α -exp). This filiation is exclusively determined by the presence or not of a gap at the origins of monoids and by the relative structure of these gaps when ruling the relationship between entropy and anti-entropy. In the frame of simplest TEISI model, characterized by an absence of inverse Fourier transform of transfer function, two methods for introducing the time must then be considered:

A direct way based on the dynamic transfer topos (basis of covering for building the dynamical site). Riemann zeta function is constructed via the transformation $1/n^\alpha \rightarrow 1/n^{\alpha+i\theta}$ with the partition $n=\omega\tau$. θ matches or dualizes the time constant τ . REE Communication [22] shows that this operation corresponds to a shift of complex scales with a fractional of order $s=\alpha+i\theta$ on one hand and to a stacking of n via $\theta \in \mathbf{N}$ for a set associated with a quadratic self-similarity $\mathbf{N}=\mathbf{N} \times \mathbf{N}$ on the other hand.

An indirect way: as indicated elsewhere, the only contribution of the TEISI model with respect to Benoit Mandelbrot's starting point of view was the introduction (1978) and for engineering reasons [16] of the complex content $1 \sim \eta(in)^\alpha$. This choice immediately raised the question of the root of the unit namely $1 \sim e^{i\theta \log(n)} \times e^{-ki\theta \log(n)}$, and then open the question of the covering of the topos using $\pm\theta$ as a free external clock with respect to the process. This expression of the root of the unit, leads the introduction of the Riemann zeta function $\zeta_\alpha(s)$ by using the spectral geodesic extension based on the generic term $1/(n)^{\alpha \pm i\theta} = (i)^\alpha \eta n^{\pm i\theta}$ [21]. We can note the “circular” character of the formulation with the complex term “i” appearing both linearly and in the power law therefore when the sign of θ changes, the presence of a fix $(i)^\alpha$ leads a phase shift which can only be balanced if $(i)^\alpha \rightarrow (i)^{1-\alpha}$ namely if $\zeta_\alpha(s) \rightarrow \zeta_{1-\alpha}(1-s)$. Thus, in the framework of scaling site with $\alpha \neq 1/2$, the sign of θ plays a role which is no more irrelevant. Then distinct of the possible time constants τ , it becomes

irreversible, involving both the ontological richness of the zeta function and all the difficulties in solving Riemann hypothesis using only analytical approach [23].

The understanding of what temporality means, not only as a flow but also as memory, -here imposed via α -, is then backed to the understanding of the physical meaning of the dual ambiguity concerning the position of the starting axis for the enumeration¹⁹ of cycle, ambiguity also contained in the dual arithmetical expression of the zeta function $\zeta_\alpha(s)$ as additive and product series *in fine* expressed through the functional relation $\zeta_\alpha(s)=F[\zeta_{1-\alpha}(1-s)]$. It is a very old problem of geometry but which we take here in a doubly singular way: first (i) by transforming it into a dynamic problem in d -fractal media with $d=1/\alpha$ (arithmetic approach) whose spectral content is linked to self-similar scaling laws, scaling transposed in a second time, (ii) into the zeta function using a scaling with a complex order $s=\alpha+i\theta$. Let us observe that the θ -unfolding along the stalk plays a role similar to the product $n=\omega\tau$ on the manifold supporting the dynamics. Due to the prime numbers decomposition of n , entangling the variables in a particularly complex way normalization plays here a major role. Let us observe that, θ -stacking then plays a role similar to a magnetic field in the Stern-Gerlach experiment. It helps to unfold the degeneracies hidden in the multi temporal unity of the dynamic. But the stacking along θ -stalk is itself dualized because the straight line expressing the monoid $(\mathbf{N},+)$ and its inversion yields in the framework of α -fractional bi-partition, namely with two distinct parts. A part related to the inversion of $\mathbf{N}$ with respect to the point A outside the line to which must be adjunct the spreading of $m=\omega\tau$, giving a distribution of subset τ by using the fundamental law of arithmetic $m=\prod_i p_i^{r_i}$ with p_i primes number and $r_i \in \mathbf{N}$, namely new form of inversion $[n \rightarrow 1/k]$. This last unfolds in the gap $[0,0]$ located at the root of the straight line to be inversed. The origin is no more a point but an enriched structure that can only be expressed arithmetically and, as Cantor did, unveils a complex set of infinites. The phase angle defines this gap. Due to the existence of a memorization factor that characterizes the boundary, this enlargement at the origin, this rooting is obviously fundamental and deconstructs the "Newtonian time" associated to a squeezing of this gap to a simple singular point. This n -spreading using primes bases the structures of the memory and manifests itself dynamically through the nonzero value of all fractional derivatives $[x (1/i\omega t)^\alpha]$ of the

¹⁹ This position requires the extension of the field of $\mathbf{N}$ to $\mathbf{Q}$, then to the **p-adic** field and finally to $\mathbf{R}$.

constants [18]. What is important is to understand the dynamic content of this gap which obviously has a direct link both with the quadratic arithmetic self-similarity $\mathbf{N}=\mathbf{N} \times \mathbf{N}$, and with the order α of the fractional metrics (locking of the angle phase)²⁰.

To sum up, the temporal issue in a self-similar bi partition context (α and $1-\alpha$) can be reduced now to give a couple of straight line site parametrized by $n \in \mathbf{N}$ and compactified by using an inversion: $[n \rightarrow n + 1] \rightarrow [n \rightarrow n \pm f_n(\pi)]$ with respect to any point A located outside of the line such that the norm (namely the 1/2 entropy) be pinned not by means of a single loop (N-Norm: $im=i\omega\tau$) but using a couple of loops ($\mathbf{N} \times \mathbf{N}$ -Norm: η_α , and $\eta_{1-\alpha}$). Both dynamics give birth together to a Grothendieck's topos. The needful matching of both norms reveals the origin of the degeneracies of the fractional dynamic, here multi-valuated through the unfolding of the complex planes along a set of stalks giving rise to a sheaf. While Newton time, linearly related to the hyperbolic spectral distance η_α is flowing, many criss-crossings on stalks gives birth to a memory (a multi-valuation due to the discrete self-similarity). The need for steady reference call for an external $i\theta$ -parameter ensuring the slicing into an infinite number of stacked washers. Due to the functional relation $\zeta_\alpha(s)=F[\zeta_{1-\alpha}(1-s)]$, the couple of Riemann zeta functions, namely $\zeta_\alpha(s)$ and $\zeta_{1-\alpha}(1-s)$, don't play the same role. The role of synthetizing above issue is ensured via the zeta function, either analytically via its non-trivial zeros (Riemann hypothesis: $\zeta_{1/2}(s)=0$, $\alpha=1/2$) leading a reversible time that can be assimilate to a Newton time, or categorically via a Grothendieck's topos, taking into account fractional dualization and dissymmetries. They radically exclude the reversibility of the proptime (excluding non-trivial zeros $\zeta_\alpha(s) \neq 0$, $\alpha \neq 1/2$) with regard to any type of referential. It will be observed that both loops building the topos, are linked through a dual monadic structure based on the *Scale-Duration* duality, a new fuzzy differential form discretized along scaling.

7. Conclusion about an arrow of time

Paradoxically, thinking about time requires first ignoring it to focus attention on the universal unity of any evolution. Any change must be tamed

²⁰ Note that the structure of the point at infinity joins the categorical approach concerning the role of ω_1 the first level of non-countable set. The point at infinity of the spectrum is countable in scale but it is uncountable in frequency. It is exactly by reaching this issue that the zeta function acquires its status as an intermediary between the continuous and the countable.

within the framework of category theory using the covering of a Grothendieck topos (initial/sheaves/final) for thinking the related dynamical site. For simplifying due to arithmetic recursion, the topos can be thought as a monadic structure (ultimately as an intermediated causal predicate). Nevertheless, in accordance with Cartesian practices, consciousness enjoins us to consider the the unfolding of this unit. This fragmentation must be ordered according to an indexation in the field of natural numbers N , but the ontological unity of the change imposes compactness and therefore the Cauchy convergence of the series leading categorical limits countable and representable. All of these constraints lead to analyze the role and the duality of succession and inversion operators:

- the succession is expressed by means of a normalized half-line $[0, \text{potential-infinity}]$ whose additive subsets can be indexed on the fields $\mathbf{N}$, $\mathbf{Q}$ or even but not without cautions via the real set $\mathbf{R}$. In $\mathbf{N}$ the simplest indexing operator is the succession operator $t \rightarrow (t, t + 1)$ but in other field a simple order relation $p < q$ is required. In all fields, a function $f(t)$ can be analyzed from its local derivative $df(t)$ if this last can be defined. This ordinal variable will be termed “the time”. It is the most intuitive parameter of the dynamical automorphism for a change controlled via a function.
- Nevertheless, a main duality in discrete indexing in $\mathbf{N}$ requires the *distinguo* between ordinal (extensive) and cardinal (intensive) character of arithmetic index. This duality gives birth to a complementary of “temporal called “spectral” approach. It consists in to distinguish a counting and a weighting. Since the works of Fourier and Laplace such approach was extended for instance recently in the frame of algebraic geometry, operator algebra and category theory. Roughly it consists in sharing any finite function of $f(t)$, or operator into an indexed sum of cycles (or measuring ball) with increasing normalized radius, namely $\nu: \omega\tau=2\pi\nu\tau$ so that the automorphism splits the global unity by means of a series of inclusive sets possibly under-additives. According to a projective categorical approach based on the monoid $(\mathbf{N}, x)$ the spectral representation operates using an inversion of the ordinal order featured by half-line by using any singular point A outside of it pointing out the duality. Virtually this approach consists in cutting the referencing straight line according to a scaling view $[[[\infty]]]$ infinitely far] so that it implies without additional hypothesis distinguishing two infinities: an infinitely far and countable limit ($t \rightarrow \infty$) but also in addition and beyond an infinitely small, associated with the product operator $\lambda \rightarrow (\lambda, k\lambda)_{k \rightarrow \infty}$ division which

involves the set of subsets and thus open the question of Cantorian infinities ($2^t \rightarrow \infty_c$). The role of indexation of these subsets with regard to degeneracies and ambiguities resulting of the covering between these sets must be considered using additional tool for example the Riemann zeta function which extend the scanning of indexation using the complex field by bridging countable set and continuous set..

Both approaches are obviously only partially independent and can lead to occurrence an involution. This is the case of the direct and inverse Fourier transforms. However, as seen above, we are much more interested by the absence of involution which ultimately leads to consider a categorical square scheme. Such kind of scheme brings in question Kan extensions and diagonal functors. This note asserts that the understanding of temporality can only really be understood from the dynamic site approach as defined above like the dynamical bi-category and its extension within a square diagram. The "time" as dynamical automorphism parameter only comes second. It emerges if commutativity and involution become relevant.

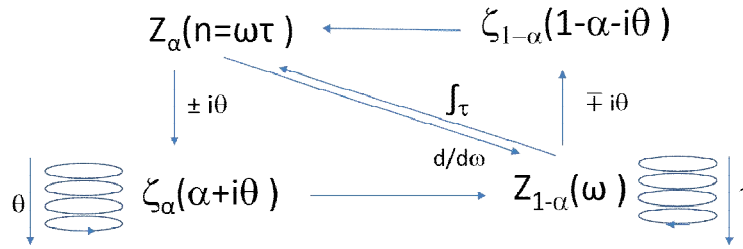


Figure 4. Square diagram characterizing the dynamical site. Causal process is represented at left. The a-causal dual is at right. The foliations below are associated to time like parameters. The upper foliations are spectral like. The diagonal gives rise to a Grothendieck's topos. The diagram commutes if and only if $\alpha=1/2$ (Riemann hypothesis). Due to a phase angle based on α , the proper time of the system θ is reversible if and only if $\alpha=1/2$.

Beyond this, the morphisms should be analyzed through sheaves weaved above Grothendieck's topos here makes up from the sum of dynamical spectra Z_α and $Z_{1-\alpha}$. For the unique dynamic automorphism parameter, called "time" τ to emerge, there must first exist a norm on the half-line on which the homogeneous ordinal indexing is built. In addition it must exist a cardinal weight $\Lambda \in X$ capable to give birth to an intensive measure of scale $1/\Lambda$. Thus the norm results from an intensity (measurement)/extensity (count) linear relationship. This dual approach gives birth to the *distinguo* usually called "space" and "time". The unit of this dual category can usually be

It will be observed that the laws of arithmetic make possible to write the self-similar quadratic relation $\mathbf{N}=\mathbf{N}\times\mathbf{N}$ without any change of indexing therefore it exists at least one other class of so-called probabilistic norms such as $1\sim\tau/\Lambda^2$. This paradigmatic case characteristic of quantum mechanics and statistical thermodynamics relates to $\alpha=1/2$. Referred to the deterministic case using linear space-time relationship, this form expresses a change of octave for the dial size thus a simple change of frame (real axis $\rightarrow$ imaginary axis) expressed by a symmetry with regard to $\pi/4$ axis; a simple change of point of view with $\pm (i\tau)^{1/2}\sim \pm\pi\theta$ if $\theta\in\mathbf{N}$. Subject to taking the hyperbolic distance, i.e. the inverse of the intensive scale, as a measure of length, θ and $n=\omega\tau$, are commensurable to $\mathbf{N}=\mathbf{N}\times\mathbf{N}$ up to the numerable infinity and thus θ can play the absolute Newton time role (Kan extension is valid). At this stage the model is quite poor closed and in many respects simplistic. Indeed the strength of probabilistic models of uncertainty and white chaos is to fit Noether's principles by adjusting the complex plane axes by implicitly turning the aera of the dial of the clock into a Peano curve. Peano's arithmetic follows. The quadratic nature of arithmetic expressions justifies the adequacy of mechanical action as relevant variable, just as the quadratic nature of energy justifies the invariance of physical laws whatever the sign of the time variable. However, it remains that the process acquires this characteristic only within the framework of a perfectly balanced duality which merges a causal action (affected by an entropy) to an a causal complement. Gaussian thermal fluctuations closes the spectral system upon itself by turning quadratic correlations into numerable and ordered uncertainties, hence the emergence of the Planck factor as categorical limit unit $\hbar=h/2\pi$. Beyond, it will be noted that the uncertainty generated by the fluctuations leads to an uncertainty induced on the metric the underlying dimension being then almost surely itself tainted with uncertainty with $\alpha=1/2\pm\epsilon$. This metrical uncertainty justifies the Connes-Rovelli hypothesis, their *thermal time arrow* taking its meaning in the analogy between Tomita's theorem applied to von Neumann algebras and the KMS model, asserting herein that temperature T is the unique fluctuations/correlations parameter of thermomechanical issues. The Riemann hypothesis related to the pure metric $\alpha=1/2$ leads to analytical exchange processes, as the set of countable zeros of the zeta function being doubly probabilistic locally via $\hbar$ and globally via $\epsilon(T)$. Represented by the synthetic diagram below, the case $\alpha=1/2$. is paradigmatic because it expresses both the dynamics of a bounded and complete chaos and the link of this type of chaos with

deterministic rules. The time to be used is ultimately standardized over integer metrics in a unique way and the arrow of time can only be “thermal”.

Subject to a great deal of precautions, the approach can be generalized by writing $1 \sim m^\alpha / \Lambda$ for any geometry with metric $d=1/\alpha$ integer or not (fractal geometry). The extension of figure 5 into Figure 6 highlights precisely the need to distinguish an indexing (extensive-time) from a measurement (intensive-space) even if the quantities must both be written within $\mathbf{N}$ (TEISI model). While the Newtonian approach implies $\alpha=1$ and the probabilistic approach requires $\alpha=1/2$ the fractal approach for example in the simplest general case corresponds to $1 < \alpha \leq 1/2$. The precautions not to be ignored then relate to two items:

- (i) The requirement for considering the power of the unity which is expressed by means of a dual stacking $1 = e^{-i\pi\theta \log(m)} e^{+i\pi\theta \log(m)}$. This relationship involves the dummy variable $\pm\theta$ and the calibration of the dial of the associated clock via $\log(m)$. This calibration adjusts the covering of the dynamic site (topos plus sheaves) onto the design and the building of the zeta function.
- (ii) By taking into account the adjunction $\alpha \rightleftharpoons 1-\alpha$ we implicitly introduce completion into the topos by referencing both the Euclidean norm and the probabilistic norm in the framework of a topos by means of pure stochastic dynamics: $\alpha+1/2 \rightleftharpoons -\alpha-1/2$; indeed the above generalization with $m \in \mathbf{N}$ allows to write (TEISI model) the relationship $1 \sim m^\alpha / \Lambda \times m^{(1-\alpha)} / \Lambda = m / \Lambda^2$.

It is only at this stage that the importance of the Riemann zeta function really appears as the theoretical intercessor between a given real dynamic (Z_α) and the whole field of possibilities attached to it ($Z_{1-\alpha}$) in an arithmetically coherent way. However, we will see that this intersession is not direct. We are going to show that the physical meaning of this real/possible completion leads to the emergence of an arrow of time defined by the behaviour upon the categorical limits. Indeed due to the relative ordered positioning of the dynamic functors Z_α versus $Z_{1-\alpha}$, as soon as the complex plane, equipped by a dual spectral structure $\alpha \neq 1-\alpha$ and a critical phase angle becomes a Grothendieck Topos and even before unfolding this topos via sheaves for implementing zeta Riemann function, the sign of θ can no longer be considered as free regarding the choice of the complex plane orientation. Indeed, the unfolding can turn

clockwise or anticlockwise even if the sign of θ is given. More precisely if $s = \alpha \pm i\theta$, used for the stacking above Z_α gives rise to $\zeta_\alpha(s)$ it follows that $s = 1 - \alpha \mp i\theta$. The stacking above of $Z_{1-\alpha}$ gives rise to $\zeta_{1-\alpha}(1-s)$ with inversion of the initial orientation of folding. Hence, the sign is not free. The constraint is due to completion of folding rotation in the spectral adjunction $Z_\alpha \rightleftharpoons Z_{1-\alpha}$ and the clock discrete temporal analysis (different from TF-1) carried out using the operator of rotation $\pm i\theta$, or more precisely the screening operator $\exp(\pm i\theta\pi \log(m))$.

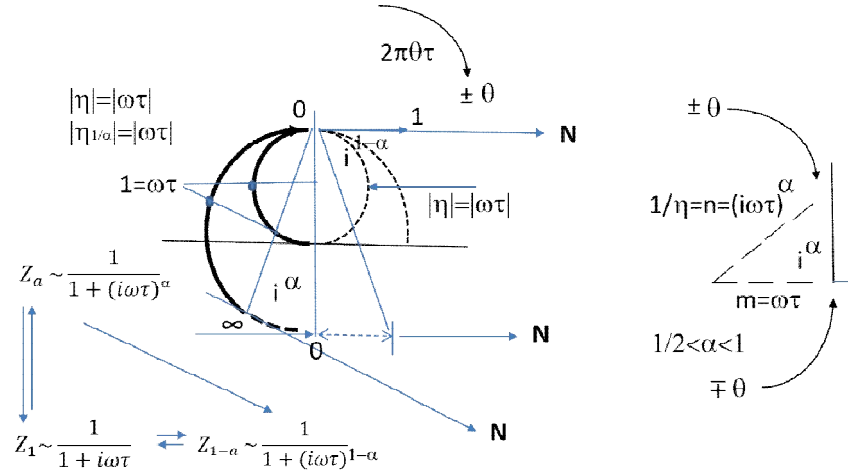


Figure 6. Extension the figure 5 via a symmetry breaking introduced into the Grothendieck's topos for ametric $\alpha : \frac{1}{2} < \alpha < 1$. The extrinsic foliation of the dynamic site seen as a stacking of oriented plan complexe parametrized by $\theta \in \mathbf{Z}$ then challenged the intrinsic foliation based on the temporality $2\pi\nu\tau = \omega\tau$ attached to $Z_{1-\alpha}$ via a parametrization using a proper time τ . The commutativity of natural transform the square diagram is then broken, as long as the arithmetic rules are fulfilled.

Moreover the related morphisms require, among other things, to put in phase infinities and zeros, operation which requires to adjust specifically the current infinity carried by the phase angle and the space-time units. It is at this stage that the adjunction between the spectral α -dynamics according to the order of the scales η , Z_α and the set of time constants τ becomes interesting accordingly with Kan extension. Indeed, by taking into account the bi-partition of $m = \omega\tau$ associable to the dynamic functor $Z_{1-\alpha}$, the set of τ is associated with a set of great number of fractal structures in $\mathbf{N}$ under the constraint $d = 1/\alpha$.

If $\pm\theta$ account for the Newton's time, that of an external clock, the geometric structure of adjunction $\alpha \rightleftharpoons 1-\alpha$ (Internal/external; real/possible)

imposes the existence of a field of correlations which goes be explored using $\pm\theta$. In the discrete logic of the TEISI model which distinguishes the hyperbolic distance as a measure $1/\eta \sim (\omega\tau)^\alpha$ and the indexing of this measure by a quantity analogous to a time τ it is important to count and qualify the set of the bipartitions $m=\omega\tau$ likely in $\mathbf{N}=\mathbf{N} \times \mathbf{N}$ to give rise to an index τ . It is via the completion hidden in the adjunction $\alpha \rightleftharpoons 1-\alpha$ therefore via the functional relationship characterizing the zeta function that the arrow of time is then surreptitiously introduced. Indeed, knowing that then $dm^\alpha/d\omega = \alpha\tau^\alpha(1/\omega)^{1-\alpha}$, the fundamental equation of the model gives, $\forall\theta: i^\alpha \cdot \eta \cdot e^{\mp i\theta \log(m)} \sim m^{-s}$ with $s=\alpha \pm i\theta$, i.e. the generic term of the zeta function $\pm i\theta$, has the role of leafing through in a trivial way over the only field of octaves the real dynamics (Z_α) . To do this, we use the order relation concerning η^d , and in a subtle way by decomposition on ideal structures, those of possibilities $Z_{1-\alpha}$. In order to ensure a zeta coherence, the foliation of the real spectral dynamics is ensured in the field of integers $m \in \mathbf{N}$ normed by the factor $\log(m)$ which fixes the extension of the spectrum reduced to the modulus of rotation in the complex plane. The factor θ systematically tests the set $\{m\}$ considered as integers while keeping the phase angle constant $\varphi=(1-\alpha)\pi/2$. This foliation matches the covering of disks of increasing size without the interest of the zeta function appearing clearly at this stage. However, every disk can itself be leafed in response to the possible bi-partitions of m . This second level of foliation must be represented by means of $(Z_{1-\alpha})$ and ordered by means of $\{\tau\}$ associated with a new hyperbolic distance $\eta=\square(1/\omega)^{1-\alpha}$. This is tested via the discontinuous map: $m \rightarrow \{1/\omega\}^{1-\alpha}$. Given this constraint, the power law $\mp\theta$ is then used as the suitable clock for the dynamics (internal clock). The question is then to find the number of cycles $\{\tau\}_m$ necessary to phase all the possible m bipartitions. The catenation of the sheaf parameterized by $m \in \mathbf{N}$, leads to the replacement of the stack $\{m\}$ by a sheaf of tangled "spaghetti" whose sections give the levels of correlations for a certain set of m . The $\aleph_0$ -countable infinity of cycles parametrized by $\pm i\theta$ will give rise to an uncountable $2^{\aleph_0}$ infinity of cycles parametrized by $\mp\theta$, so that the ontological unity of the final/sheaf/initial site is guaranteed. This is exactly what Voronin's theorem and Bagchi's lemma also teach us about to approximate the distribution of zeros of the zeta function. The non-trivial zeros of the zeta function arise precisely here as an ability to deal analytically with probabilistic chaos in accordance with a symmetrization of the dynamic automorphism parameter. This property suggests treating the general case in such a way as to make $\alpha=1/2$ appear as the reference metric. Thus we assert here that the reference time is always the stochastic time $\pm\theta$ associated with $\alpha=1/2$ capable to take into account the phase angle $\pi/4$. It is then from this axis of the complex plane that the

introduction of the irreversibility of the clock time $\tau = \pm i\theta_{1/2} \mp \Delta\theta$. is explained. More explicitly in the case $\alpha \neq 1/2$, we can write $\alpha = 1/2 + \beta$ et $1 - \alpha = 1/2 - \beta$ the function $\zeta_\alpha(s)$ introduces the factor $n^{\pm i\pi\theta}$, namely $\exp[\pm(2/(1+2\beta))i\pi\theta \log(m^\alpha)]$ while $\zeta_{1-\alpha}(1-s)$ introduces the factor $\omega^{\mp i\pi\theta}$, that is $\exp[\pm(2/(1-2\beta))i\pi\theta \log(\omega^{1-\alpha})]$. Thus, due to the fact that the dynamics cannot be referred to an axis of symmetry for $\alpha \neq (1-\alpha)$ any change of orientation of the complex plane with respect to an axis of symmetry $\pm i\theta \rightarrow \mp \theta$ must take into account has a phase shift angle $|2\psi| = |2\alpha - 1|\pi/2$. Admittedly the beat of an external clock $\exp(2i\pi\theta)$ therefore $\pm i\theta$ always makes it possible to qualify and count the commensurable space/time correlations but this beat is affected by a phase which subsists at the limits of the basic Topos $\psi(\pm\theta) = -\psi(\mp\theta)$. The inversion of the sign θ , of scanning without consequences in terms of commensurability in the adequate metric $\alpha = 1/2$, acquires in fractal metric $\alpha \neq 1/2$ a decisive impact by causing a symmetry breaking. Not only do the direct and inverse rotations not concern the same categorical object (real dynamics Z_α / possible dynamic $Z_{1-\alpha}$) but also do not concern the same class of set ($\aleph_0$ -countable versus $2^{\aleph_0}$ -continuous). This “dual duality” is hidden, in the case $\alpha = 1/2$ which led one of the authors (P. Riot) to equate Riemann's hypothesis to Martin's axiom in set theory namely $\omega_1 : 2^{\aleph_0} \sim \aleph_0$ [30]. This axiom amounts to assuming that the gap between the countable and the continuous can be reduced to a Peano surface in other words a bi-partition based on $\mathbf{N} = \mathbf{N} \times \mathbf{N}$. Any complete symmetric bipartition is thus associated with the Cantorian metric $\alpha = 1/2$, each of the components then being fiberized in a numerable way, giving rise to an analytical approximation (Voronin's theorem and Bagchilemma [22]).

The general case $\alpha \neq 1 - \alpha$ based on the monadic structure schematized in figure 7 could lead to think on the one hand that the ordered gap at infinity (between countable and continuous) could be bi-partitioned asymmetrically with a mirror ω_1 and on the other hand that a permutation of the direction of rotation due to a functorial reflection could be take place in the gap with respect to ω_1 mirror. This reflection is not thinkable because it would suppose an inversion of the relation of order of complexity between the countable and the continuous, able to remove the arbitrary character of the sign of θ while preserving at the limits the categorical distinction between *countable limit* $\alpha \neq 1 - \alpha$ *co-limit continuous* but by forcing the system to adopt a statistical chaos. Thus a quantum of entropy ε_s appears at infinity which practically manifests itself by the appearance of an arrow of time that propagates the monad at all scales of functorization through the *distinguo* θ (reference clock time) and τ (the parametrization of the proper time). The difference is the universal *engine of the time*. Irreversibility is exhibited by the absence of

commutativity of the square diagram linking the categories of dynamic Topos and the Riemann functions associated to that of the site based on it (Figure 4). This absence is due more precisely to the characteristics of the diagonal of the square diagram which, like in the case of Cantor's demonstration highlights the existence of several classes of infinities, points the absence of equivalence between the test of the distribution of τ by means of θ , and the reverse test of the using- θ by means of the same τ . As it had already been pointed out the functional commutator is then different of zero: $[\text{hom}(Z_\alpha, Z_{1-\alpha}) - \text{hom}(Z_{1-\alpha}, Z_\alpha)] \neq 0$. This absence is precisely due to the difference in correlations reducible to transpositions to octaves, i.e. $\mathbf{N} = \mathbf{N} \times \mathbf{N}$, and correlations linked to the fractional metric associated with a tensorial monad over scaling. The commutativity is restored by Martin's axiom when ω_1 is a mirror between countable and continuous namely for $\alpha = 1/2$ (homogeneous bi partitions), then justifies [30] the reduction of the Riemann hypothesis to Martin's axiom and the transfer between quantum level to a countable statistic over the field of prime numbers.

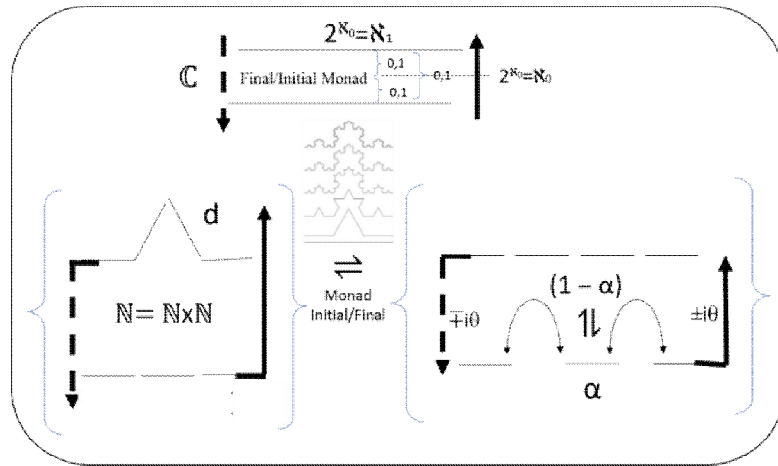


Figure 7. Synthesis of the geometrical origin of dual foliation created by fractal dynamic (TEISI model) expressed using the Grothendieck topos based on Z_α and unveiled via $\zeta_\alpha(s)$ Riemann function. This schematization highlights and expresses the open issue at infinity and significance of Martin's axiom for matching the Riemann hypothesis.

To sum up, as Rovelli points out, any change only makes sense relative to an external reference. It follows that time must be dual. This duality is revealed through dynamic sites capable of distinguishing between a reference clock beat and its dual basis as Grothendieck's topos. The double duality that emerges from the multiple bi-partitioning capacities of the real is veiled in deterministic systems but it emerges in complex systems through the tensor

structure between the proper time/clock time. Analyzed in terms of clock time, the proper time reveals a contra-variant component corresponding to the irreversibility of time as observed experimentally, in particular in recursive dynamics and self-similar media. This irreversibility is not only linked to the phenomenon serving as a test but partly linked to the topology and more precisely to the metric of dual-time structure analogous to a space-time. Constraint at categorical limits, this topology leads to the existence of an arrow of time which must be associated with a necessary entropic adjustment of the complexity over a dual infinity (countable and continuous). Indeed, at this stage the clock time and the proper time can only be constrained by the self-similarity of the countable $\mathbf{N}=\mathbf{N}\times\mathbf{N}$ which arithmetically assimilates probabilistic time and deterministic time as for example in Quantum Mechanics. Mathematically, the set of zeros of zeta Riemann function and the Martin's axiom make this constraint explicit, bringing thus the arrow of time back to a still open hypothesis.

In Memoriam: This note was written in memory of my friend José Tenreiro Machado, whose dynamism and scientific relevance I have always admired [31-35]. Rest in peace!

Author Contribution: PR is currently studying the pure categorical content of zeta function design [30]. ALM develop the idea of time starting unfolding of Grothendieck's topos through zeta function building and has written the draft of this note. AG has help him to relate fractal geometry to atomic topology and its link with quantum mechanics [11]. These studies was developed in the frame of Complex System Group implemented by DT in the Physic department of KFU [36].

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Conflict of interest: no conflict.

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NATURAL GAS PRICES AND THE CHAOTIC GROWTH MODEL

Vesna JABLANOVIC*

Abstract. *The basic aims of this paper are: firstly, to create the simple chaotic natural gas prices growth model that is capable of generating stable equilibrium, cycles, or chaos; secondly, to discover a sequence of Elliot waves in natural gas prices movements in the period 2001-2009; and thirdly, to analyze the local stability of natural gas prices in the period 2001-2009. This paper confirms the existence of a two-point attractor. Namely, natural gas prices oscillated between two values.*

Keywords: *Natural Gas, Prices, Chaos, Elliot Waves.*

Introduction

Hulshof, Mat, and Mulder (2016) analyze the day-ahead spot price at the Dutch gas hub over the period 2011–2014. Also, they find that the oil price had a small positive impact on the gas price. Changes in the concentration on the supply side did not affect the movement in gas prices. The availability of gas in storages and the outside temperature negatively influenced the gas price. Further, they find that the gas price was related to the production of wind electricity. Finally, they conclude that the natural gas prices are predominantly determined by gas-market fundamentals.

Hartley, Medlock and Rosthal (2008) investigate the relationship between the prices of natural gas and crude oil, and the factors that cause the short run equilibrium price and the long run equilibrium price. They find out that the link between natural gas and crude oil prices is indirect. They find that technology is important to the long run relationship between fuel prices. On the other hand, the short run equilibrium prices are influenced by product inventories, weather, other seasonal factors and supply shocks.

Ascari (2021) confirms that a price control is much preferable to the politically motivated price regulation. Also, Ascari (2021) discusses the theory of gas price control, provides some empirical analysis of its

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usefulness. He discusses examples of (positive and negative) regulatory practices.

Using an error-correction model, Yücel and Brown (2008) show that movements in crude oil prices have a prominent role in shaping natural gas prices. The findings of Yücel and Brown (2008) imply a continuum of prices at which natural gas and petroleum products are substitutes.

Further, Shi, X. and S. Yifan (2021) construct homogeneous macroeconomic uncertainty measures for three major natural gas markets and investigate the dynamic causal effects of macroeconomic uncertainty shocks on the formation of natural gas prices. They show that macroeconomic uncertainty plays an important role in determining natural gas price variations. Compared to oil price, the uncertainty impact on gas pricing is relatively moderate but significant. Market-specific supply and demand analysis shows the drivers of gas prices are substantially different in three different markets.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can explain very complex behavior. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Day (1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2020) etc.

The Model

Increases in natural gas supply generally result in lower natural gas prices, and decreases in supply tend to lead to higher prices. Increases in demand generally lead to higher prices, and decreases in demand tend to lead to lower prices. In turn, higher prices tend to moderate or reduce demand and encourage production, and lower prices tend to have the opposite effects.

The chaotic natural gas price growth model is presented by the following equations:

$$D_t = \alpha - \beta P_t \quad \alpha > 0, \beta > 0 \quad (1)$$

$$S_t = \gamma + \delta P_t \quad \gamma > 0, \delta > 0 \quad (2)$$

$$\frac{P_{t+1} - P_t}{P_t} = \mu(D_t - S_t) \quad \mu > 0. \quad (3)$$

Where P_t – natural gas price; S_t – supply function of natural gas; D_t – demand function for natural gas; μ – the adjustment coefficient; α, β – the

coefficients of the demand function; γ , δ – the coefficient of the supply function.

(1) defines demand function for natural gas; (2) defines supply function of natural gas; (3) determines the relation between natural gas price growth rate and surplus of demand for natural gas.

By substitution one derives:

$$P_{t+1} = [1 + \mu (\alpha - \gamma)] P_t - \mu (\beta + \delta) P_t^2, \alpha, \beta, \mu, \gamma, \delta > 0. \quad (4)$$

Further, it is assumed that the current value of the natural gas price is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the natural gas price growth rate depends on the current size of the natural gas price, P , relative to its maximal size in its time series P^m . We introduce p as $p = P/P^m$. Thus p range between 0 and 1. Again we index p by t , i.e., write p_t to refer to the size at time steps $t = 0, 1, 2, 3, \dots$. Now growth rate of the natural gas price is measured as

$$p_{t+1} = [1 + \mu (\alpha - \gamma)] p_t - \mu (\beta + \delta) p_t^2, \alpha, \beta, \mu, \gamma, \delta > 0. \quad (5)$$

This model given by equation (5) is called the logistic model. For most choices of α , β , γ , δ , and μ there is no explicit solution for (5). This is at the heart of the presence of chaos in deterministic feedback processes. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The map was popularized by the biologist Robert May (1976). The logistic model was originally introduced as a demographic model by Pierre François Verhulst.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t), \pi \in [0, 4], z_t \in [0, 1] \quad (6)$$

is equivalent to the iteration of growth model (5) when we use the identification

$$z_t = \frac{\mu (\beta + \delta)}{[1 + \mu (\alpha - \gamma)]} p_t \quad \text{and} \quad \pi = [1 + \mu (\alpha - \gamma)] \quad (7)$$

Using (5) and (7) we obtain:

$$\begin{aligned} z_{t+1} &= \frac{\mu (\beta + \delta)}{[1 + \mu (\alpha - \gamma)]} p_{t+1} = \frac{\mu (\beta + \delta)}{[1 + \mu (\alpha - \gamma)]} \{ [1 + \mu (\alpha - \gamma)] p_t - \mu (\beta + \delta) p_t^2 \} = \\ &= \mu (\beta + \delta) p_t - \left[\frac{\mu^2 (\beta + \delta)^2}{[1 + \mu (\alpha - \gamma)]} \right] p_t^2 \end{aligned}$$

On the other hand, using (6) and (7) we obtain:

$$\begin{aligned} z_{t+1} &= \pi z_t (1 - z_t) = [1 + \mu (\alpha - \gamma)] \left[\frac{\mu (\beta + \delta)}{1 + \mu (\alpha - \gamma)} \right] p_t \left\{ 1 - \left[\frac{\mu (\beta + \delta)}{1 + \mu (\alpha - \gamma)} \right] p_t \right\} = \\ &= \mu (\beta + \delta) p_t - \left[\frac{\mu^2 (\beta + \delta)^2}{1 + \mu (\alpha - \gamma)} \right] p_t^2 \end{aligned}$$

Thus we have that iterating (5) is really the same as iterating (6) using (7). It is important because the dynamic properties of the logistic equation (6) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that :

- For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- For $1 < \pi < 3,57$ there exist fixed points the number of which depends on μ ;
- For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$;
- For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- For $3 < \pi < 4$ all solutions will continuously fluctuate;
- For $3,57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.
- If $3 < \pi < 1+\sqrt{6}$ (approximately 3.45), z may oscillate between two values forever. These two values are dependent on π . In this case, we have a two-point attractor.
- if $3.45 < \pi < 3.54$, then z may oscillate between four values forever.
- If $3.54 < \pi < 3.57$ (approximately), then z will probably oscillate between 8 values, then 16, 32, etc.
- For $3,57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever. For example, an increase or decrease in the fifth or sixth decimal place will have a drastically large effect. This is known as sensitivity to initial conditions. It means that a chaotic system, even one determined by a simple rule, is in principle unpredictable. It is unpredictable,

because in order to predict its behavior into the future we must know its current value *precisely*. Namely, a slight difference, in the decimal place, resulted in prediction failure. For $3,57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

- If $\pi > 4$, then the values eventually leave the interval $[0,1]$ and diverge for almost all initial values.

Empirical Evidence

The main aim of this analysis is to present the natural gas prices growth stability in the period September 2001-August 2009 by using the logistic model (8).

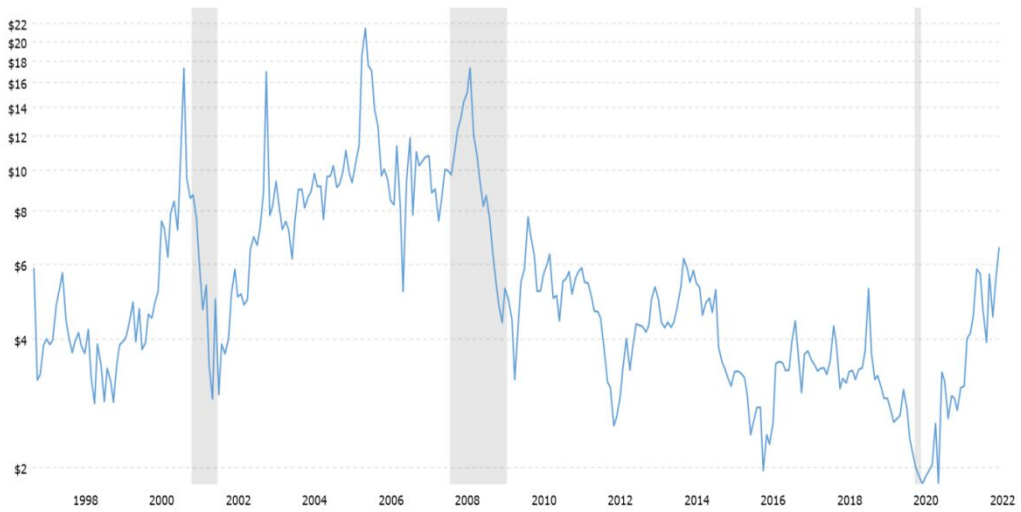


Figure 1. Interactive chart illustrating the history of Henry Hub natural gas prices. The prices shown are in U.S. dollars. The current price of natural gas as of April 12, 2022 is **\$6.59**.

Source: <http://www.macrotrends.net/2478/natural-gas-prices-historical-chart>

In this sense,

$$p_{t+1} = \pi p_t - v p_t^2 \quad \pi \in [0, 4], \quad v > 0 \quad (8)$$

where p – natural gas price, $\pi = [1 + \mu (\alpha - \gamma)]$ and $v = \mu (\beta + \delta)$.

It is supposed that the basic Elliott wave pattern exists in the period September 2001 – August 2009. (see Table 1) This wave pattern consists of an impulse wave and a corrective wave. Impulse waves consist of 5 waves and moves in the direction of the trend. Waves 1, 3, and 5 moves in the direction of the trend, while Waves 2 and 4 move opposite to the trend. Corrective waves can be simple or complex. A simple correction consists of 3 waves (Wave A, B and C) which retrace a portion of impulse. The Elliot Wave Theory identifies (Frost A. J. & R. P. Prechter, 2006): (i) impulse waves that set up a pattern; and (ii) corrective waves that oppose the larger trend. The stock price movements are divided into: (i) trends (five waves in the direction of the main trend); and (ii) corrections (three corrective waves).

Table 1.
Natural Gas Prices

Sep-2001	2.82
April-2002	5.66
July-2002	4.67
Feb-2003	16.46
Oct-2003	6
Sep-2005	20.81
Sep-2006	5.03
Jun-2008	16.8
Aug-2009	3.13

Source: <http://www.macrotrends.net/2478/natural-gas-prices-historical-chart>

Further, it is important to transform data on natural gas price from 0 to 1, according to our supposition that actual value of natural gas price stock market index, P , is restricted by its highest value in the time-series, P^m . Further, we obtain time-series of $p = P / P^m$. Now, we estimate the model (8). The results are presented in Table 2. (Source: <http://www.macrotrends.net/2478/natural-gas-prices-historical-chart>)

Table 2.*The estimated model (8): Natural Gas Prices, September 2001-August 2009.*

R=0.53602		
N=8	π	υ
Estimate	3.26797	3.298749
Std.Err.	0.762994	0.884807
t(6)	4.283086	3.728212
p-level	0.005188	0.009757

Because $\pi=3.26797$, then natural gas prices oscillated between two values. Namely, according to the logistic map, if $3 < \pi < 1+\sqrt{6}$ (approximately 3.45), z may oscillate between two values forever. These two values are dependent on π . In this case, we have a two-point attractor.

Conclusion

This paper creates the chaotic natural gas price growth model. This model has to rely on specified parameters α , β , μ , γ , δ , and initial value of the natural gas price, p_0 . But even slight deviations from the values of parameters: α , β , μ , γ , δ , and initial value of the natural gas price, p_0 , show the difficulty of predicting a long-term natural gas price.

A key hypothesis of this work is based on the idea that the coefficient $\pi = [1+\mu(\alpha-\gamma)]$ plays a crucial role in explaining local stability of the natural gas price, where, μ is the adjustment coefficient; α is the coefficient of the demand function, γ is the coefficients of the supply function. The estimated value of the coefficient π is **3.26797**. This result confirms continuously fluctuations of the natural gas price between two values in the observed period.

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A CONSISTENT AND COMPREHENSIVE ANALYSIS FOR ACCRETION AND DILUTION OF EPS DURING M&A PROCESSES BASED ON RIGOROUS MATHEMATICAL ANALYSIS

Dr. Udayan Kumar BASU*

Abstract: *Mergers and acquisitions (M&As) constitute an important part of investment banks' business portfolios. While dealing with such processes, they often employ the accretion-dilution model.*

This model endeavours to give a tentative view about how a merger or acquisition impacts the acquirer's Earnings Per Share (EPS). Some analysts believe that an accretion of EPS would be good for the acquirer company.

It is rather strange that a set of analysts, based on certain assumptions which – as per their own admission – may very well be wrong, should try to predict if a merger would be accretive or dilutive, and then conjecture that an accretive merger may augur well for the acquirer company. This exercise does not even try to explore whether such a merger would be feasible. If a merger is not feasible and cannot be implemented, there is no point trying to conjecture whether it could have been good or not for the acquirer company. In order to provide a firm footing to this concept for a systematic analysis of various M&A processes, we formulate here a consistent and comprehensive framework based on rigorous mathematical analysis. We derive necessary and sufficient conditions for Break Even Exchange Ratios for both zero and non-zero synergies and systematically study the impact of accretion-dilution on earnings, price and value. We also integrate this precise analysis with results of other M&A theories to predict and explain decisively all happenings on the ground, including the feasibility of mergers, for various kinds of scenarios.

Keywords: *Synergy; Break Even Exchange Ratio; Zones of Accretion & Dilution; Feasibility of a Merger*

Introduction

As per the Accretion – Dilution model used by investment banks and M&A analysts, any merger or acquisition deal is defined as accretive if the acquirer company's Earnings Per Share (EPS) increases following

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execution of the deal. On the other hand, if the deal leads to decrease of the acquirer company's EPS, the deal is defined as dilutive.

The accretion – dilution model appears to rely on the following rules of thumb:

(a) An accretive deal is to augment shareholder value for the acquirer company, while a dilutive deal is likely to destroy its value.

(b) An M&A deal would be accretive if Price/Earnings Ratio (P/E Ratio) of the acquirer company is greater than the P/E Ratio of the target company. On the other hand, if the target company has a P/E Ratio greater than that of the acquirer, the deal is to be dilutive.

However, these thumb rules often do not hold true even under rather conventional scenarios^{1,2}. It has been pointed out that every accretive deal need not necessarily augment shareholder value, and not every dilutive deal needs to be bad.

Accretion-Dilution Analysis allows investment banks and M&A analysts to see whether the post-merger combined EPS of the absorbing and absorbed companies (also termed as pro-forma EPS) is expected to be higher or lower than the standalone EPS of the acquirer company before merger.

Accretion-Dilution of EPS is normally measured by the parameter $[(\text{post-merger EPS}/\text{acquirer's standalone EPS}) - 1]$.

In this context, the Thomson Reuters earnings estimates are known to offer data for a large number of companies in quite a few established and emerging markets around the globe.

One may come across the following 3 possible scenarios:

(a) EPS of the post-merger entity > acquirer's stand-alone EPS, and it is a case of accretion [i.e. the above parameter > 0].

(b) EPS of the post-merger entity < acquirer's stand-alone EPS, and it is a case of dilution [i.e. the above parameter < 0].

(c) EPS of the post-merger entity = acquirer's standalone EPS, and it is a case of Break Even [i.e. the above parameter = 0].

The next question is what the impact of accretion or dilution of EPS would be on the absorber/acquirer company or the absorbed/target company.

Some analysts suggest that, in the absence of any synergy, an acquisition that is 100% paid for by way of shares would be dilutive if the Price/Earnings Ratio (P/E Ratio) of the target is higher than that of the buyer. Conversely, if the P/E Ratio of the target happens to be lower than that of the buyer, the acquisition would be accretive. This fact, the analysts

point out, has been usually a driver for many M&A activities with public companies absorbing privately held companies having lower P/E multiples.

These analysts also point out that their predictions are based on a presumption that the P/E ratio of the post-merger entity would remain at the same level as that of the acquirer, an assumption which may not hold true in reality and is admittedly a potential flaw for this type of analysis. After all, the P/E Ratio of the acquirer can decrease after merger following absorption of a target having a lower P/E Ratio.

According to these analysts, if an acquirer with a lower P/E Ratio acquires a target company with a higher P/E Ratio, the transaction would generally be dilutive in nature. This, the analysts argue, is due to the fact that an acquirer is to receive fewer dollars of earning for each dollar used to acquire the target company.

It is stated that many on Wall Street do not at all like dilutive transactions. If a deal has a likelihood of turning accretive from at least the second year after merger, it is treated as much more acceptable for the acquirer.

In sum, the accretion-dilution analysis appears to be considered by a set of analysts as an indicator of whether a merger/acquisition deal is to create or destroy shareholder value.

Various examples used by the analysts to illustrate their above assertions can be captured in short by the following simple instance of absorption with zero synergy and Exchange Ratio of 2 (based on relative prices of the shares of the two companies):

	Acquirer	Target
Earnings (E)	\$10 Mio	\$10 Mio
Number of shares (N)	1 Mio	1 Mio
EPS	\$ 10	\$ 10
Price	\$ 40	\$ 80
P/E Ratio	4	8

Here, the target has a higher P/E Ratio than the acquirer. Exchange Ratio = 2.

Post-merger: earnings = \$ 20 Mio and no of shares = 3 Mio.

Post-merger EPS = \$ 20/3 = \$6.67 < \$ 10, i.e. EPS of the acquirer. The transaction is thus dilutive (33.33% dilution).

Now, if we interchange the acquirer and the target, and change the Exchange Ratio to 1/2 (based on relative share prices), we get the following position:

	Acquirer	Target
E	\$ 10 Mio	\$ 10 Mio
N	1 Mio	1 Mio
EPS	\$ 10	\$ 10
P	\$ 80	\$ 40
P/E	8	4

Here, P/E Ratio for the target company is smaller than that of the absorbing company. Exchange Ratio = 0.5 (based on relative share prices).

Post-merger: earnings = \$ 20 Mio and no of shares = 1.5 Mio.

EPS = \$ 13.33 > \$ 10, and the transaction is thus accretive (33.33% accretion).

As per such analysts, an accretive acquisition is perceived to be creating value for the shareholders of the acquirer company. However, not all dilutive transactions need necessarily be bad. For example, acquisition of a company with a growth rate for earnings higher than that of the acquirer may push up the acquirer's P/E ratio and over time increase its earnings significantly even if it may be dilutive in the short-term.

As a matter of fact, a study by McKinsey & Company also corroborates that the accretive or dilutive aspects of acquisitions can have a limited impact on share prices.

Weakness of the Current Model:

As we have seen, in its current form, the Accretion and Dilution model is rather tentative in nature. Although used by a set of practitioners, it appears to be based primarily on some elementary observations and surmise rather than any rigorous logical analysis. The model is thus not reliable at all and hardly finds any role during any serious and systematic study of M&A processes.

There is no precise logical analysis to place its assertions and expectations on a sound footing within the robust framework of a proper mathematical formulation. There is even no clue as to how to compute the Break Even Exchange Ratio and what truly separates the zone of accretion from the zone of dilution.

This model makes no attempt to address the fundamental issue of feasibility of a merger/acquisition process. There is no comment or consistency check regarding how this model fares vis-a-vis other established and useful theories like the P/E based Analysis, let alone any effort to integrate itself with such theories for M&A.

There is also no exploration to find out how accretion or dilution of EPS of the acquirer may impact the earnings, price and value of the acquirer and target companies.

It is thus abundantly clear that, in its present form, the accretion - dilution model does not make itself amenable to any precise and systematic analysis and, therefore, cannot serve much useful purpose. It badly requires a consistent and comprehensive formulation based on a proper mathematical framework, which would infuse a fresh lease of life into it and change its status altogether by elevating it to the level of a serious and front-ranking theory for all M&A related studies.

Aim of this Article:

Our aim in this article is to provide a consistent and comprehensive formulation of the concept of accretion and dilution based on rigorous mathematical analysis so that it can be systematically employed to explore logically all cases of M&A, and can work in conjunction with other established theories of M&A to provide an all-encompassing approach to the study of all kinds of M&A processes.

Towards this end, we derive, for zero value of synergy as well as for non-zero synergies, the necessary and sufficient conditions for Break Even Exchange Ratios that clearly demarcate the zones of accretion from the zones of dilution.

Thereafter, we examine at length the impact of accretion and dilution of EPS on earnings, share price, value and feasibility for different kinds of merger. We also integrate completely with the established M&A theory based on P/E Ratio to analyse and explain entire features for various types of M&A deals.

Once we accomplish the aforesaid tasks, Accretion – Dilution of EPS would secure its position as a reliable and analytically precise theory for consistent and systematic analysis of all M&A related processes.

Analysis:

A. Derivation of Exact Equation for Break Even Exchange Ratio (ER) for Zero Synergy:

Let us consider a case where company “1” absorbs company “2” via a share swap [i.e. $1 + 2 \xrightarrow{12}$, where “12” represents the post-merger entity]

We follow the undernoted nomenclature for our analysis.

E_1 = Earnings of company “1”

E_2 = Earnings of company “2”

S = Synergy

E_{12} = Earnings of post-merger entity (i.e. “12”) = $E_1 + E_2 + S$

N_1 = No of shares of company “1”

N_2 = No of shares of company “2”

ER = Exchange Ratio (number of new shares of “1” issued per share of “2”)

N_{12} = No of shares of post-merger entity = $[N_1 + ER * N_2]$

$EPS(1)$ = EPS of company “1” = E_1 / N_1

$EPS(2)$ = EPS of company “2” = E_2 / N_2

$EPS(12)$ = Post-merger EPS = $[E_1 + E_2 + S] / [N_1 + ER * N_2]$

For $S = 0$, $EPS(12) = [E_1 + E_2] / [N_1 + ER * N_2]$

At Break Even EPS, we require that $EPS(12) = EPS(1)$

i.e. $[E_1 + E_2] / [N_1 + ER * N_2] = E_1 / N_1$

Or, $(E_1 + E_2) * N_1 = E_1 * [N_1 + ER * N_2]$

Or, $E_1 * N_1 + E_2 * N_1 = E_1 * N_1 + E_1 * N_2 * ER$

Or, $E_1 * N_2 * ER = E_2 * N_1$

Or, $ER = (E_2 / N_2) / (E_1 / N_1) = [EPS(2) / EPS(1)]$

So, for $S = 0$, Break Even EPS demands that $ER = [EPS(2) / EPS(1)]$.

Similarly, it can be shown easily that $ER = [EPS(2) / EPS(1)]$ would ensure that $EPS(12) = EPS(1)$.

(i) In other words, for $S = 0$, an Exchange Ratio $ER = [EPS(2) / EPS(1)]$ is both a necessary and a sufficient condition for Break Even of EPS.

(ii) As we shall show later in our analysis, such a scenario of no accretion or dilution of EPS for the absorber/acquirer company may witness an accretion or dilution of share price and/or of earnings.

(iii) Since synergy is nil, total earnings and total value would remain constant during the M&A process. This can be propounded as the principle of conservation of total earnings and total value during a M&A process in the absence of any synergy.

Therefore, any accretion/dilution of earnings and/or value of the absorber/acquirer company would have to be accompanied by an equal dilution/accretion for the shareholders of the absorbed/ target company.

(iv) For a merger deal between “1” and “2” to be feasible, the terms of the deal are to be acceptable to the shareholders of both “1” and “2”, i.e. $NPV(1) \geq 0$ and also $NPV(2) \geq 0$.

(v) Thus, for synergy = 0, in case of any accretion or dilution of share price and value of the absorber/acquirer company, the proposed merger deal would turn out to be not feasible.

B. Zones of Accretion and Dilution of EPS:

(a). If the selected exchange ratio is lower than the Break Even Exchange Ratio derived above, there will be an accretion of EPS for the absorber/acquirer company.

This is because an exchange ratio lower than the Break Even Exchange Ratio means that less earnings and value has been passed on to the target company than was necessary for break even. This enables the acquirer company to retain a greater portion of the aggregate earnings which leads to a hike of EPS for the acquirer.

(a) (i) Such an accretion of EPS may be accompanied by an accretion of price and value of the acquirer.

(a) (ii) However, the merger may not be feasible as, in view of zero synergy, the accretion of value for the acquirer will come with an equal dilution of value for the target .

(b) On the other hand, if the selected exchange ratio is higher than the Break Even Exchange Ratio derived above, there will be a dilution of EPS for the absorber/acquirer company. This is because a greater part (compared to what it would have been at Break Even) of the aggregate earnings has been passed on to the target company in this case.

(b) (i) However, the problem with distribution of price and value may still continue.

(b) (ii) In other words, possibility of the proposed merger deal turning not feasible would still linger.

C. Impact of Accretion and Dilution of EPS on Earnings of Acquirer and Target Companies, for Zero Synergy:

Let us consider the scenario analysed in paragraph A above i.e. Break Even EPS with zero synergy, and try to explore how it may impact the earnings of the absorber/acquirer and absorbed/target companies.

Let us consider a case where Company X is contemplating to absorb company Y. Company X has 0.3 million shares with a market price of \$ 30 per share, while company Y has 0.2 million shares with a market price of \$20 per share. Synergy of the merger deal is nil.

EPS for X is \$ 4.00 and that for Y is \$ 2.25.

The managements of companies X and Y are discussing the following two alternative proposals for exchange of shares:

(i) exchange ratio commensurate with the relative Earnings Per Share (EPS) of the companies X and Y (i.e. 2.25:4 or 9:16)

(ii) 0.5 share of company X for each share of company Y (i.e. 0.5:1)

For both the above proposals, we work out/examine below:

(a) the post – merger EPS

(b) accretion/dilution of EPS

(c) impact of the merger on the shareholders of X & Y.

Earnings E(X) = \$ 1.2 million (0.3million*\$4)

Earnings E(Y) = \$ 0.45 million (0.2million*\$2.25)

Synergy S = 0.

Then, E (XY) = (1.2 + 0.45) = \$ 1.65 million

For proposal number (i) ER = 9:16 [2.25:4 = 9:16]

Post-merger number of shares = (0.3 + 0.2*9/16) million = (0.3 + 0.1125) million = 0.4125 million

(a). Post-merger EPS i.e. EPS (XY) = \$ 1.65 million/0.4125 million = \$16.5/(33/8) = \$ 4

(b) Thus, EPS remains unchanged at \$4, i.e. no accretion or dilution of acquirer's EPS during the M&A process (9:16 being the Break Even Exchange Ratio).

(c) (I) Since the number of shares for the original shareholders of the acquirer company X remains unchanged at 0.3 million during the merger, a constant EPS would mean that E(X) has to remain unchanged at \$ 1.2 million (0.3 million*\$4).

In other words, post-merger, earnings linked to the original shareholders of X remains unchanged at \$ 1.2 million.

(II) Earnings attributed to the shareholders of Y i.e. the target company, post-merger, is (\$ 4*0.1125 lakh) = \$ 0.45 million, which is

identical with its pre-merger level of (\$ 2.25*0.2) million i.e. \$ 0.45 million. In other words, E(Y) too remains constant.

Since S = 0 and total earnings has thus to remain constant, earnings for the shareholders of X remaining unchanged during the merger demands that earnings for the shareholders of Y too must remain unchanged.

For proposal number (ii), the position is as under:

ER = 1:2 (below Break Even Exchange Ratio of 9/16)

(a) EPS post-merger = \$ 1.65 million/[0.3+ {0.2*(1/2)}] lakh = \$ 1.65/0.4 = \$ 4.125 (accretion of EPS, as exchange ratio is below its Break Even level).

(b) EPS increases from \$4 to \$4.125 and there is accretion of \$ 0.125 (3.13%). It is important to note that this accretion of EPS takes place in contravention of the thumb rule as per the accretion – dilution model i.e. in spite of the P/E Ratio for the absorbed/target company (i.e. 20/2.25 = 8.89) exceeding that of the absorber company (i.e. 30/4 = 7.5).

(c) (I) Following accretion of EPS, earnings linked to the original shareholders of X increases from (\$4*0.3 million) to (\$4.125*0.3 million) i.e. a hike of \$0.0375 million (accretion of 3.13%).

(II) Earnings linked to the shareholders of Y decreased from \$0.45 million (i.e. \$ 2.25*0.2 million) to \$ 0.4125 million (\$4.125*0.1 million) i.e. a fall of \$ 0.0375 million.

For synergy = 0, total earnings has to be conserved. In other words, change of E(X) and change of E(Y) must be equal and opposite. At the Break Even Exchange Ratio, changes of both E(X) and E(Y) happened to be nil.

D. Impact of Accretion and Dilution of EPS on Share Price, Values of the Acquirer and Target Companies and Feasibility of the Merger, for Zero Synergy:

Prior to merger, values for X and Y are:

V(X) = \$30*0.3 million = \$ 9 million

V(Y) = \$ 20*0.2 million = \$ 4 million

In the absence of synergy, i.e. for S = 0,

V (XY) = \$ (9 + 4) million = \$ 13 million, which is distributed uniformly among the total number of shares of XY.

Proposal No (i): For this case, we have ER = 9/16.

Post-merger, total number of shares = (0.3 + 0.2*9/16) million = (0.3 + 0.1125) million = 0.4125 million.

So, share price for the post-merger entity = \$13 million/0.4125 million = \$31.515 > \$30 i.e. the pre-merger price of the absorber/acquirer company (price accretion).

Hence, post-merger value for the original shareholders of X = \$ 31.515*0.3 million = \$ 9.4545million

Thus, for the original shareholders of X, there will be a value accretion of \$0.4545 million over and above their pre-merger value of \$ 9 million. In other words, during the merger, NPV (X) = \$0.4545 million.

At the same time, the value passed on post-merger to the shareholders of Y = \$ 31.515*0.1125 million = \$3.5455 million.

Thus, for the shareholders of Y, there is a value dilution of \$ 0.4545 million from their pre-merger value of \$ 4 million. In other words, NPV (Y) = (-) \$ 0.4545 million.

Since synergy = 0, total value during the M&A process is conserved. Hence NPV(X) and NPV(Y) must be equal and opposite. This makes the merger non-feasible because a negative NPV for the shareholders of Y would not be acceptable to them.

The merger deal must have $NPV(X) \geq 0$ and also $NPV(Y) \geq 0$ in order that the terms of the merger are acceptable to the shareholders of both X and Y.

For $S = 0$, therefore, non-accretion/dilution of EPS during a merger deal (i.e. at the Break Even Exchange Ratio) does not hold out any promise for feasibility of the deal.

Proposal number (ii): For this case, we observe that:

ER = 0.5. Post-merger number of shares = (0.3 + 0.1) million = 0.4 million

Post-merger share price = \$ 13 million/0.4 million = \$ 32.5 (8.33% price accretion from its pre-merger price of \$ 30).

Post-merger value for the original shareholders of X = \$32.5*0.3 million = \$9.75 million, i.e. a value accretion of \$0.75 million from its pre-merger level of \$9 million. In other words, NPV (X) = \$0.75 million.

Post-merger value for the shareholders of Y = \$32.5*0.1 million = \$ 3.25 million i.e. a value dilution of \$ 0.75 million from its pre-merger value of \$ 4 million. In other words, NPV (Y) = (-) \$0.75 million.

Since $S = 0$, total value must remain conserved during the M&A process. In other words, NPV (X) and NPV (Y) must be equal and opposite.

The negative NPV for Y would once again make the merger non-feasible because the shareholders of Y are not to accept such a proposal.

Thus, for $S = 0$, even an accretion of EPS cannot hold out any guarantee for feasibility of the concerned merger deal.

E. Derivation of Exact Equation for Break Even Exchange Ratio (ER) for Non-Zero Synergy:

After carrying out Accretion Dilution Analysis without synergy, let us examine the impact of non-zero synergy on the Break Even Exchange Ratio (ER).

At the Break Even Value of ER, there would be neither any accretion nor any dilution of EPS for the acquirer i.e. $EPS(12) = EPS(1)$.

In other words, $(E1+E2+S)/(N1+ER*N2) = E1/N1$

Or, $E1(N1+ER*N2) = N1(E1+E2+S)$

Or, $E1N1+ER*N1*N2 = N1E1 + N1*E2 + N1*S$

Or, $ER*N1*N2 = N1*E2 + N1*S$

Or, $ER = (N1*E2)/(E1*N2) + N1*S/E1N2 = [EPS(2)/EPS(1)] + [S/[N2*EPS(1)]]$

(1) Thus, for non-zero Synergy, Break Even EPS requires that $ER = [EPS(2)/EPS(1)] + [Synergy \text{ per target share/ } EPS(1)]$.

Moreover, an $ER = [EPS(2)/EPS(1)] + [Synergy \text{ per target share/} EPS(1)]$ ensures that $EPS(12) = EPS(1)$.

(2) In other words, for non-zero synergy ($S > 0$), the above equation for Exchange Ratio is both a necessary and a sufficient condition for Break Even of EPS.

(3) The Break Even Exchange Ratio (ER) for non-zero synergy ($S > 0$) will always be higher than that for $S = 0$. This is logically understandable because now there is some latitude for the acquirer company to share a portion of the so called “merger cake” (i.e. Present Value of Synergy) with the absorbed/acquired company.

(4) Further, the additional Break Even Exchange Ratio for non-zero synergy would (a) increase with increase of synergy per target share and (b) fall with increase of initial (pre-merger) EPS of the absorber/acquirer company.

F. Zones of Accretion and Dilution of EPS for Non-Zero Synergy:

(a) If the selected exchange ratio is lower than the Break Even Exchange Ratio derived above, there will be an accretion of EPS for the absorber/acquirer.

(a)(i) Such an accretion of EPS may be accompanied by an accretion of price and value of the acquirer.

(a)(ii) However, the merger may not necessarily be feasible in such a case. A detailed exploration would be done in a subsequent paragraph to comment on the feasibility of the merger process.

(b) On the other hand, if the selected exchange ratio is higher than the Break Even Exchange Ratio derived above, there will be a dilution of EPS for the absorber/acquirer company.

(b)(i) However, the problem with distribution of price and value may still continue. In other words, possibility of the proposed deal turning not feasible would still linger.

(b)(ii) A detailed exploration would be done in a subsequent paragraph to know about feasibility of the merger.

G. Impact of Accretion and Dilution of EPS on Earnings of Acquirer and Target Companies for Non-Zero Synergy:

Let us consider the scenario analysed in paragraph E above i.e. Break Even EPS with Non-Zero Synergy, and try to explore how it is to impact the earnings of the absorber/acquirer and absorbed/ target companies.

Let us consider a case where Company X is contemplating to absorb company Y. Company X has 0.3 million shares with a market price of \$ 30 per share, while company Y has 0.2 million shares with a market price of \$20 per share. EPS for X is \$ 4.00 and that for Y is \$ 2.25.

Additionally, Synergy = \$ 0.2 million. In other words, while the other figures are identical with the case of zero synergy, S = \$ 0.2 million now instead of nil.

The managements of companies X and Y are discussing the following two alternative proposals for exchange of shares:

(i) Break Even Exchange Ratio so that post-merger there is neither accretion nor dilution of EPS of the acquirer.

(ii) 0.5 share of company X for each share of company Y (i.e. 0.5:1)

For both the above proposals, we are to work out below:

(a) the post – merger EPS

(b) accretion/dilution of EPS

(c) impact of the merger on the shareholders of X & Y.

Earnings E(X) = \$ 1.2 million (0.3million*\$4)

Earnings E(Y) = \$ 0.45 million (0.2million*\$2.25)

S = \$ 0.2 million.

Then,

$E(XY) = [E(X) + E(Y) + S] = (1.2 + 0.45 + 0.2) = \$ 1.85 \text{ million}$

Proposal no (i): According to the formula derived in chapter E above,
Break Even ER = $9/16 + [(0.2/0.2)/4] = 9/16 + 1/4 = 13/16$

Post-merger, total number of shares = $(0.3 + 0.2 \cdot 13/16) = (0.3 + 0.1625)$ million = 0.4625 million

(a). Post-merger, EPS i.e. EPS (XY) = \$ 1.85 million / 0.4625 million
= $\$18.5 / (37/8) = \$ 4$

(b). As expected, EPS remains unchanged at \$ 4 during the merger.

No accretion or dilution of acquirer's EPS (Break Even ER).

(c). Post-merger number of shares = $(0.3 + 0.2 \cdot 13/16)$ million = $(0.3 + 0.1625)$ million = 0.4625 million.

EPS (XY) = \$ 1.85 million / 0.4625 million = $\$18.5 / (37/8) = \$ 4$.

Thus, EPS remains unchanged at \$ 4, i.e. "no accretion or dilution" of EPS during the M&A process (Break Even Situation).

(I) Since the number of shares for the original shareholders of the acquirer company X remains unchanged at 0.3 million during the merger, a constant EPS would mean that E(X) has to remain unchanged at \$ 1.2 million ($0.3 \text{ million} \cdot \4).

(II) Post-merger, holdings of the original shareholders of X and the shareholders of Y are in the ratio of 3: (13/8) or 24:13. Thus, post-merger, earnings linked to shareholders of Y = $(13/37)$ of \$ 1.85 million = $\$ 1.85 \text{ million} \cdot (13/37) = \$1.85 \cdot (1.3/3.7) \text{ mio} = \0.65 million

This is \$ 0.2 million over and above the pre-merger value of earnings of Y i.e. E (Y) of \$ 0.45 million.

In other words, the entire synergy of merger (\$ 0.2 million) is linked to the shareholders of the target company.

Since number of shares for the shareholders of the acquirer company X remains unchanged during the merger, EPS remaining constant means E(X) has to remain unchanged. In other words, post-merger, earnings linked to original shareholders of X remains unchanged at \$ 1.2 million. At the same time, earnings linked to shareholders of the target increases by \$0.2 million (entire synergy), which is quite natural to expect considering that there is no addition to the earnings of the absorber/acquirer company.

So, irrespective of the synergy, at the Break Even Exchange Ratio, earnings linked to the original shareholders of X cannot but remain constant during the merger. Hence, increase of earnings due to synergy will reflect entirely in the earnings linked to the shareholders of Y. For X to have a share of the additional earnings due to synergy, the exchange ratio must be lower than the Break Even Exchange Ratio (i.e. the zone of accretion).

For proposal number (ii), the position is as under:

ER = 1:2 (below Break Even Exchange Ratio of 13/16)

(a) EPS post-merger = \$ 1.85 million/[0.3+ {0.2*(1/2)}] million = \$ 1.85/0.4 = \$ 4.625 (accretion of EPS, as exchange ratio is below its Break Even level)

(b) EPS increases from \$4 to \$ 4.625 and there is accretion of \$ 0.625 (15.63%). It is important to note that this accretion of EPS is contrary to the thumb rule of accretion – dilution model and in spite of the fact that P/E Ratio for the absorber/acquirer company (30/4 = 7.5) is lower than that of the absorbed/target company (20/2.25 = 8.89).

(c) Post-merger, total earning = \$ 1.85 million

Post-merger, total number of shares = [.3 + 0.2(1/2)] = 0.4 million

(I) Post-merger, earnings linked to original shareholders of X = \$ 1.85 million*(3/4) = \$ 1.3875 million. This is greater than the pre-merger earning for X i.e. E(X) of \$ 1.2 million by \$ 0.1875 million (exchange ratio below Break Even level).

(II) Post-merger earnings linked to the shareholders of Y = \$ 1.85 million*(0.1 /0.4) = \$1.85 million/4 = \$ 0.4625 million. This is higher than pre-merger earning E (Y) by \$ 0.0125 million.

Thus, aggregate accretion of earnings linked to the original shareholders of X and Y = \$ (0.1875 + 0.0125) million = \$ 0.20 million which, as expected, tallies with the synergy of the case (S = \$ 0.2 million).

H. Impact of Accretion and Dilution of EPS on Share Price, Values of the Acquirer and Target Companies and Feasibility of the Merger, for Non-Zero Synergy:

EPS (X) = \$ 4

Price(X) = \$ 30

So, P/E Ratio for X = PE (X) = \$ 30/\$ 4 = 7.5

EPS (Y) = \$2.25

Price (Y) = \$20

P/E Ratio for Y = PE (Y) = \$20/\$2.25 = 8.89

For the merger to be feasible with S = \$ 0.2 million, as per the PE Theory for M&A, the threshold P/E Ratio of the post-merger entity i.e. “XY” must be $PE(XY) = [PE(X) E(X) + PE(Y) E(Y)] / [E(X) + E(Y) + S]$ = [9 + 4]/[1.85] = 130/ 18.5 = 7.03.

Let us assume that PE Ratio for the post-merger entity i.e. PE (XY) = 8 (above threshold level).

Pre-merger values for X and Y are:

$V(X) = \$30 \times 0.3 \text{ million} = \9 million

$V(Y) = \$20 \times 0.2 \text{ million} = \4 million

With $S = \$0.2 \text{ million}$ and PV of Synergy = \$1.8 million [i.e. post-merger value $V(XY)$ at \$14.8 million is 8 times the post-merger earnings of \$1.85 million], we observe as follows:

$V(XY) = \$ (9+4+1.8) \text{ million} = \14.8 million

Post-merger, total number of shares = $(0.3 + 0.1625) = 0.4625 \text{ million}$

Price per share = $\$14.8 / 0.4625 = \32

This is greater than the original share price of X i.e. \$30 per share (Price Accretion of 6.67%).

Post-merger $V(X) = (0.3 \times 32) \text{ million} = \9.6 million

This is \$0.6 million over and above \$9 million (Value accretion of \$0.6 million i.e. 6.67%)

In other words, NPV (X) on account of the merger = \$0.6 million

Post-merger, value to shareholders of Y = $(0.1625 \times 32) = \$5.2 \text{ million}$.

This is \$1.2 million over and above the pre-merger value of Y of \$4 million.

In other words, NPV(Y) on account of the merger = \$1.2 million.

Thus, $NPV(X) + NPV(Y) = \$1.8 \text{ million} = \text{PV of Synergy}$

Since both NPV(X) & NPV(Y) are +ve, the merger deal is acceptable to the shareholders of both X and Y and is thus feasible.

Thus, with a suitable non zero synergy, it is possible to structure a feasible merger deal, which is neither EPS accretive nor EPS dilutive (i.e. for Break Even Value of ER).

Without any synergy, this was not looking possible.

So, feasibility of the merger in this case can be traced to synergy associated with the merger rather than to the Break Even Exchange Ratio.

For proposal at (ii) i.e. ER = 0.5 or 1:2 (less than Break Even ER and hence EPS accretive),

Post-merger, number of shares = 0.4 million

Post-merger, total earning = \$1.85 million

Post-merger, $EPS = \$1.85 / 0.4 = \$4.625 > \text{pre-merger EPS of } \4 (accretion of EPS).

Post-merger, total earnings = \$1.85 million

Assuming PE for post-merger entity to be 8, and PV of Synergy to be \$ 1.8 million, post-merger V (XY) = \$ 14.8 million.

V (XY) = \$ 14.8 million

Total number of shares = 0.4 million

Post-merger share price = \$ 37 (price accretion)

Post-merger value of X = \$ (\$37*0.3) million = \$ 11.1 million

This is higher than pre-merger V(X) of \$9 million by \$2.1 million (value accretion)

Thus, NPV (X) = \$ 2.1 million.

Post-merger value for Y = \$ 3.7 million.

There is a dilution of value for Y of \$ 0.3 million.

Thus, NPV (Y) = (-) \$ 0.3 million.

NPV (X) + NPV (Y) = \$ 1.8 lakh = PV of Synergy.

Since NPV (Y) is negative, the merger is not acceptable to the shareholders of Y and is thus not feasible.

So, by lowering the Exchange Ratio below its Break Even level, we find that some portion of addition to the earnings due to merger has been shared with the original shareholders of X. However, in this scenario, the merger is not feasible.

Accretion of EPS, therefore, does not hold out any guarantee as to feasibility of the merger. We shall explore the reason therefor by using the PE Analysis.

I. Integration of Accretion & Dilution of EPS with the results of PE Analysis:

We can explore the above two scenarios, viz. Break Even ER of 13/16 and ER = 1/2 for non-zero synergy, further using the P/E Analysis for feasibility of the merger. We can then extend such analysis for the case of zero synergy as well.

The P/E theory for a merger (whereby company “1” absorbs company “2” through a share swap) tells us that, for the merger to be feasible 3,4,

$ER \geq P_2N_1/[PE_1(E_1+E_2+S) - P_2N_2]$ for the target, and

$ER \leq - (N_1/N_2) + \{[PE_1(E_1+E_2+S)]/P_1N_2\}$ for the acquirer.

The above two equations encapsulate the requirements of the absorbed/target and absorber/acquirer companies respectively.

It is quite clear that the acquirer cannot go beyond a certain maximum exchange ratio (ER) that, as expected, increases with increase of synergy. In case the acquirer does cross this upper bound for ER, it would end up passing on to the target more than what the synergy can afford, thereby

turning its own NPV from the merger negative – something which would not be acceptable to the shareholders of X, the acquirer company.

On the other hand, the target company needs to have a minimum exchange ratio (ER), which decreases with increase of synergy. Any ER below this lower bound would lead to a negative NPV from merger for the target company – a scenario which would not be acceptable to the shareholders of the target company.

Feasibility of a merger, which warrants that NPV from the merger for both the entities are non-negative, would thus require that both the above two equations are satisfied concurrently. The exchange ratio would thus need to lie between the lower bound set for the target and the upper bound set for the acquirer in order that the merger deal is feasible.

In other words, the benefit from synergy of the merger should be fairly distributed between the two entities in order that the merger becomes feasible.

Let us now explore our cases/examples based on these equations from the PE model.

Substituting for the relevant parameters of this case (with X & Y replacing 1 and 2 respectively) in the above two equations, we get:

$ER \geq 5/9$ (requirement for Y, the target) and $ER \leq 29/30$ (requirement for X, the acquirer).

The scenario with $ER = 13/16$ (Break Even ER) satisfies both these conditions and hence the merger is feasible.

On the other hand, $ER = 0.5$ does not satisfy the twin bounds on ER set for feasibility of the merger. So, in this scenario, the merger is not feasible.

To be more specific, $ER = 0.5$ fails to meet the lower bound set for the target company Y (i.e. $ER \geq 5/9$). As a result, NPV (Y) turns out to be – ve, thereby rendering the merger not feasible.

Now, let us extend the P/E Analysis to the domain of Synergy = 0 also.

The P/E theory tells us that, for $S = 0$, for the merger to be feasible (with X & Y replacing 1 & 2 respectively),

$$ER \geq P_2N_1/[PE_{12}(E_1+E_2) - P_2N_2] \text{ and}$$

$$ER \leq -(N_1/N_2) + [PE_{12}(E_1+E_2)]/P_1N_2$$

Or, $ER \geq 60/92$ (required by Y, the target) & $ER \leq 21/30$ (required by X, the acquirer)

For both the scenarios considered i.e. $ER = 9/16$ and 0.5 , the ERs are $\leq 21/30$ i.e. 0.7 but not complying with the condition $ER \geq 60/92$.

So, for both scenarios, NPV for the acquirer company i.e. NPV(X) is + ve, while NPV for the target company i.e. NPV (Y) turned out to be – ve. The merger was thus not feasible for both these scenarios.

We now extend the domain of our analysis further to the two illustrations included in “Impact on Acquirer & Target companies” i.e. the second chapter after “Introduction”.

For the first illustration, we observe that:

	Acquirer	Target
Earnings (E)	\$10 mio	\$10 mio
Number of shares (N)	1 mio	1 mio
EPS	\$ 10	\$ 10
Price	\$ 40	\$ 80
P/E Ratio	4	8

P/E Ratio for the acquirer company is half of that for the target company.

Exchange Ratio = 2 (based on relative share prices).

Since synergy = 0, for the post-merger entity, EPS = \$ 6.67 (dilution of EPS by as much as 33.33%).

However, since synergy = 0, as per PE theory, the threshold P/E Ratio for the post –merger entity is the weighted average of the two pre-merger P/E Ratios where the weight factors are the respective earnings. Accordingly, post-merger P/E Ratio = $[4 \times 10 + 8 \times 10] / [10 + 10] = 6$.

Thus, share price of the post-merger entity = \$ 6.67*6 = \$40 (no accretion or dilution of price). Hence, price and value undergo no change during merger in spite of quite a large dilution of EPS. So, even a large dilution of EPS need not necessarily result in any change of shareholder wealth of either company.

For the second illustration, we observe that:

	Acquirer	Target
E	\$ 10 mio	\$ 10 mio
N	1 mio	1 mio
EPS	\$ 10	\$ 10
P	\$ 80	\$ 40
P/E	8	4

P/E Ratio for the acquirer company is double of that for the target company.

Exchange Ratio = 0.5 (based on relative share prices).

Since synergy = 0, for the post-merger entity, EPS = \$ 13.33 (accretion of EPS by as much as 33.33%).

However, since synergy = 0, as per PE theory, the threshold P/E Ratio for the post –merger entity is the weighted average of the two pre-merger P/E Ratios where the weight factors are the respective earnings. Accordingly, post-merger P/E Ratio = $[8*10 + 4*10]/[10 + 10] = 6$.

Thus, share price of the post-merger entity = \$ 13.33*6 = \$80 (no accretion or dilution of price). Hence, price and value undergo no change during the merger in spite of quite a large accretion of EPS. So, even a large accretion of EPS may not necessarily result in any change of shareholder wealth of either company.

In other words, the belief that an accretion of EPS is good for the acquirer and a dilution of EPS is bad for it is not necessarily corroborated by the happenings on the ground.

For accretion dilution model, things could happen in such a tentative manner simply because there was no clue as to the way for a definitive answer to various issues and questions related to the underlying merger process. However, our fresh and consistent formulation based on rigorous mathematical analysis allows for systematic exploration of various aspects and gives precise answers to all merger related issues and questions.

J. Conclusion

As we have observed, whether without any synergy or with some finite synergy, it is possible to derive exact equations for computation of the Break Even Exchange Ratio separating clearly the zones of accretion and dilution of EPS. Further, we observe that accretion of EPS for the acquirer holds no guarantee for augmentation of shareholder wealth for the acquirer or for feasibility of the underlying merger process.

There is no doubt that existence of a finite synergy opens up the prospects for a feasible merger in a big way. As a matter of fact, our analysis demonstrates that existence of adequate synergy and a balanced distribution of the benefits from such synergy between the acquirer and the target companies seem to be the most important issues for clinching the feasibility of a merger. This is corroborated by the PE theory for M&A also.

Moreover, our Break Even Analysis has been used in conjunction with the PE theory to have a comprehensive and composite analytical

basis. This would provide a robust and consistent framework as well as a sound and inclusive footing for our Break Even Analysis for the purpose of exploring various kinds of M&A processes.

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NEW ECONOMY Section

THE ECONOMIC IMPACT OF COVID-19 ON THE LOW-TECHNOLOGY MANUFACTURING INDUSTRY IN ROMANIA. A COMPARATIVE ANALYSIS

Andrei Silviu DOSPINESCU*

Abstract. *The global economy experienced one of the most significant economic contractions due to the COVID-19 pandemic. The Romanian economy, which is strongly integrated into the European Union, contracted by over 10 percent in the second quarter of 2020, the strongest quarterly contraction in the last 25 years. Moreover, Romania's industrial structure still has to converge with that of the European Union, as it has a higher weight of low-technology manufacturing compared to other EU countries making it more vulnerable to the COVID-19 shock. In this context, this paper analyzes the dynamics of production and employment in the low-technology activities specific to the Romanian manufacturing industry and compares the dynamics in Romania with the main economies in the region and the EU average. The impact of the COVID-19 shock is also captured through an autoregressive vector model, which indicates a strong impact, especially for low-technology activities, which are more vulnerable to mobility restrictions and the acceleration of digitalization trends.*

Keywords: *manufacturing industry, low-technology activities, exogenous shock, VAR model, impulse response function.*

1. Introduction

The manufacturing industry was significantly affected by the COVID-19 pandemic through a constellation of factors, including the impact of global value chain disruptions, sharp increases in commodity prices, increased volatility of domestic and foreign demand. The pandemic accelerated some of the global tendencies as the reallocation of economic activity to more productive companies and the digitalization of the economy. Less productive firms and low-technology manufacturing

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experience stronger output declines compared to high productivity firms and high-technology manufacturing (Bruhn et al. 2021). The weight of low-technology manufacturing continues to be higher in Romania compared to the EU average making the Romanian economy more vulnerable to the COVID-19 shock.

The paper analyzes the impact of the COVID-19 crisis on the low-technology manufacturing industry. The analysis uses quarterly data for 2020 and 2021, data on production, and employment. The economic shock induced by COVID-19 revealed and accentuated several trends prior to the health crisis, including higher digitalization. These trends negatively impacted low-technology manufacturing. In this context, the present analysis facilitates the identification of the current trends in the analyzed activities of the manufacturing industry.

The paper also investigates the impact of the COVID-19 shock through an econometric model, namely a vector autoregression model that groups the specific activities of the low-technology manufacturing industry and generates impulse response functions to estimate the shape and magnitude of the shock induced by COVID-19 on the production of the low-technology manufacturing industry.

1. Methodology

Autoregressive models are frequently used in the econometric analysis of economic shocks, including the impact of COVID-19 (Sims 1980, Hamilton 1994, Bobeica and Hartwig 2021).

Vector autoregression models (VARs) are statistical models that capture the relations between economic variables without a priori assumptions about the causal relations between these variables. Mathematically, the model is described below:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + e_t, \quad (1)$$

Where Y is a vector of the analyzed variables, t is the number of lags used, e is a vector of errors of mean 0 and finite variation.

VAR models capture the structural relations in the economy and doesn't impose assumptions about the causal structure of the investigated data. The impact is usually summarized using the impulse response function.

The paper estimates a vector autoregression model that groups the specific activities of the low-technology manufacturing industry and

generates impulse response functions to estimate the shape and magnitude of the shock induced by the COVID-19 pandemic on output.

2. COVID-19 impact on low-technology activities from the perspective of output and employment

The Romanian economy contracted by 10.1% in the second quarter of 2020, the highest quarterly contraction since 1995. Services were strongly impacted by the sanitary restrictions. The services sector contracted by 13.4% in the second quarter of 2020, while hotel and restaurant business which are vulnerable to mobility restrictions recorded a contraction of 65.2%. Against the background of the reduction of international trade and the shock induced on the global value chains, the industrial production contracted by 19.1% in the same analyzed period. The manufacturing industry was more strongly affected by the above-mentioned factors, contracting by 29.1% in the second quarter of 2020. In what follows will analyze the impact of COVID-19 on the low-technology activities of the manufacturing industry, highlighting the main specificities in terms of production and employment.

Manufacture of food products; beverages and tobacco products

Food, beverage and tobacco retailing proved more resilient than other product categories. The stability of retail sales is confirmed in the mobility data. If we look at the period of time when the containment measures were the most severe, the period April-May 2020, we see the weaker correlation between the strictness of the restrictions and the demand for food. Sales through HORECA, which historically generate the highest profits, have decreased significantly in the context of health restrictions. Restaurant capacity has been drastically reduced with the implementation of social distancing rules. Delivery companies have seen significant expansion in major cities, partially offsetting the decline in restaurant sales. In this context, production in the food industry, the manufacture of beverages and tobacco products contracted by 9.8% in the 2nd quarter of 2020, significantly less compared to the average recorded in the case of the manufacturing industry as a whole, which contracted by 29.1% in the same analyzed period. Also, the production dynamics of the food industry, the manufacture of beverages and tobacco products in Romania was comparable to the EU27 average (see Figure 1).

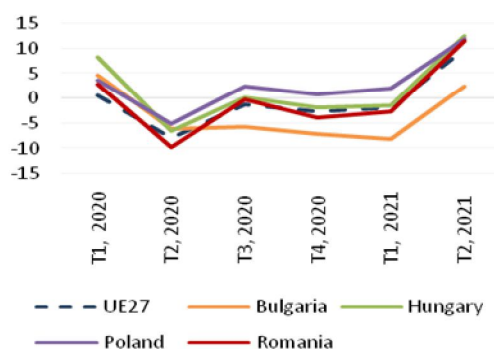


Figure 1. COVID-19 impact on the output of the Manufacture of food products; beverages and tobacco products (% y-o-y)

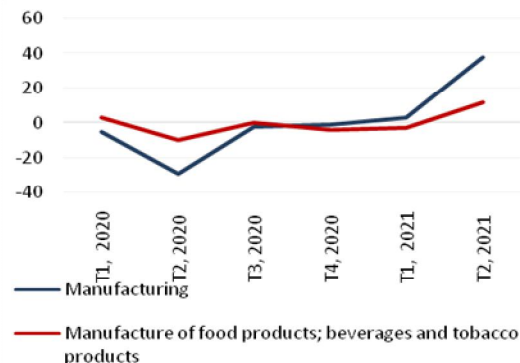


Figure 2. COVID-19 impact on the output of Manufacturing vs Manufacture of food products; beverages and tobacco products (% y-o-y)

Source: Eurostat

Note: Index calculated using chain linked volumes 2015 prices

Manufacture of textiles, wearing apparel, leather and related products

The textile industry was strongly affected by the economic shock induced by the COVID-19 pandemic. Globally the industry has been predominantly affected by delays in the supply of raw materials since the beginning of the pandemic in China. After factories in China were closed, many brands and retailers moved production to Turkey. According to the Employers' Federation of Textiles, Garments and Leather in Romania, the negative effects of the COVID-19 pandemic were reflected on several levels. Difficulties were recorded in the supply of raw materials and accessories both due to the decrease in transport between Romania and Italy and China, respectively, and due to the slowdown in the activity of many Italian manufacturers. During the first state of emergency, March – May 2020, production stagnated, with a 30% reduction in the export capacity of Romanian factories. Since the beginning of the pandemic, there has been a drop in orders of over 40% compared to the previous year. Almost 20% of businesses in Romania were forced to either reduce or cease operations during the COVID-19 pandemic. Only 19% of the companies in Romania did not at least partially suspend their activity and had money to pay salaries.

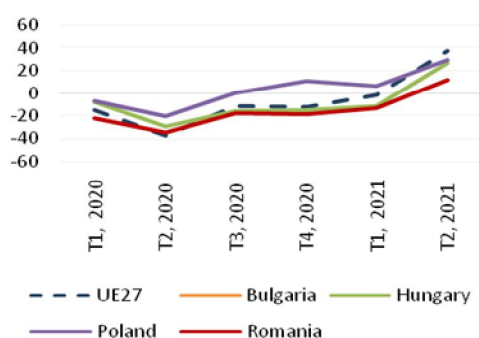


Figure 3. COVID-19 impact on the output of the Manufacture of textiles, wearing apparel, leather and related products (% y-o-y).

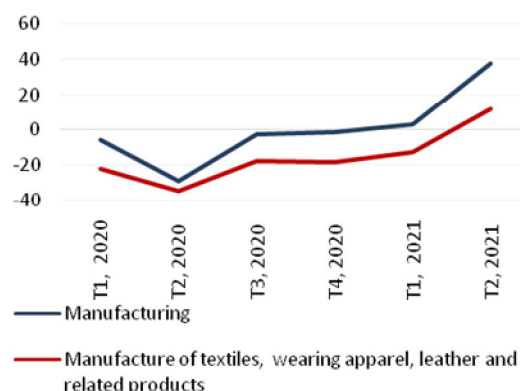


Figure 4. COVID-19 impact on the output of Manufacturing vs. Manufacture of textiles, wearing apparel, leather and related products. (% y-o-y).

Source: Eurostat

Note: Index calculated using chain linked volumes 2015 prices

It can be seen in Figure 3 the extent of the output contraction in the analyzed activity. In the 2nd quarter of 2020, production contracted by 35.2%. Also, the dynamics of production in the textile industry following the initial COVID-19 shock was lower than the dynamics recorded in the manufacturing industry as a whole (see Figure 4).

Employment in the textile industry was more strongly affected compared to the manufacturing industry as a whole. The dynamics of employment remained negative throughout 2020. Historical data indicates that employment in the textile industry returns more slowly to values before an economic crisis. In this sense, after the initial shock of the financial crisis of 2008-2009, employment in the textile industry returned to pre-crisis values in approximately four years.

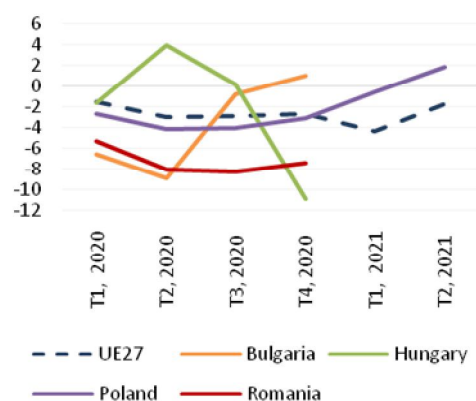


Figure 5. COVID-19 impact on the employment of the Manufacture of chemicals and chemical products (% y-o-y).

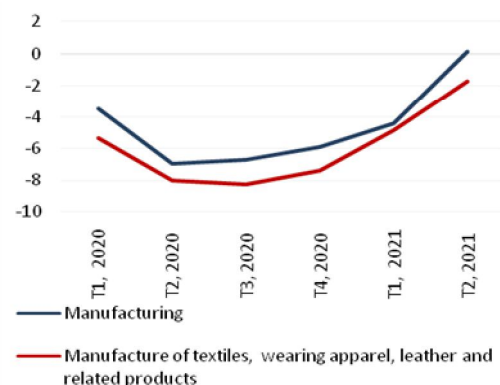


Figure 6. COVID-19 impact on the employment of Manufacturing vs Manufacture of chemicals and chemical products (% y-o-y).

Source: Eurostat

Note: Index calculated using chain linked volumes 2015 prices

Manufacture of wood, paper, printing and reproduction

The restrictions introduced in the context of the COVID-19 pandemic had a significant impact on the Manufacture of wood, paper, printing and reproduction. The transition of school activities to an online environment and telecommuting have significantly reduced the production of printing, writing and newsprint paper. The reduction of mobility, in the context of the pandemic, led, instead, to the increase in the production of paper and cardboard for packaging and of paper for household and sanitary use. The increase in packaging and toilet paper production is likely due to increased online shopping in the early stages of the pandemic, as well as increased demand for toilet paper in homes and hospitals. In the context of the COVID-19 pandemic, the demand for disposable paper, disinfectant wipes, face masks, paper underwear and other related hygiene products has increased significantly. In the context of the pandemic, it is estimated that printing, writing and newsprint paper will represent about 10% of the sales of the business analyzed worldwide compared to about 70% a decade ago (Financial Times 2021). Production of the activity Manufacturing of paper and paper products; Printing and reproduction on media of records decreased by 18.9% in the 2nd quarter of 2020. After the initial shock generated by COVID-19, the activity experienced an output recovery in Romania comparable to that recorded in the main countries in the region

(see Figure 7). However, at the level of the 2nd quarter of 2021, the output remained below the level before the crisis triggered by COVID-19.

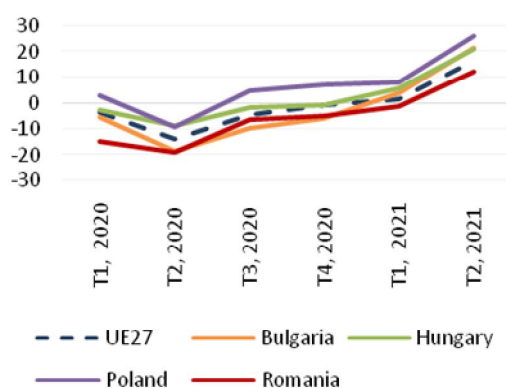


Figure 7. COVID-19 impact on the output of the Manufacture of machinery and equipment n.e.c. (% y-o-y).

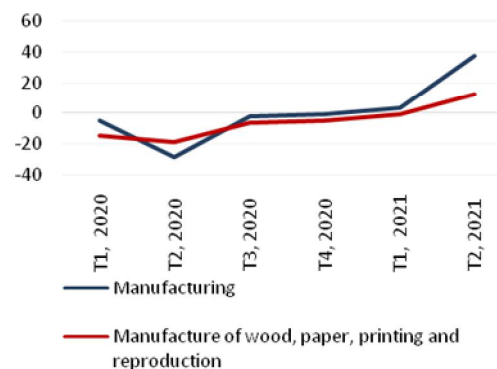


Figure 8. COVID-19 impact on the output of Manufacturing vs. Manufacture of machinery and equipment n.e.c. (% y-o-y).

Source: Eurostat.

Note: Index calculated using chain linked volumes 2015 prices.

Manufacture of furniture; jewellery, musical instruments, toys; repair and installation of machinery and equipment

Furniture manufacturers faced several difficulties in the context of the COVID-19 shock such as the difficulty of finding specialized labor and raw materials and a significant increase in transportation costs. The restrictions imposed by the health crisis led to an output reduction reflecting the closure of some production units as well as social distancing impacting the production lines. Manufacturers faced a reduced supply of specialized labor, especially for those activities that require high skills. Significant pressures on supply chains, especially in 2021, have affected raw material availability. This has led to significant delays in the ability of manufacturers to meet the demand for furniture, especially in 2021. The activity was strongly affected in the first wave of COVID-19. Production contracted by 37.5%, in Romania, in the 2nd quarter of 2020 (see Figure 9 and 10). In the context of high uncertainty, consumers postponed the purchase of non-perishable goods. Later, against the background of the recovery of economic activity, especially construction, the dynamics of production at the level of the analyzed activity returned to positive territory starting from the 1st quarter of 2021.

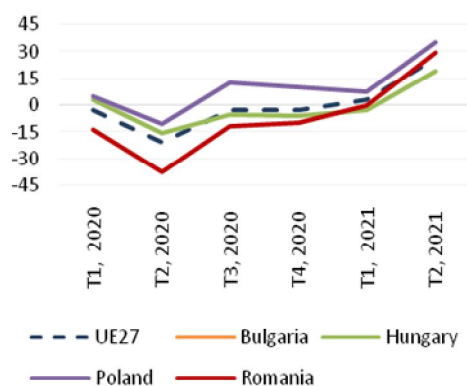


Figure 9. COVID-19 impact on the output of the Manufacture of machinery and equipment n.e.c. (% y-o-y)

Source: Eurostat

Note: Index calculated using chain linked volumes 2015 prices

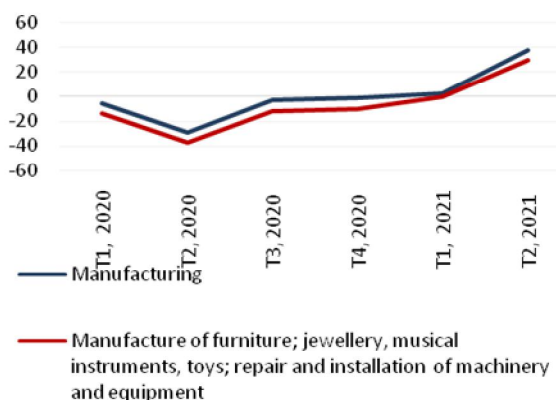


Figure 10. COVID-19 impact on the output of Manufacturing vs Manufacture of machinery and equipment n.e.c. (% y-o-y)

The increased demand for high-skilled workers from furniture manufacturers suggests a rebound in employment in the sub-segments of the labor force engaged in high-skilled activities. At the same time, the analysis of Figures 11 suggests that production at the branch level experienced a recovery starting in 2021 and an increase in the ability to adapt to the restrictions imposed by the authorities. This suggests that activity-level production should register positive dynamics, which should lead to an increase in labor demand.

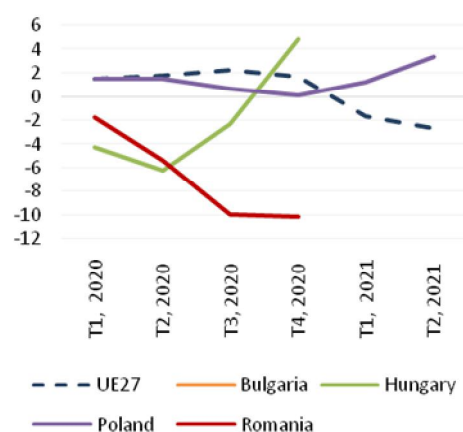


Figure 11. COVID-19 impact on the employment of the Manufacture of machinery and equipment n.e.c. (% y-o-y).

Source: Eurostat; Note: Index calculated using chain linked volumes 2015 prices

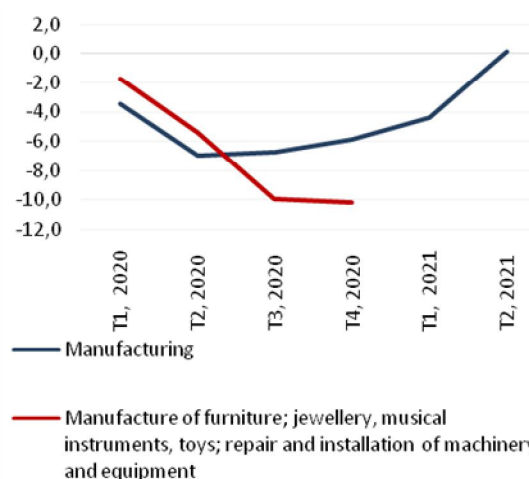
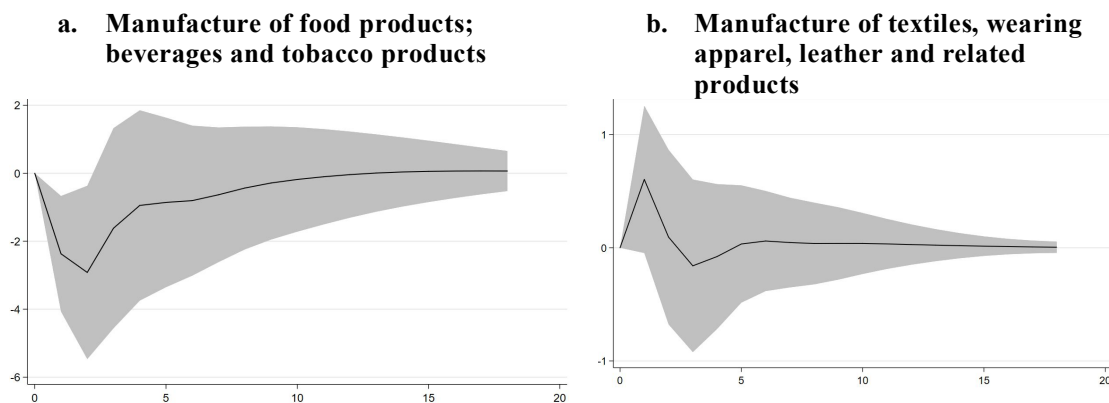


Figure 12. COVID-19 impact on the employment of Manufacturing vs Manufacture of machinery and equipment n.e.c. (% y-o-y).

3. Econometric analysis of the impact of the COVID-19 shock on low-technology activities

The response of low-technology activities to a shock on the manufacturing industry is negative and statistically significant for two of the analyzed activities, namely Manufacture of food products; beverages and tobacco products and Manufacture of wood, paper, printing and reproduction (see Figure 13). In the case of the food industry, the manufacturing of beverages and tobacco products, output recovered in approximately 4 periods, subsequently registering a positive evolution. The result is consistent with the recovery of the analyzed activity starting from the second part of 2020 and in 2021 against the background of the relaxation of health restrictions and the increase of the economy's capacity to adapt to these restrictions. Recovery of the activity Manufacture of wood, paper, printing and reproduction took longer. The result is consistent with the broader changes in the case of the analyzed activity, changes that were accelerated by the economic shock induced by COVID-19. In this sense, it is estimated that printing, writing and newsprint paper will account for about 10% of sales worldwide compared to about 70% about a decade ago (Financial Times 2021).



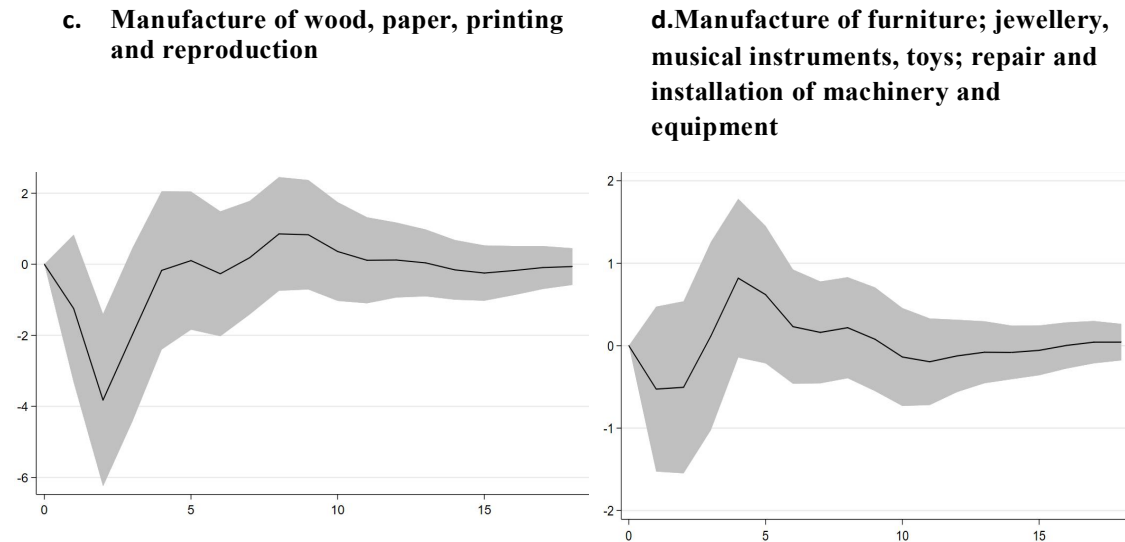


Figure 13. The response of low-technology activities to a shock to manufacturing

Source: Author's calculations using STATA 15

Legend: The shock response function measures the impact of a shock equal to one standard deviation of the residuals of the manufacturing industry-specific regression equation. The shock response is followed for 18 periods equivalent to 18 months.

4. Conclusions

Low-technology manufacturing was strongly affected by the COVID-19 shock and the acceleration of several trends triggered by the pandemic. Mobility restrictions and telecommuting affected activities like manufacturing wood, paper, printing, and reproduction. The acceleration of digitalization trends in the context of COVID-19 dealt a significant blow to the above-mentioned activity. The textile industry was strongly affected by delays in the supply of raw materials, with almost 20% of textile firms in Romania forced to either reduce or cease operations during the COVID-19 pandemic.

The econometric analysis of the low-manufacturing response to the economic shock induced by the COVID-19 pandemic illustrates that, on average, the impact is negative, especially in 2020. The result reflects the impact of a mixture of COVID-19 factors, chiefly mobility and sanitation restrictions, but also the acceleration of key trends such as digitalization.

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FOREIGN DIRECT INVESTMENTS (FDI) IN THE CENTRAL AND EASTERN EUROPE (CEE) REGION

Liviu C. ANDREI^{*}, Dalina M. ANDREI^{**}

Abstract: *This paper will be for describing the specific of this part of international capital that is FDI for one of well designed word multi-country region in the view of the UNCTAD Organization. Plus, it will follow previous systematic analyses of ours basically observing the same international capital at the world level, i.e. for which $FDI=DIA$, where FDI and DIA(direct investments abroad) are the two opposite senses of this capital flows and reference data time is the 1990-2015 quarter of century. This paper's results set will so include: levels of FDI&DIA stocks accumulated during this period, certainly by the whole region and individual countries, the same for stock balances (FDI minus DIA), FDI&DIA regional dynamics, sources and destinations of capital flows, as well as some comparison between the CEE region and other such world regions identified by UNCTAD on the same international capital bases.*

Keywords: *foreign direct investments (FDI), direct investments abroad (DIA), international capital, FDI stock balances, world multi-country regions, flows & stocks of international capital.*

JEL classification: *F20, F21, O11.*

1. Introduction

Basically, it is about a double aiming in our paper below: (i) some description on the CEE region and (ii) directly relating to our recent and larger research on *international investments*. So that the 2016 number of the “*World Investments Report*”(WIR 2016) periodical of UNCTAD – i.e. with its annual FDI flows on 215 UNCTAD member countries so reporting and then so world multi-country regions distinctly formed in the same world Organization's view

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– and all our recent papers called *Foreign Direct Investments*” form the very references of this current paper (see the Annex). The latest affords to account some extended conclusions of our previous FDI related research on the CEE region. It is about in *WIR 2016*, besides 6 important FDI-related world countries – the US, UK, Canada, Japan, Australia and New Zealand, the so-called “off the world region” countries --, a number of 14 such multi-country regions, here including the “West Europe”, our other name for what *WIR 2016* calls “Other developed Europe”, and our “Middle Africa” region reuniting the three: East Africa, Central Africa and West Africa regions, as here considered by *WIR 2016* either. The CEE region is one of these and refers to 11 countries: Poland, Czech and Slovak Republics, the three Baltics, Estonia, Latvia and Lithuania, then Hungary, Romania, Bulgaria and the ex-Yugoslav Slovenia and Croatia[1].

2. Literature review and methodology

The complexity of *international investments* issue comes to be strengthened also by an interesting list of *theories* about their origins and sources. According to the *Production Factors* theory, capital, labour and resources “do follow and search for each-other” in both macroeconomic and international economic areas; the capital factor proves in context more physically mobile than the others about moving about and this would be international investment [2]. The old classic and neoclassic theory of *International trade* – i.e. especially the Heckscher, Ohlin and Samuelson (HOS) variant -- concludes for FDI that since there is no competition on the international markets, countries prefer to put into value and profit their own available/ abundant resources/production factors. Capital, be it a derivative and more sophisticated production factor, makes no exception to this whatsoever. *Comparative and competitive advantages* in international areas play the essential roles [3] & [4]. The *Product's life cycle* theory belongs to Vernon [5&6]. That is from a product entering the consumer's market demand, then enlarging it, going to exportation, but finally, when *exporting production* itself, instead of just goods produced -- i.e. direct investment abroad (DIA) -- this is the decline already. Ozawa (1992) here adds the scenario of just moving entire industries from a country to the neighbouring international areas [7]. The *Ownership (O), Location (L) and Internationalization (I)* theory prefers to accuse *the firms* for producing international capital through assuming such trio of requirements to give birth to other firms in the international area [8]&[9]. There are also authors here accusing existing big *multinationals* world-wide. They might form an international investment *oligopoly* and the peremptory

result of this would be forming a new category of *international entities* to work at the same with countries (nations) and do equal parts business with national governments where the case [10;11;12]. On the contrary and finally our recent theory about international investments re-evaluates the role of *national economies* in this respect, similarly with the old theory of international trade; the international capital would be formed by components that could be named *cooperation capital* and *long-way flows* according to the criteria of existing or not back flows and involving or not the Third World in the investment process [13].

The methodology of this paper undertaking of ours [1;13;14] ,bases on the very simple FDI=DIA equality for two here applied models or model hypotheses. The previous sees the world level FDI (the international directly invested capital) and then engenders an appropriate set of other equalities – e.g. the null sum of world countries' FDI stock balances; equal dynamics for the world level FDI and DIA. The latter sees all individual capitals, capital flows and even capital stocks as concomitantly invested (DIA) by one identified country and received (FDI) by another also identified country – i.e. our model disposes of a limited ability to detect such individual investment flows and stocks and this in a well determined set of circumstances, i.e. model restrictions. Technically, this latter model turns the previous binary FDI-DIA evaluation system into a unitary evaluation system of the same capitals moving between world countries. And finally the two models meet their own specific results, among which a few lines of world regions' FDI behavioural description – i.e. here including for the CEE countries.

3. The FDI basing description of the CEE region

3.1. FDI level and sources

An amount of 440,391 million US dollars, as high as 2.0% of world-international capital stocks, is estimated to be the dominant *long-way flows* entering the 11 member countries of the CEE region during the 1990-2015 year interval. They are alternatively estimated to come from: (a) the Euro-Zone, as entirely, e.g. from Netherlands – i.e. 414,582 million US dollars, the same as 1.7% of world FDI/DIA stocks – and Luxembourg – i.e. 46,047 million US dollars, 0.2% of the same world capital stocks – or (b) the Euro-Zone and West Europe together [14].

In any of its estimating alternatives our modelling finds the CEE region as part of the much larger Eurasia – i.e. a very *section* of this international capital -- that appears to be all fed by long-way flows from these same two

international investment entities. An *international capital section* is assumed to be a pretty closed market area – i.e. as multi-region world space -- here found to have accumulated larger and larger amounts of FDI&DIA for corresponding narrower and zero tending cumulative *FDI stock balances* of countries. Or, Eurasia, with its 9 regions, not only behaves like this, but also qualifies as the largest world space of this kind, i.e. about 57% of the same world capital stocks as FDI/DIA. Plus, this isn't any exceptional state, but just a trend of the same international directly invested capital [1;13;14]. The Eurasian specific doesn't even limit to this, but it includes that it is about entire regions, rather than countries here investing in other regions.

The truth here might be a double one: on the one hand the Euro-zone and West Europe stay the lonely FDI long-way flow sources of the CEE region (and, of course, not only); on the other the last comes to be the 3rd preferred DIA target of the Euro-zone and West Europe investor entities to the rest of 7 region-entities in the whole Eurasia – i.e. after just East Asia(680,262 million US\$/ 2,9% of world stocks) and South-East Asia (619,941 million US\$/ 2,7% of world stocks).

By the criterion of average FDI amount per individual country CEE accounts for 51,467 million US dollars per individual CEE member country during the 1990-2015 year interval, so 0.22% of world capital stocks that makes the annual average of 1,980 million US dollar/ 0.01% of world capital per the same CEE member-nation and so the region ranks the 7th position in the total of 9 regions of Eurasia – i.e. it leaves behind just South Asia and South-East Europe region¹. Or, this might equally relate to the (e) international investment status of the CEE – i.e. the same as of its neighbouring South-East Europe little region of Balkans --, this type of regions is likely to stay relatively lower level FDI. However, just concomitantly CEE comes on top of the four regions of the (e) type world-wide – i.e. off Eurasia -- with its same performance².

3.2. *The FDI stock balances*

The *FDI stocks balance* is made by the FDI-DIA difference for the country, region and certainly continent, i.e. is the most direct result of this

¹ This is while the East Asia comes on the top of Eurasia with 442,067 million US dollars/ 1.83% of world capital stocks on average per nation, over-passing the Euro-Zone, with its 362,024 million US dollars / 1.5 % of world capital stocks on average per nation.

² Besides CEE, see Northern Africa (30,894 millions US dollars/ 0.14% of world capital stocks), Middle Africa (7,974 million US dollars/0.02% of world capital stocks) and Oceania (1,315 million US dollars/0.00% of world capital stocks).

opposite flows' encounter on individual country areas. Our modeling assumes that such negative difference ($FDI < DIA$) equals the DIA predominance as an entity obviously and systematically investing in other regions and throughout the world – i.e. this is roughly assumed to be about well developed economies as much as negative FDI stock balances verify for rather larger international capital amounts implied. On the contrary, when positive difference ($FDI > DIA$) it is even more obvious than in the previous case that it is about developing and emerging economies – i.e. wherever positive, such balance relates to lower FDI/DIA flows and stocks. And the CEE region sees itself as typical for, as seen in this following text [1] .

According to our modeling, *positive FDI stock balances* are the very symptom of developing and emergent economies – i.e. when the region is positive FDI stock balances, as for the (e) type regions and especially for the case of CEE, it is even about an economic homogeneity similar to the Euro-Zone's economic integration on the other side of our evaluation for the international capitals. See Table 1 for the CEE region's and its member countries' FDI stock balances – i.e. actually, these table data are the same with those regarding long-way flows discussed in the here above previous paragraph³.

Table 1.

C&E Europe's countries FDI stock balances in 2015

Ranking position	Member country	Millions of US\$ 2015	% of world stocks
i	Poland	142200	0.6
ii	Romania	78816	0.3
iii	Czech Republic	64100	0.3
iv	Bulgaria	52632	0.2
v	Slovakia	38455	0.2
vi	Croatia	22076	0.1
vii	Hungary	11704	0.1
viii	Latvia	10568	0.1

³ Not as a general, but as a specific rule for an (e) type region with positive FDI stock balances all over.

ix	Lithuania	9596	0.1
x	Estonia	5647	0.0
xi	Slovenia	4596	0.0
-	C&E Europe	440391	2.0

Source: own calculation based on UNCTAD data (World Investments Report WIR 2016)

The CEE⁴ region's cumulative FDI stocks surplus of 2.0% of world stocks actually also is as high as 26.0% of the Euro-zone FDI stocks deficit and as 22.2% of the one of world top-16 countries as correspondingly, up to 2015. As by individual member countries, Poland actually reaches the highest positive FDI stocks balance in the whole Europe during the 1990-2015 year interval, followed by Malta (in the Euro-Zone), and Romania here might compare to Belgium (in the Euro-Zone, as well) – i.e. it is though true that positive FDI stock balances are rather not quite appropriate to (b) type regions like Euro-Zone or West Europe [1].

These above are about some competition of positive FDI stock balance economies inside a region of types (e) and (a) that are basically FDI recipient regions, as rather appropriate [14]. The idea – i.e. another double one -- is that CEE, as an (e) type region, (i) seems to keep on with the old “FDI attracting” type competition among member countries, but then (ii) the basic country ranking for FDI inflows rather stays the same for FDI stock balances and FDI dynamics.

4. FDI dynamics

The FDI&DIA dynamic of an entity, in our modelling once more, is viewed as the difference between the flows'/stocks' growth and the average world FDI&DIA growth. A country/region meets positive dynamic when its FDI&DIA stocks' growth is higher than the corresponding world average growth — the contrary for what is here called negative dynamic. The difference between percentages of the same country in total stocks in different moments simply results into this dynamic – i.e. it so expresses in positive or negative percentage points. Dynamics of all world countries as cumulated for the same period make the null sum for both FDI and DIA flow senses.

⁴ i.e. that include five Euro-zone member countries that are: Estonia, Latvia, Lithuania, Slovakia and Slovenia [15]. As authors of this study, we found that these countries' FDI&DIA aggregate behaviour rather belongs to the CEE country group [1]

Then another aspect comes up here in context: a dynamic calculated for the whole 1990-2015 interval risks some irrelevance since the basic stocks of 1990 really and conceptually are as low as the flows of the same year. This is why another basic moment was settled as such a basis – so that was chosen for 1994. And concomitantly such a solution for dynamics defined might have made place for some results discrepancy between dynamics calculated and the other ones, i.e. the so called static results for countries and regions. As in general, of course, dynamic and static performances of a country stay related to each other – e.g. high positive FDI dynamic leads to high positive FDI stocks balance; the same for high DIA dynamic leading to highly negative such balance; a highly positive FDI stocks balance puts up-pressure on next following DIA flows (and so on). Last, but not least emerging economies – as the ones of the CEE region – are expected for positive dynamics at least for the FDI flows sense [1].

According to its (e) region type and special characteristics – i.e. all positive FDI stock balances within the region area – the CEE's dynamics are expected to come positive on both FDI and DIA flow senses and all over within -- i.e. on member countries. For the CEE region's and its member countries' flow dynamics within the 1994-2015 year interval, such a thesis does confirm, except for Hungary, on FDI, and Romania, on DIA, both for negative dynamics.

4.1. FDI and DIA, as long-way flows, versus cooperation capital

Equally as an (e) type world region – i.e. of emergent economies in their capital accumulation phase --, the CEE has -- besides its uneven international capital distribution among member countries that actually belongs to all types of world regions -- different FDI and DIA leader countries (Andrei & Andrei 2020). Poland is the FDI country leader with its 167,603 million US dollars, 0.7% of world capital stocks and about 1/3 of the whole basic long-way flows feeding the region, while Hungary tops the DIA of the region that equals the region's cooperation capital: 45,391 million US dollars that is 0.2% of world capital stocks of the 1990-2015 year interval and 1/2 of all CEE countries' international invested capital (i.e. in their own region/[1]).

This paragraph is for deepening about the international capital outflows (DIA) in this region since already viewed above as less important – i.e. than the international capital inflows (FDI) and this for both its quantitative level and its economic significance (qualitative aspect). The CEE region's DIA, the

same as *cooperation capital* – i.e. the short-way and intra-region one⁵ --, but then let us notice not only the Hungarian supremacy on the region's *DIA-cooperation capital*, but equally that three countries of the *Visegrad-4 Treaty*⁶ obviously make nearly the whole *DIA-cooperation capital* of the region. Though, the cooperation capital story ends on the other capital flow sense – i.e. it is also by definition that cooperation capital rises both DIA and FDI stocks in the CEE whole region at the same.

So, finally let us have our whole idea about cooperation capital in the region of CEE even when its role, as international capital, stays secondary, as compared to the received long-way flows from the west of the continent. First, Hungary comes on top of regional cooperation capital despite its long-way flows received were less important than the ones of other member countries – i.e. besides, the Hungary's FDI stocks balance is even suffering this way. Second, Poland and Czech Republic do have a different capital story together – i.e. they stay on top for both DIA and FDI flows of the region and this comes to be explained by inter-investment flows among the Visegrad-4 countries. These two remarks so claim the undeniable role of this Organization for the international capital developing in the area. And then third, Romania seems equally to owe its high FDI position in the region to Hungarian capital inflows.

5. Conclusions

CEE appears as an *(e) type FDI&DIA region* with an obviously typical and homogeneous behaviour this way. Its international capital is dominated by long-way flows entries from the Euro-Zone and possibly also from the West Europe that here make their third level amount destination in the whole rest of Eurasia. As a typical region for emergent member economies, CEE either proves an international investment really dynamic – i.e. above the world dynamic of investing average --, or shows positive FDI stock balances on all its member countries – i.e. member countries do not invest significant amounts off the region as concomitantly so far -- and so its cooperation capital makes as low as 1/5 of total international capital working around and rather originates from the restrained Visegrad-4 group of countries, especially from Hungary. Finally, there are in the CEE region two country FDI behaviours “breaking” its ensemble homogeneity as such: Romania, too much complying with the *(e)* type rule of keeping its FDI home and not (yet) investing abroad, and Hungary,

⁵ i.e. since cooperation capital does equal the lower stocks of FDI- inflows and DIA-outflows and all FDI stock balances are positive in the CEE region [14]

⁶ *Ibidem*

the opposite country case behaviour, with enlarging its investing abroad (DIA) rather indicating a next future behaviour of all these countries.

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INDIA'S INDUSTRIAL GROWTH SCENARIO DURING COVID-19 PANDEMIC – A TIME SERIES ANALYSIS

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Abstract. Covid-19 pandemic has adversely affected the different industrial sectors of Indian economy. It has thus adversely disturbed the parameters of the sustainable development in this country. This paper shows how different macroeconomic sectors of India have reacted to the adverse impact in the Indian Economy differently. Some sectors are highly affected while some sectors are not appreciably affected by the pandemic. Few major sectors have been affected badly by the pandemic. For example the highest impact on industrial growth during the pandemic has been assessed for the sectors like, mining –26.9%, manufacturing –66.6%, infrastructure –37.9%, steel - 65.2%, cement – 40%, crude oil –5.22% etc. I have studied in this paper the impact of Covid-19 pandemic on different sectors and got different findings for different sectors. The study involved with the Cointegration and Vector Error Correction Estimation of Time Series Econometrics. It may be noted that the pandemic has been going on for one and half year and therefore it is expected to be a transitory phenomenon. We, therefore, cannot expect a very long run relationship between pandemic and other sectors of the economy. That's why, the Cointegration study has been indirectly established through the estimation of Vector Error Correction model. In the estimated VEC model it has been found that there is a strong relation between changes in macroeconomic parameters and the pandemic in the short run. I have found that the growth rate in manufacture, infrastructure and mining etc have been affected adversely by the pandemic. Growth rate of different sectors have been significantly related to the 1st and 2nd lag variation of the Covid-19 dataset. The Covid-19 dataset have been found to exert negative impact on the sectors under study.

Keyword: Covid-19 Pandemic, Sustainable Development, Unemployment, Time Series Econometrics, Cointegration, and Vector Error Correction Estimation

JEL Code: C01, C22, E2, L6.

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Introduction

The world has witnessed several epidemics such as the Spanish Flu of 1918, Ebola 1976, MERS (Middle East Respiratory Syndrome) of 2012 and SARS (Severe Acute Respiratory Syndrome) of 2002. In the past, India has had to deal with diseases such as the small pox 1972, polio 1985, outbreak of HIV/AIDS of 1986 and Plague 1994 (Kolli, R., & Sinharay, A. 2014). However, the Novel Corona virus Covid-19 which originated in China in November-December 2019 and over the next few months rapidly spread to almost all countries of the world has turned out to be one of the biggest health and economic crises in our history. India recorded the first positive case of the disease on 30th January, 2020. Since then the highest new positive case 414188 has been found on 6th May 2021 and still it is continuing (Source: JHU CSSE COVID-19).

The government of India has been taking different restrictive steps to overcome the situation since the first case found on January and announced full 'Lockdown' on 25th March 2020. The 1st 'Lockdown' had been continued up to 31st May 2020. This restrictive step is still continuing with 'Unlock' process. There are four nationwide full 'Lockdown' and fifteen 'Unlock' already has been taken but the daily fluctuation of new cases are still continuing (Source: JHU CSSE COVID-19). Moreover presently it has been further noticed that the few states like Kerala and 7th sister's responding increasing trend daily cases (Source: JHU CSSE COVID-19).

The outbreak of the Covid-19 pandemic is an unprecedented shock to the Indian economy (Dev and Sengupta 2020, FICCI 2020). This shock has been shown on various sectors like manufacturing, infrastructure, mining, steel, cement and crude oil etc. (Kapur, Dev and Subramanian, Arvind (2020). There are 83% labour are involved in informal sectors and 17% are in involved in formal sectors who are directly or indirectly involved with these sectors (Indian Economic Survey 2019-20). Ongoing this Pandemic the situation of growth rate of the industrial sectors and the positive growth rate of new cases over the period of study represents the clear scenario of Indian economy.

Data and Different Sources

The monthly time series data of growth rate from January 2020 to March 2021 has been taken from the different sources. The source of the

data is Office of the Economic Adviser to the Government of India, Ministry of Statistics and Program Implementation (MOSPI), World Steel Association and U.S. Energy Information Administration.

The monthly average growth rate of Covid-19 infection data has been taken from the source JHUCSSE COVID-19.

Objective of the Study

More specifically, we seek to examine in this study

1. If there exists any unique long-run relationship between the Covid-19 and the different sectors growth rate over the period of study.
2. If the short-run relationship, so identified, were stable.

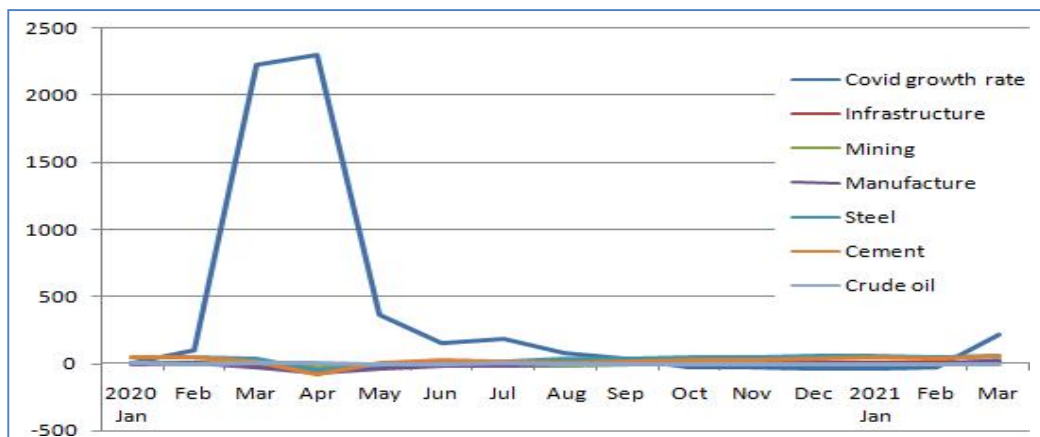


Figure 1.1. Time Plot of Industrial Sectors and Covid-19 Growth Rate
[Period: January 2020 to March 2021].

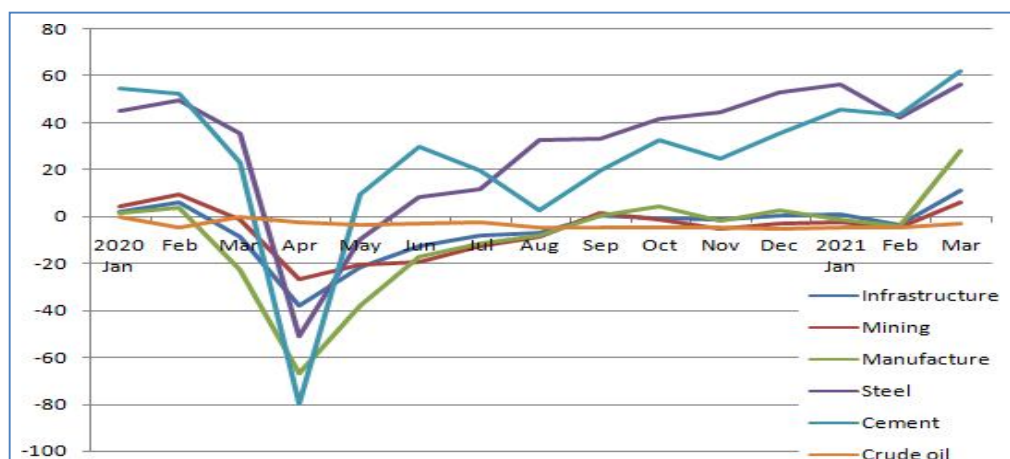


Figure 1.2. Time Plot of Industrial Sectors Growth Rate
[Period: January 2020 to March 2021].

SECTION – 1

1.1. Different Test results of Infrastructure and Covid-19 Growth Rate

In this section we observe the existence of cointegration between infrastructure growth rate and Covid-19 growth rate. This is shown in the following table.

Table 1.1
*Johansen Cointegration Test of Infrastructure
and Covid-19 Growth Rate [Period: January 2020 to March 2021]*

Series: INFRASTRUCTURE COVID Lags interval: 1 to 1			Sample: 1 15 Included observations: 13		
Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	1	1	2	1	2
Max-Eig	1	1	2	1	2
Log Likelihood by Rank (rows) and Model (columns)					
0	-142.8185	-142.8185	-142.7817	-142.7817	-140.3380
1	-131.8651	-131.8505	-131.8140	-128.4549	-127.8173
2	-130.7194	-128.8201	-128.8201	-122.4447	-122.4447
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	22.58747	22.58747	22.88950	22.88950	22.82123
1	21.51771	21.66931	21.81754	21.45460	21.51036
2	21.95684	21.97232	21.97232	21.29919	21.29919*

The corresponding regression equation is given below:

Table 1.2
Regression of Infrastructure and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Sample: 1 15 Included observations: 15		Dependent Variable: INFRASTRUCTURE Method: Least Squares			
Variable	Coefficient	Std. Error	t-Statistic	Prob.	
COVID	-0.010190	0.003210	-3.174769	0.0073	
C	-1.434063	2.684719	-0.534158	0.6022	
R-squared	0.436721	Mean dependent var		-5.160000	
Adjusted R-squared	0.393392	S.D. dependent var		12.00713	
S.E. of regression	9.351756	Akaike info criterion		7.432571	
Sum squared resid	1136.920	Schwarz criterion		7.526978	
Log likelihood	-53.74428	F-statistic		10.07916	
Durbin-Watson stat	1.176533	Prob(F-statistic)		0.007314	

It is observed that the coefficient of 'Covid-19' level variable is – 1.998914 which is significant at 6% level and the coefficient is negative. This indicates that the current 'Covid-19' growth rate adversely affected

the current growth rate of the infrastructure sector.

We have also tested if lagged rates of ‘Covid-19’ growth affect infrastructure growth rate. The regression equations are given below.

Table 1.3.

Regression of Infrastructure and lagged Covid-19
Growth Rate [Period: January 2020 to March 2021]

Dependent Variable: INFRASTRUCTURE				
Method: Least Squares				
Sample(adjusted): 2 15				
Included observations: 14 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID(-1)	-0.013178	0.002198	-5.995964	0.0001
C	-0.723559	1.898693	-0.381083	0.7098
R-squared	0.749748	Mean dependent var	-5.685714	
Adjusted R-squared	0.728893	S.D. dependent var	12.27993	
S.E. of regression	6.393904	Akaike info criterion	6.680131	
Sum squared resid	490.5841	Schwarz criterion	6.771425	
Log likelihood	-44.76092	F-statistic	35.95158	
Durbin-Watson stat	2.162136	Prob(F-statistic)	0.000063	

It is observed that the coefficients of one-period lagged ‘Covid-19’ growth rate [Covid-19 (-1)] and two-period lagged ‘Covid-19’ growth rate [Covid-19 (-2)] are not statistically significant. This indicates that lagged growth rate of ‘Covid-19’ has not statistically significant affects the infrastructure growth rate.

VEC Analysis Results

Short term adjustment of growth rates of the sectors concerned to those in ‘Covid-19’ variable is represented through VEC model.

The estimated VEC model for infrastructure vis-à-vis ‘Covid-19’ variables is given below.

Table 1.4.

VEC Results of Infrastructure and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Cointegrating Eq: CointEq1		Vector Error Correction Estimates			
INFRASTRUCTU...		Date: 08/17/21 Time: 11:57			
COVID(-1)		Sample(adjusted): 4 15			
		Included observations: 12 after adjusting endpoints			
		Standard errors in () & t-statistics in []			
C		C			
Error Correction: D(INFRASTRUC...		D(COVID)			
CointEq1		R-squared			
D(INFRASTRUC...		Adj. R-squared			
D(INFRASTRUC...		Sum sq. resids			
D(COVID(-1))		S.E. equation			
D(COVID(-2))		F-statistic			
		Log likelihood			
		Akaike AIC			
		Schwarz SC			
		Mean dependent			
		S.D. dependent			
		Determinant Residual Covariance			
		Log Likelihood			
		Log Likelihood (d.f. adjusted)			
		Akaike Information Criteria			
		Schwarz Criteria			

It is observed from the table that coefficients of [D Covid-19 (-1)] and [D Covid-19 (-2)] under [D (infrastructure)] are both statistically significant. This indicates that infrastructure sector displays the significant result in the short run in order to ensure any adjustment to variations in short-run variations in 'Covid-19' growth rate.

SECTION – 2

2.1. Different Test results of Manufacture and Covid-19 Growth Rate

In this section we observe the existence of cointegration between manufacture growth rate and Covid-19 growth rate. This is shown in the following table.

Table 2.1.
Johansen Cointegration Test of Manufacture and Covid-19 Growth Rate [Period: January 2020 to March 2021]

Series: MANUFACTURE COVID Lags interval: 1 to 2			Sample: 1 15 Included observations: 12		
Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	2	2	2	1	0
Max-Eig	2	2	2	0	0
Log Likelihood by Rank (rows) and Model (columns)					
0	-129.5620	-129.5620	-127.0628	-127.0628	-119.9075
1	-120.1386	-119.1424	-117.6751	-117.6663	-115.6329
2	-116.2480	-114.3844	-114.3844	-113.7151	-113.7151
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	22.92701	22.92701	22.84379	22.84379	21.98458
1	22.02310	22.02373	21.94585	22.11105	21.93882*
2	22.04133	22.06407	22.06407	22.28585	22.28585

The corresponding regression equation is given below:

Table 2.2.
Regression of Manufacture and Covid-19 Growth
Rate [Period: January 2020 to March 2021]

Sample: 1 15 Included observations: 15		Dependent Variable: MANUFACTURE Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID	-0.020010	0.005414	-3.696168	0.0027
C	-1.209936	4.528359	-0.267191	0.7935
R-squared	0.512408	Mean dependent var	-8.526667	
Adjusted R-squared	0.474901	S.D. dependent var	21.76782	
S.E. of regression	15.77376	Akaike info criterion	8.478139	
Sum squared resid	3234.551	Schwarz criterion	8.572546	
Log likelihood	-61.58604	F-statistic	13.66166	
Durbin-Watson stat	1.224046	Prob(F-statistic)	0.002690	

It is observed that the coefficient -1.953159 of 'Covid-19' level variable is significant at 7% level and the coefficient is negative. This indicates that the current 'Covid-19' growth rate adversely affected the current growth rate of the manufacture sector.

We have also tested if lagged rates of 'Covid-19' growth affect manufacture growth rate. The regression equations are given below.

Table 2.3.
Regression of Manufacture and lagged Covid-19 Growth
Rate [Period: January 2020 to March 2021]

Sample(adjusted): 2 15 Included observations: 14 after adjusting endpoints		Dependent Variable: MANUFACTURE Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID(-1)	-0.023435	0.004292	-5.460421	0.0001
C	-0.439477	3.707854	-0.118526	0.9076
R-squared	0.713030	Mean dependent var	-9.264286	
Adjusted R-squared	0.689116	S.D. dependent var	22.39415	
S.E. of regression	12.48630	Akaike info criterion	8.018705	
Sum squared resid	1870.894	Schwarz criterion	8.109999	
Log likelihood	-54.13094	F-statistic	29.81620	
Durbin-Watson stat	1.795458	Prob(F-statistic)	0.000145	

It is observed that the coefficients of one-period lagged 'Covid-19' growth rate [Covid-19 (-1)] and two-period lagged 'Covid-19' growth rate

[Covid-19 (-2)] are not statistically significant. This indicates that lagged growth rate of 'Covid-19' has not statistically significant affects manufacture growth rate.

VEC Analysis Results

The estimated VEC model is given below.

Table 2.4.
VEC Results of Manufacture and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Cointegrating Eq: CointEq1		Vector Error Correction Estimates			
MANUFACTURE...		Date: 08/17/21 Time: 12:14			
COVID(-1)		Sample(adjusted): 4 15			
C		Included observations: 12 after adjusting endpoints			
		Standard errors in () & t-statistics in []			
Error Correction: D(MANUFA...		D(COVID)	C	3.245058 (3.56819) [0.90944]	-139.9099 (53.0450) [-2.63757]
CointEq1	-0.493271 (0.44094) [-1.11867]	-26.90623 (6.55509) [-4.10463]	R-squared	0.786075	0.943998
D(MANUFACTU...	-1.215363 (0.64606) [-1.88118]	28.58566 (9.60443) [2.97630]	Adj. R-squared	0.607804	0.897329
D(MANUFACTU...	-0.109372 (0.47986) [-0.22793]	-23.18924 (7.13359) [-3.25071]	Sum sq. resids	896.3527	198094.5
D(COVID(-1))	-0.018836 (0.01577) [-1.19424]	1.408452 (0.23447) [6.00697]	S.E. equation	12.22261	181.7024
D(COVID(-2))	-0.007465 (0.01283) [-0.58188]	-0.104633 (0.19072) [-0.54863]	F-statistic	4.409438	20.22777
			Log likelihood	-42.90783	-75.29682
			Akaike AIC	8.151304	13.54947
			Schwarz SC	8.393758	13.79192
			Mean dependent	4.258333	-167.5133
			S.D. dependent	19.51696	567.0714
			Determinant Residual Covariance	4515633.	
			Log Likelihood	-117.6751	
			Log Likelihood (d.f. adjusted)	-125.9929	
			Akaike Information Criteria	23.33214	
			Schwarz Criteria	23.89787	

It is observed from the table that

(i) The coefficients of [D Covid-19 (-1)] and [D Covid-19 (-2)] under [D (Manufacture)] is significant statistically. The coefficient is positive.

It means that manufacturing sector display any short-run adjustment against short-run variations in 'Covid-19' growth rate.

SECTION – 3

3.1. Different Test results of Mining and Covid-19 Growth Rate

In this sector we observe the existence of cointegration between mining growth rate and 'Covid- 19' rate. This is shown in the following table.

Table 3.1.
Johansen Cointegration Test of Mining and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Series: MINING COVID Lags interval: 1 to 2			Sample: 1 15 Included observations: 12		
Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace Max-Eig	2 2	2 2	2 2	1 1	1 1
Log Likelihood by Rank (rows) and Model (columns)					
0	-122.9383	-122.9383	-121.7102	-121.7102	-116.8562
1	-116.6460	-113.9742	-113.2635	-111.7578	-106.9069
2	-111.4275	-108.4095	-108.4095	-106.6464	-106.6464
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	21.82304	21.82304	21.95170	21.95170	21.47603
1	21.44100	21.16237	21.21059	21.12631	20.48448*
2	21.23792	21.06825	21.06825	21.10773	21.10773

The corresponding regression equation is given below.

Table 3.2.
Regression of Mining and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Sample: 1 15 Included observations: 15		Dependent Variable: MINING Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID	-0.005251	0.003382	-1.552544	0.1445
C	-3.706731	2.828903	-1.310307	0.2128
R-squared	0.156413	Mean dependent var	-5.626667	
Adjusted R-squared	0.091522	S.D. dependent var	10.33845	
S.E. of regression	9.854000	Akaike info criterion	7.537198	
Sum squared resid	1262.317	Schwarz criterion	7.631604	
Log likelihood	-54.52898	F-statistic	2.410393	
Durbin-Watson stat	0.799118	Prob(F-statistic)	0.144530	

It is observed that the coefficient – 2.521515 of ‘Covid-19’ level variable is significant at 2% level and the coefficient is negative. This indicates that the current ‘Covid-19’ growth rate adversely affected the current growth rate of the mining sector.

We have also tested if lagged rates of Covid-19' growth affect manufacture growth rate. The regression equations are given below.

Table 3.3.
Regression of Mining and lagged Covid-19
Growth Rate[Period: January 2020 to March 2021]

Sample(adjusted): 3 15 Included observations: 13 after adjusting endpoints		Dependent Variable: MINING Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID(-2)	-0.007079	0.002771	-2.555088	0.0268
C	-4.681860	2.483914	-1.884872	0.0861
R-squared	0.372450	Mean dependent var	-7.569231	
Adjusted R-squared	0.315400	S.D. dependent var	9.639017	
S.E. of regression	7.975378	Akaike info criterion	7.131233	
Sum squared resid	699.6732	Schwarz criterion	7.218149	
Log likelihood	-44.35302	F-statistic	6.528475	
Durbin-Watson stat	1.989103	Prob(F-statistic)	0.026752	

Sample(adjusted): 2 15 Included observations: 14 after adjusting endpoints		Dependent Variable: MINING Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID(-1)	-0.009885	0.002351	-4.204013	0.0012
C	-2.620481	2.031420	-1.289975	0.2214
R-squared	0.595602	Mean dependent var	-6.342857	
Adjusted R-squared	0.561902	S.D. dependent var	10.33535	
S.E. of regression	6.840865	Akaike info criterion	6.815269	
Sum squared resid	561.5692	Schwarz criterion	6.906563	
Log likelihood	-45.70688	F-statistic	17.67373	
Durbin-Watson stat	1.387573	Prob(F-statistic)	0.001223	

It is observed that the coefficients of one-period lagged 'Covid-19' growth rate [Covid-19 (-1)] have adversely affected to mining growth rate with 6% level of significant value and the coefficient are negative. But two-period lagged 'Covid-19' growth rate [Covid-19 (-2)] are not statistically significant. This indicates that 1 lagged growth rate of Covid-19' has statistically significant affects on mining growth rate.

VEC Analysis Results

In order to examine the short-run adjustment process we estimated an appropriate VEC model. The estimated VEC model is given below:

Table 3.4.
VEC Results of Mining and Covid-19 rowth Rate
[Period: January 2020 to March 2021]

Cointegrating Eq:	CointEq1	Vector Error Correction Estimates			
		Date: 08/17/21 Time: 12:26			
		Sample(adjusted): 3 15			
		Included observations: 13 after adjusting endpoints			
		Standard errors in () & t-statistics in []			
MINING(-1)	1.000000				
COVID(-1)	0.015287 (0.00397) [3.84674]				
C	1.086320				
Error Correction:	D(MINING)	D(COVID)			
CointEq1	-0.802685 (0.20700) [-3.87775]	29.61215 (24.9972) [1.18462]	R-squared	0.775321	0.561408
			Adj. R-squared	0.700428	0.415210
			Sum sq. resids	252.0351	3675467.
			S.E. equation	5.291871	639.0503
D(MINING(-1))	-0.399291 (0.18671) [-2.13860]	75.82869 (22.5468) [3.36316]	F-statistic	10.35239	3.840067
			Log likelihood	-37.71622	-100.0358
			Akaike AIC	6.417881	16.00550
			Schwarz SC	6.591711	16.17933
D(COVID(-1))	0.001159 (0.00309) [0.37438]	0.052911 (0.37375) [0.14157]	Mean dependent	-0.284615	8.680000
			S.D. dependent	9.668492	835.6708
C	-0.552111 (1.47319) [-0.37477]	60.13775 (177.903) [0.33804]	Determinant Residual Covariance		11270633
			Log Likelihood		-137.6571
			Log Likelihood (d.f. adjusted)		-142.4375
			Akaike Information Criteria		23.45193
			Schwarz Criteria		23.88650

It is observed from the table that the coefficients of [D Covid-19 (-1)] and [D Covid-19 (-2)] under [D (Mining)] are not statistically significant. This implies that the Mining sector did not display any short-run adjustment movements against short-run variations in 'Covid-19' growth rate.

SECTION – 4

4.1. Different Test results of Cement and Covid-19 Growth Rate

In this sector we observe the existence of cointegration between cement growth rate and covid- 19 growth rate. This is shown in the following table.

Table 4.1
Johansen Cointegration Test of Cement and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Series: CEMENT COVID Lags interval: 1 to 2			Sample: 1 15 Included observations: 12		
Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	1	1	1	2	1
Max-Eig	1	1	1	2	1
Log Likelihood by Rank (rows) and Model (columns)					
0	-132.0874	-132.0874	-129.6428	-129.6428	-114.2530
1	-118.9726	-114.9461	-114.2275	-110.8108	-98.60893
2	-118.3855	-113.8732	-113.8732	-96.93584	-96.93584
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	23.34790	23.34790	23.27380	23.27380	21.04217
1	21.82877	21.32436	21.37126	20.96847	19.10149*
2	22.39758	21.97887	21.97887	19.48931	19.48931

The corresponding regression equation is given below

Table 4.2.
Regression of Cement and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Sample: 1 15 Included observations: 15		Dependent Variable: CEMENT Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID	-0.028863	0.008862	-3.257047	0.0062
C	35.46692	7.412271	4.784892	0.0004
R-squared	0.449348	Mean dependent var		24.91333
Adjusted R-squared	0.406990	S.D. dependent var		33.52854
S.E. of regression	25.81937	Akaike info criterion		9.463693
Sum squared resid	8666.321	Schwarz criterion		9.558100
Log likelihood	-68.97770	F-statistic		10.60836
Durbin-Watson stat	1.639816	Prob(F-statistic)		0.006243

It is observed that the coefficient of 'Covid-19' level variable is statistically not significant at level and the coefficient is negative. This indicates that the current 'Covid-19' growth rate is not adversely affected the current growth rate of the cement sector.

We have also tested if lagged rates of 'Covid-19' growth affect cement growth rate. The regression equations are given below.

Table 4.3.
Regression of Cement and lagged Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Dependent Variable: CEMENT				
Method: Least Squares				
Sample(adj): 2 15				
Included observations: 14 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID(-1)	-0.031182	0.008043	-3.876940	0.0022
C	34.54201	6.948591	4.971081	0.0003
R-squared	0.556060	Mean dependent var	22.80000	
Adjusted R-squared	0.519065	S.D. dependent var	33.74153	
S.E. of regression	23.39958	Akaike info criterion	9.274877	
Sum squared resid	6570.484	Schwarz criterion	9.366171	
Log likelihood	-62.92414	F-statistic	15.03067	
Durbin-Watson stat	2.071193	Prob(F-statistic)	0.002200	

It is observed that the coefficients of one-period lagged ‘Covid-19’ growth rate [Covid-19 (-1)] and two-period lagged ‘Covid-19’ growth rate [Covid-19 (-2)] are not statistically significant. This indicates that lagged growth rate of Covid-19 has not statistically significant affects on cement growth rate.

VEC Analysis Results

In order to examine the short-run adjustment process we estimated an appropriate VEC model. The estimated VEC model is given below.

Table 4.4.
VEC Results of Cement and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Cointegrating Eq:		Vector Error Correction Estimates			
CointEq1		Date: 08/17/21 Time: 12:36			
CEMENT(-1)		Sample(adj): 4 15			
COVID(-1)		Included observations: 12 after adjusting endpoints			
C		Standard errors in () & t-statistics in []			
			C		
Error Correction:		D(CEMENT)	D(COVID)		
CointEq1	0.224639 (0.26519) [0.84707]	12.02721 (1.67869) [7.16462]	R-squared	0.916521	0.980806
D(CEMENT(-1))	-0.942462 (0.58717) [-1.60509]	-6.344566 (3.71683) [-1.70698]	Adj. R-squared	0.846955	0.964811
D(CEMENT(-2))	0.028050 (0.44199) [0.06346]	-9.374862 (2.79780) [-3.35080]	Sum sq. resids	1694.417	67894.48
D(COVID(-1))	-0.058038 (0.00940) [-6.17691]	0.175021 (0.05948) [2.94266]	S.E. equation	16.80485	106.3755
D(COVID(-2))	0.009479 (0.02443) [0.38804]	-0.529791 (0.15464) [-3.42606]	F-statistic	13.17485	61.31941
			Log likelihood	-46.72839	-68.87208
			Akaike AIC	8.788064	12.47868
			Schwarz SC	9.030518	12.72113
			Mean dependent	3.258333	-167.5133
			S.D. dependent	42.95610	567.0714
			Determinant Residual Covariance	2542009.	
			Log Likelihood	-114.2275	
			Log Likelihood (d.f. adjusted)	-122.5453	
			Akaike Information Criteria	22.75755	
			Schwarz Criteria	23.32328	

It is observed from the table that the coefficients of [D Covid-19 (-1)] and [D Covid-19 (-2)] under [D (Cement)] is statistically significant. This implies that the cement sector display a short-run adjustment movements against short-run variations in 'Covid-19' growth rate.

SECTION – 5

5.1. Different Test results of Crude Oil and Covid-19 Growth Rate

In this sector we observe the existence of cointegration between crude oil growth rate and covid- 19 growth rate. This is shown in the following table.

Table 5.1.

Johansen Cointegration Test of Crude Oil and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Series: OIL COVID Lags interval: 1 to 2			Sample: 1 15 Included observations: 12		
Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	1	2	2	2	2
Max-Eig	1	2	2	2	2
Log Likelihood by Rank (rows) and Model (columns)					
0	-101.0335	-101.0335	-98.81415	-98.81415	-88.20962
1	-89.63069	-86.51415	-85.55003	-85.01033	-74.42112
2	-89.63068	-79.73012	-79.73012	-72.51308	-72.51308
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	18.17224	18.17224	18.13569	18.13569	16.70160
1	16.93845	16.58569	16.59167	16.66839	15.07019*
2	17.60511	16.28835	16.28835	15.41885	15.41885

The corresponding regression equation is given below.

Table 5.2.

Regression of Crude Oil and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Sample: 1 15 Included observations: 15		Dependent Variable: OIL Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID	0.001231	0.000482	2.552167	0.0241
C	-4.096770	0.403439	-10.15461	0.0000
R-squared	0.333796	Mean dependent var	-3.646667	
Adjusted R-squared	0.282550	S.D. dependent var	1.659116	
S.E. of regression	1.405312	Akaike info criterion	3.641961	
Sum squared resid	25.67371	Schwarz criterion	3.736368	
Log likelihood	-25.31471	F-statistic	6.513557	
Durbin-Watson stat	1.635715	Prob(F-statistic)	0.024094	

It is observed that the coefficient of 'Covid-19' level variable is statistically not significant at level and the coefficient is not negative. This indicates that the current 'Covid-19' growth rate is not adversely affected on the current growth rate of the crude oil sector.

We have also tested if lagged rates of 'Covid-19' growth affect crude oil growth rate. The regression equations are given below.

Table 5.3.
Regression of Crude Oil and lagged Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Sample(adjusted): 2 15		Dependent Variable: OIL		
Included observations: 14 after adjusting endpoints		Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID(-1)	0.000527	0.000488	1.079162	0.3017
C	-4.084029	0.421611	-9.686719	0.0000
R-squared	0.088464	Mean dependent var	-3.885714	
Adjusted R-squared	0.012503	S.D. dependent var	1.428747	
S.E. of regression	1.419788	Akaike info criterion	3.670455	
Sum squared resid	24.18956	Schwarz criterion	3.761749	
Log likelihood	-23.69319	F-statistic	1.164591	
Durbin-Watson stat	1.851489	Prob(F-statistic)	0.301729	

It is observed that the coefficients of one-period lagged 'Covid-19' growth rate [Covid-19 (-1)] and two-period lagged 'Covid-19' growth rate [Covid-19 (-2)] are not statistically significant. This indicates that lagged growth rate of 'Covid-19' has not statistically significant affects on crude oil growth rate.

VEC Analysis Results

In order to examine the short-run adjustment process we estimated an appropriate VEC model. The estimated VEC model is given below.

Table 5.4.
VEC Results of Crude Oil and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Cointegrating Eq:		Vector Error Correction Estimates			
CointEq1		Date: 08/17/21 Time: 13:37			
		Sample(adjusted): 3 15			
		Included observations: 13 after adjusting endpoints			
		Standard errors in () & t-statistics in []			
OIL(-1)	1.000000				
COVID(-1)	-0.002429 (0.00034) [-7.04794]				
C	4.915578				
Error Correction:		D(OIL)	D(COVID)	R-squared	0.573224 0.578704
				Adj. R-squared	0.430965 0.438271
CointEq1		0.407950 (0.36174) [1.12775]	554.1799 (172.111) [3.21989]	Sum sq. resids	15.59573 3530526.
				S.E. equation	1.316381 626.3231
				F-statistic	4.029441 4.120877
				Log likelihood	-19.62951 -99.77425
				Akaike AIC	3.635309 15.96527
D(OIL(-1))		-0.742282 (0.23534) [-3.15405]	-296.8865 (111.974) [-2.65139]	Schwarz SC	3.809140 16.13910
				Mean dependent	0.123077 8.680000
				S.D. dependent	1.745066 835.6708
D(COVID(-1))		0.000496 (0.00060) [0.82341]	0.676415 (0.28643) [2.36153]	Determinant Residual Covariance	212935.4
				Log Likelihood	-111.8588
				Log Likelihood (d.f. adjusted)	-116.6392
				Akaike Information Criteria	19.48296
C		-0.132672 (0.37388) [-0.35485]	-92.45771 (177.890) [-0.51975]	Schwarz Criteria	19.91754

It is observed from the table that the coefficient -2.49209 of [D Covid-19 (-1)] under [D (Crude oil)] are statistically significant and [D Covid-19 (-2)] under [D (Crude oil)] are not statistically significant. This implies that the Crude oil sector display a short-run adjustment movements against short-run variations in 'Covid-19' growth rate at [D Covid-19 (-1)] lag.

SECTION – 6

6.1. Different Test results of Steel and Covid-19 Growth Rate

In this sector we observe the existence of cointegration between steel growth rate and 'Covid-19' rate. This is shown in the following table.

Table 6.1.
Johansen Cointegration Test of Steel and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Series: STEEL COVID Lags interval: 1 to 1			Sample: 1 15 Included observations: 13		
Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	0	1	1	2	2
Max-Eig	0	1	1	2	2
Log Likelihood by Rank (rows) and Model (columns)					
0	-154.0901	-154.0901	-154.0824	-154.0824	-152.5740
1	-149.2771	-141.0241	-141.0219	-139.0966	-137.7290
2	-149.0886	-139.4750	-139.4750	-128.4142	-128.4142
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	24.32155	24.32155	24.62807	24.62807	24.70369
1	24.19648	23.08063	23.23414	23.09179	23.03523
2	24.78286	23.61153	23.61153	22.21757*	22.21757*

The corresponding regression equation is given below.

Table 6.2.
Regression of Steel and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Sample: 1 15 Included observations: 15		Dependent Variable: STEEL Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID	-0.022448	0.008379	-2.679003	0.0189
C	38.08793	7.008675	5.434398	0.0001
R-squared	0.355704	Mean dependent var		29.88000
Adjusted R-squared	0.306143	S.D. dependent var		29.30861
S.E. of regression	24.41352	Akaike info criterion		9.351717
Sum squared resid	7748.256	Schwarz criterion		9.446123
Log likelihood	-68.13787	F-statistic		7.177057
Durbin-Watson stat	1.232668	Prob(F-statistic)		0.018937

It is observed that the coefficient -2.565999 of Covid-19 level variable is significant at 2% level and the coefficient is negative. This indicates that the current Covid-19 growth rate adversely affected the steel growth rate of the steel sector.

We have also tested if lagged rates of Covid-19 growth affect steel growth rate. The regression equations are given below.

Table 6.3.
Regression of Steel and lagged Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Sample(adjusted): 2 15 Included observations: 14 after adjusting endpoints		Dependent Variable: STEEL Method: Least Squares		
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COVID(-1)	-0.032848	0.005100	-6.440830	0.0000
C	41.15499	4.406016	9.340637	0.0000
R-squared	0.775635	Mean dependent var	28.78571	
Adjusted R-squared	0.756938	S.D. dependent var	30.09531	
S.E. of regression	14.83738	Akaike info criterion	8.363741	
Sum squared resid	2641.776	Schwarz criterion	8.455034	
Log likelihood	-56.54618	F-statistic	41.48429	
Durbin-Watson stat	2.044952	Prob(F-statistic)	0.000032	

It is observed that the coefficients -1.979741 of one-period lagged 'Covid-19' growth rate [Covid-19 (-1)] is statistically significant at 7% and two-period lagged 'Covid-19' growth rate [Covid-19 (-2)] are not statistically significant. This indicates that one lagged growth rate of 'Covid-19' has affected on steel growth rate but two lag dose not statistically significant affects steel growth rate.

VEC Analysis Results

In order to examine the short-run adjustment process we estimated an appropriate VEC model. The estimated VEC model is given below

Table 6.4.
VEC Results of Steel and Covid-19 Growth Rate
[Period: January 2020 to March 2021]

Cointegrating Eq: CointEq1		Vector Error Correction Estimates Date: 08/17/21 Time: 13:28 Sample(adjusted): 4 15 Included observations: 12 after adjusting endpoints Standard errors in () & t-statistics in []			
STEEL(-1)	1.000000				
COVID(-1)	0.095552 (0.00512) [18.6687]				
C	-65.94898	C	0.941867 (2.69491) [0.34950]	-135.9281 (40.0463) [-3.39428]	
Error Correction:	D(STEEL)	D(COVID)			
CointEq1	-0.247401 (0.16999) [-1.45536]	-12.55434 (2.52610) [-4.96985]	R-squared	0.950519	0.967832
			Adj. R-squared	0.909285	0.941026
			Sum sq. resid	515.2901	113785.7
			S.E. equation	9.267237	137.7109
D(STEEL(-1))	-0.666825 (0.29953) [-2.22625]	0.124749 (4.45098) [0.02803]	F-statistic	23.05171	36.10452
			Log likelihood	-39.58620	-71.97026
			Akaike AIC	7.597700	12.99504
			Schwarz SC	7.840154	13.23750
D(STEEL(-2))	0.173457 (0.35555) [0.48786]	-21.01406 (5.28347) [-3.97732]	Mean dependent	1.758333	-167.5133
			S.D. dependent	30.76877	567.0714
D(COVID(-1))	-0.025064 (0.01389) [-1.80506]	1.232340 (0.20633) [5.97254]	Determinant Residual Covariance		636694.3
			Log Likelihood		-105.9210
			Log Likelihood (d.f. adjusted)		-114.2388
			Akaike Information Criteria		21.37313
D(COVID(-2))	0.004974 (0.00896) [0.55537]	-0.400373 (0.13309) [-3.00831]	Schwarz Criteria		21.93886

It is observed from the table that the coefficients of [D Covid-19 (-1)] is statistically significant under [D (Steel)] and [D Covid-19 (-2)] are not statistically significant. This implies that the Steel sector display a short-run adjustment movements against short-run variations in 'Covid-19' growth rate.

Conclusion and policy ecommendation

Infrastructure

It is observed from the regression equation and VEC analysis that

- (i) The coefficient of level 'Covid-19' data is significant at 6% level
- (ii) The coefficient is negative
- (iii) The coefficients of [Covid-19 (-1)] and [Covid-19 (-2)] is not statistically significant.
- (iv) The coefficients of [D Covid-19 (-1)] and [D Covid-19 (-2)] under [D (Infrastructure)] are significant statistically and coefficient value is positive.

This indicates that 'Covid-19' situation over this period concerned affected the infrastructure growth rate adversely but lagged growth rate of 'Covid-19' is not statistically significant affects on the infrastructure growth rate. Infrastructure sector also display a short-run adjustment against short-run variations in 'Covid-19' growth rate.

Manufacture

It is observed from the regression equation and VEC analysis that

- (v) The coefficient of level 'Covid-19' data is significant at 7% level
- (vi) The coefficient is negative
- (vii) The coefficients of [Covid-19 (-1)] and [Covid-19 (-2)] is not statistically significant.
- (viii) The coefficients of [D Covid-19 (-1)] and [D Covid-19 (-2)] under [D (Manufacture)] are significant statistically and the coefficient is positive.

This indicates that 'Covid-19' situation over this period concerned affected the manufacture growth rate adversely. The lagged period values of 'Covid-19' data did not affected the manufacture sector growth rate at all. Manufacturing sector also display the short-run adjustment against short-run variations in 'Covid-19' growth rate.

Mining

It is observed from the regression equation and VEC analysis that

(i) The coefficient of level 'Covid-19' data is significant at 2% level

(ii) The coefficient is negative

(iii) The coefficients of [Covid-19 (-1)] is significant at 6% level but [Covid-19 (-2)] is not statistically significant.

(iv) The coefficients of [D Covid-19 (-1)] and [D Covid-19 (-2)] under [D (Mining)] are not significant statistically.

This indicates that 'Covid-19' situation over this period concerned affected the mining growth rate adversely. The 1st lagged period values of 'Covid-19' also affected the mining sector growth rate. Mining sector does not display any short-run adjustment against short-run variations in 'Covid-19' growth rate.

Cement

It is observed from the regression equation and VEC analysis that

(i) The coefficient of level 'Covid-19' data is not significant at any level

(ii) The coefficient is negative

(iii) The coefficients of [Covid-19 (-1)] and [Covid-19 (-2)] is not statistically significant.

(iv) The coefficients of [D Covid-19 (-1)] and [D Covid-19 (-2)] under [D (Cement)] both are statistically significant.

This indicates that 'Covid-19' situation over this period concerned does not affect the cement growth rate adversely. Lagged growth rate of 'Covid-19' has also not statistically significant effects on the cement growth rate. Cement sector displays a short-run adjustment against short-run variations in 'Covid-19' growth rate.

Crude Oil

It is observed from the regression equation and VEC analysis that

(i) The coefficient of level 'Covid-19' data is not significant at any level

(ii) The coefficient is positive but at 2nd lag its negative

(iii) The coefficients of [Covid-19 (-1)] and [Covid-19 (-2)] is not statistically significant

(iv) The coefficients of [D Covid-19 (-1)] are statistically significant under [D (Crude oil)] but [D Covid-19 (-2)] are not statistically significant and coefficient is negative.

This indicates that 'Covid-19' situation over this period concerned dose not affected the crude oil growth rate adversely. Lagged growth rate of 'Covid-19' has also not statistically significant affects on the crude oil growth rate. Crude oil sector display a short-run adjustment against short-run variations in 'Covid-19' growth rate at [D Covid-19 (-1)].

Steel

It is observed from the regression equation and VEC analysis that

- (i) The coefficient of level 'Covid-19' data is significant at 2% level
- (ii) The coefficient is negative
- (iii) The coefficients of [Covid-19 (-1)] is significant at 7% level but [Covid-19 (-2)] is not statistically significant.
- (v) The coefficients of [D Covid-19 (-1)] are statistically significant under [D (Steel)] but [D Covid-19 (-2)] are not statistically significant. The coefficient is negative.

This indicates that 'Covid-19' situation over this period concerned affected the steel growth rate adversely. The 1st lagged period values of 'Covid-19' also affected the steel sector growth rate at 7% level. Steel sector display a short-run adjustment against short-run variations in 'Covid-19' growth rate at [D Covid-19 (-1)].

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