OUTLINE ABOUT A SHEAF APPROACH OF THE "ARROW OF TIME" AND CREATIVITY: FRACTIONAL OPERATORS, TOPOS AND GROTHENDIECK SCHEMES APPROACH

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Abstract *: The purpose of this note is to show why and how the fractional operators required for formalizing dynamic issues in complex environment , mobilize among the most advanced mathematical concepts : topos, site, spectrum, sheaf, ringed spaces, p-adic numbers, etc. In the context of physical exchanges in spaces crumpled by long range interaction (hyperbolic geometries), geodesics no longer respond to Noether invariance principles but to memory effects and to the categorical limits imposed by some requirements for completion. This completion is in fine based upon algebra-topology coupling. This coupling may be expressed through the zeta function which, as a bridge, ensures the formal closure of the representation and implicitly the space-time characteristics. The analysis points out the existence of a particular case corresponding to the Riemann hypothesis (and Martin's axiom). Above approach involves a relationship between quantum mechanics and 2D self-similarity. In non integer cases, time is discretized and an arrow of time naturally emerges from the analysis. The note relates this arrow to anti entropic properties of dissipative complex systems.*

Introduction

Despite its apparent abstraction the problem we address here is practical issues. Its purpose is to identify the part of the entropy which must be attributed to geometry when a physical process whose formalization is locally simple (transfer or transport of extensive objects), is immersed in a complex geometry which is heterogeneous and as crumpled as can be those involved in materials engineering (Electrode of batteries; composites materials; catalyzers, etc). The technical object at the origin of the present questions have been the first lithium batteries, while it was still being developed in the mid-seventies [LMA82-20]. The dynamics of its macroscopic flow/force behavior led to the use of non-integer differential equations [OlS74] originated in fractal geometry of electrodes [LMA82,83,84]. The use of such operator was quickly generalized at the time, giving rise in particular to a new school in automatics and robotics [OUA83]. Linked to fractal geometries [BMA77,83],[TRC03,08],[GAA19] this use was contested by some scientists either for empirical reasons [Liu85], or for analytical reasons [COS86] [SAB97], [JSa20)]. The arguments of the objectors being scientifically defensible but the use of such operators and fractal models proving surprisingly very effective [LMA82], the understanding of apparent contradictions has required new tools and a mathematical examination that was formally much more extended than it was at that time and even until recently. It was nevertheless quickly recognized that the possible explanations of the observed efficiency and the irreversibility of time would be based on non-commutative geometries [COA90] related to scaling in fractal algebra, but at that time these ideas were beyond the reach of the abstraction capabilities of engineering [LMA96]. Nearly half a century of mathematical and educational progress allows us to have today the tools of thought necessary for a reconciliation of the opposite points of view: fractional operators make sense not within the framework of set theory but within the framework of category theory [MLS71], [PSE06], [LAW97,12], [LET14]. By their criticized incompleteness these operators naturally fit into a class of dualities referred *in fine* as Isbell duality [VAA19]

Before addressing the theoretical questions, let us observe that the concept of entropy is mainly based on issues of symmetries, combinatorics, indistinguishability, ambiguities, circular reasoning and stability [LET22]. According to modern physics, since temperature gives rise to limit fluctuations and characteristic lengths, it is not conceivable to obtain meaningful physical measurements for length less than a given characteristic. In quantum mechanics temperature leads to a non-optimal filling of the energy states which then interfere in accordance to von Neumann algebras which justify the fundamental structure of the tree of states. Then these states lose part of their singularity and give rise to entropic factor. This loss can be approached using methods based on homotopy. More simply, we can use homology therefore the orderings associated with internal surjections between different states. By using quotient groups ($^kH=Ker\delta_n/Im\delta_{n\pm1}$) we can then justify logarithmic formulations of so-called entropic state functions based on probability measure [BAP15],[VJP19]. Beyond the need for physical measurements, contemporary physics leads us to assert that any phenomenon recorded in time having an asymptotic cyclic representation (integrals space basis) , for example in Fourier space, gives rise to a series of congruent approximations leading to spread out in space-time a set of entropy sources whose summation will manifest itself by a production of heat, a loss of efficiency and more surprisingly the possible emergence of fractal structures, as for example in aggregative processes of electro-plating [SAB97]. In the best case, the loss of efficiency ensures stability and leads to equilibrium [BRC23]. Conversely and provided that uncertainty may be controlled, engineers, -using imbalances and non-stationarities- make it possible, to build machines which can be thermal but also mechanical, electrical or now even quantic. These machines contribute to innovative action which then appears as an anti-entropic dual of the irreducible dissipative effect [CNM23], [LMA20]. The question here is to give a mathematical meaning to the notion of anti-entropy to be distinguished from Brillouin's neg-entropy [BRL56], [BON68].

In the concrete case that here concerns us, fractional dynamics now requires a categorical approach of the limits of morphisms, -i.e. physically dynamic long range discrete correlations -, through dualities capable of offsetting the divergence over singularities. All the questions we ask, can be addressed by means of the theory of ideals of rings (by duality the theory of filters [RIP23] or distributive lattices [SEM13]). It is appropriate to refer the reader to the conditions of validity of ring theory [RBT11] and to all its subtle topological and algebraic extensions and consequences [open/closed, ring spectrum, sheaf of rings, functorial correspondences (Galois, Poincaré, Langland), groups (cyclic, Galois, etc.), topos, sites, local frames, patterns, fixed points] to understand where the blind spot of the analytical and local approach to reality is located and the relevance of certain criticisms about the operators of non-integer order when manipulated outside duality.

By handling unusual notions in engineering, we will try to give to these abstract concepts an intuitive approach more suited to practical purposes than those done in works intended for professional mathematicians [COP08],[COA12],[COA22]. To summarize, when using fractional operators, ring ideal theory formalizes the basic question of divisibility of categories, with morphisms as objects and natural transform as morphisms. In particular this approach may explain the thickness of the local. More precisely, by mimicking the characteristics of type natural rings $\mathbb{Z}p$ with p ∈ $\mathcal{P}_\tau$, the theory of prime ideals seeks to find the chains of subsets associable with quotient rings, capable of fitting together its modules in a finite manner as perfectly Russian dolls can do [1]. We denote <x> ideal « x » the set of multiples of x. x may be a morphism. The main idea fits a generalization of the concept of octave in music namely « $n \sim n^2$ » and the condensation of all global information in a single octave, namely locally. This approach is related to the notion of scale changes (scaling) when cycles are involved and able to resonate between them.

In order to be explicit, we will implement the fractional operator through a particular unclosed impedance (transfer function) noted $Z_\alpha(n)=1/[1+(i\omega\tau)^\alpha]$ namely Cole and Cole impedance related to fractional differential equation. We show below that the duality required for closing, namely completion/adjunction $Z_{1-\alpha}(\tau,p_i)$: $Z_\alpha(n) \rightleftharpoons Z_{1-\alpha}(\tau,p_i)$ may a basis to a fibered structure giving birth

[1] We call Ideal of an ordered set (X,<), the set of subsets of a stable part A of X for the change of scale. In certain respects, ideals are therefore similar to vector subspaces — which are additive subgroups stable by external multiplication; in other respects, they behave like distinguished subgroups — they are additive subgroups from which a quotient ring structure can be constructed. For example <60>/2 ⊂ <30>/2 ⊂ <15>/3 ⊂ <5>/5 ⊂ <1>= $\mathbb{Z}$ gives an idea of the nestings we are talking about. The dual notion of that of ideal, that is to say where the ≤ are inverted and ⋁ into ⋀ inverted, is that of filter.

to a new duality concerning zeta function $\zeta_\alpha(s) \rightleftharpoons \zeta_{1-\alpha}(1\text{-}s)$. We shall note ${}^nZ_{1-\alpha}(\tau) = Z_{1-\alpha}(\tau,p_i)$, the foliated structure and $Z_{1-\alpha}(\tau)$ its support as geometrical completion of $Z_\alpha(n)$ .This approach do not imply simple elliptical varieties -matching Galois groups, but Grothendieck schemes which involve hyperbolic groups [SEC91] capable of taking into account the existence of open points at infinity and torus punctuations [SEC02], [LMA05],[RIP19].

The duality required to complete the dynamic problems associated with intrinsically open fractional differential operators, here featured through what we name α-exponential ($Z_\alpha(n) \rightleftharpoons Z_{1-\alpha}(\tau,p_i)$), always comes from a macroscopic approach to a reality approximated via inclusion order [ROG64] of open sets. This approach is ultimately backed at category theory because it clearly distinguishes the objects considered from the internal links (morphisms and natural transformations) which define the overall identity of the process at stake. In practice, it is these links that we are trying to reach; links that our experiences disclose and that engineers implement in technological applications. Our experience being quantified, these links are for instance woven as the relationship between different numbers that share the same set of prime numbers, or related rings based on these. The generalization to categories of morphisms and to algebras associated with prime ideals, is natural for building a “vectorial space” of thinking but the approach is then necessarily non-local. Physically, this particularity means that a process, even characterized by a time constant defined by a numerical quantity, can be used to test several classes of experimental contexts; in particular a given geometries will then respond to a given request of a process in different ways. Conversely, a context may give rise to various responses depending on the spectrum of the signal applied to it. What then about the dependence of inclusion morphisms on the level of approximation of the objects considered, namely the geometrical meaning of observed dynamical responds? How does the connection between levels of distinct scales operate dynamically, in other words what are the graded algebras (by duality the derived scheme) which define the relationships between objects and their sub-objects? Given empirical data are flows of extensities (QE) and intensities related (QE) [LAW14] therefore continuous functions here not differentiable, parameterized not temporally but from spectrum, how can we think about the existence of possible invariants far from Noether's principle approach (absence of velocity; memory effect etc). These are the questions that engineers and physicists are required to answer if they choose to refuse the usual paradigms (continuous functions, variables in $\mathbb{R}$, use of Laplacian and d'Alembertian operators in heterogeneous and discrete environments, kinetic or potential energy, forced convergence at the edges, etc [SAB97 and scientific personal reférences]). In this context, noting the renormalization properties of almost all experimental responses understood as convolution products and test functions (and not composed functions [SCL50]), the introduction of self-similarity of transfer process initially appeared as a natural way for simplifying the geometrical approach [LMA82]. We understand later that significant renormalizable dynamic endomorphisms arise from monadic category (limitation to 3 objects. 2 morphisms, a unit and a co-unit ensuring the normalization of duality, adjunct functors). In this hypothesis, thanks to the monadic operator, open infinity can be reduced to a closed/open dual quasi local representation (clopen). Thus, if a quantum of geometrical entropy emerges, as suggested by experiments, it can only depend on the properties of monadic symmetries expressed, self-similarity helping, under the constraint of the stability of the product/coproduct functors (inclusion/affinization), the minimization of the velocity of the production of entropy then resulting from the diagonal character of the optimal representation [TOD87], [BAP15] and stabilization [BRC23].

Due to cross link resonance associated with the divisibility of a dual structure (frequency "n" and amplitude "1/η"), the octave changes the frequency (extensity "n") but retains the note of the scale (intensity "1/η"). Here we underline Lawvere advices [LAW12] regarding the irreducible distinction between quantity of space (QE: Extensity "n") and space of quantities (EQ: intensity "η") within categorical representation approach and empirically. Both concept being involved within a local algebra based on number, the concept of ideal leads to manage carefully this type of distinction when local structures are nested within an unique category, indexed using $\mathbb{Z}$ then ringed {$\mathbb{Z}$p | p $\in \mathcal{P}_r$ }. We call Noetherian ring, a ring in which any ascending chain of ideals $<I>_1 \subset <I>_2 \subset <I>_3 \subset \ldots <I>_{n-1} \subset <1>_n$ is finite and there exists a maximal ideal. Any of such rings then the fundamental theorem of arithmetic applies and any set of objects related together according to the laws of rings (A,+,x) can be featured in a basis of irreducible prime ideals into finite numbers. It is obviously these ideals which interest the engineer because they lead to closed systems, then acquiring a structural identity characterized by invariants (Noether Theory [YKS04]). However, the entanglement that characterizes complex systems (physics of heterogeneous environments, quantum mechanics, social and living systems, etc.) shows that there is an obstruction to this radical objectification [BAI11], [KUT85] [GOF37,45]. For example, duality algebra /topology featured as $Z_\alpha \rightleftharpoons {}^nZ_{1-\alpha}$ is shape by the difference $\lim_{ind} Z_\alpha \neq \lim_{proj} {}^nZ_{1-\alpha}$ and that matching is only ensured for $\alpha=1/2$ related to the quadratic self-similarity $\mathbb{N}=\mathbb{N} \times \mathbb{N}$ mimicking octave characteristics. This obstruction is related to specific entropic properties (here Boltzmann and Planck constants) characterized by a breaking of symmetry at infinity. We can, however, go beyond this break by observing that in the case of general self-similarity ($1>\alpha>1/2$) the physical dual behavior may be squeezed in an unique octave itself divided in two physical parts based at categorical limit on a monadic form related to the extensity (EQ) versus intensity (QE) as ring and ring spectrum. This distinction leads back to a dual behavior allowing, in spite of local divergence, an overall stable structure capable to be unfolded.

Preliminary approach : α-exponentielle and uncompleteness

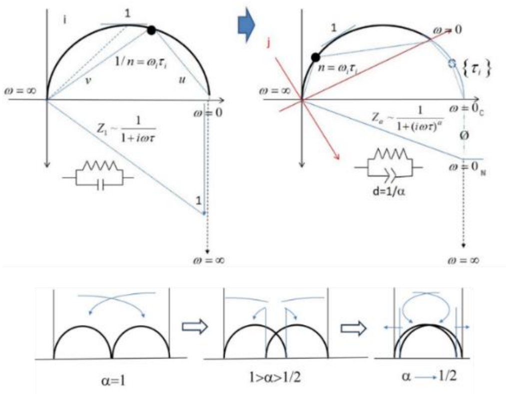

Figure 1: Shift from exponential in the complex plane to α–exponential. The spectrum is characterized by a phase angle. It is related to the origin of the plane (zero and hole set Ø) and to the unit of arithmetic. The renormalization characteristics model is featured by a fractance (->>-) instead of the usual capacity (-||-). This presence is attributed to the self-similar character of the fractal geometry of dimension $d=1/\alpha$ characterizing the geometry. Schematic approach of the analysis of the fundamental group allowing, in the manner of Poincaré, to relate dynamics and non-Euclidean geometries and to tessellate the complex plane. Paradoxically the overlapping pointed out, furthermore related to phase angle, gives rise to a Hausdorff space and Stone approach.

The fundamental questions raised in this note are based empirically on the study of universal transfer functions in an arc of a circle (hyperbolic geodesic) called Cole and Cole $Z_\alpha \sim 1/1+(\iota\omega\tau)^\alpha$. We chose them for their links with the concept of exponentiation (given by $\alpha=1$). These complex functions are parameterized spectroscopically between infinity and zero. They are symmetrical with respect to the parameter "1" They constitute a compact domain of representation. Resulting from the expression of a fractional differential equation of type $d^\alpha(flux)/dt^\alpha \sim force$ with $\alpha \leq 1$ and $\alpha \in \mathbb{R}$ [ALM82], these kind of dynamics were named *α–exponential*. The authors consider this function as an the empirical representation in the complex plane of the concept of topos [COA16,19],[DOA09] [VJP10], herein a place of quasi universal physical entanglements. It represents the normalized hyperbolic geodesic of a fundamentally simple transfer process (to be distinguished from transport processes nevertheless one can choose exponential relaxations but also, in a more advanced although archetypal way, Brownian diffusion processes or not as an empirical basis) immersed in fractal geometry. While, the macroscopic characteristics depend on this geometry spectral geodesics are often observed (TEISI Model [LMA82]) or may be used as fix point for thinking. A point located on the arc defines a non-Euclidean metric based on the scale $\eta=u/v \in \mathbb{Q}$ or $\mathbb{R}$., if "u" and "v" point out the Euclidean distance to the limits of the compact domain $[0,\infty]$. The arc of a circle representative of the macroscopic process can be protectively obtained via an inversion of a straight line (Fig.1) pointing on the abscissa (0,1) of the complex plane. This line is parameterized in $\mathbb{N}$ or $\mathbb{Z}$ from zero to infinity. The line is equipped with a gap (hole segment) at the origin of the parameters. This hole set ${}^{1-\alpha}\varnothing_Z$ reflects an incompleteness of the representation whose closure is expressed through a geometrical completion noted ${}^{n}Z_{1-\alpha}$. We call the disjoint sum $Z_\alpha \bigoplus Z_{1-\alpha}$ the dynamical site and $Z_\alpha \bigoplus {}^{n}Z_{1-\alpha}$ the dynamic topos[2] [CAO16], that is to say a spot of the discrete dynamics when dynamical expression is completed by the set of pre-sheafs reduced to partition of dynamical data according to the fundamental theorem of arithmetic and characterized. At this stage of analysis this spot may be characterized by a scratch, a groove, at infinity which marks the presence of a dual singularity. Nevertheless this Topos is a bridge between Z_α and ${}^{n}Z_{1-\alpha}$ this last stacking structure being a virtual completion setting up the topology associated to the dynamical algebra.

The reasoning which drives the initial model known as TEISI model [LMA82] is based on the coupling between the Fourier component ("$\omega\tau$" as open set) of the physical process and the level of approximation "k" of a fractal geometry on which physically implements the transfer of extensive entities (charge, momentum, molecule...) of the physical phenomenon considered. Due to the very nature of fractal geometry, this transfer is discretized and summited to inclusion order. As in every topological approach the model squeezes in only one octave all the set of frequencies which mathematically amounts to progressively compacting the response of the system (intensity flow F(QE,t) to the physical solicitation (extentive flow (F'(EQ,t)), Based on fractal geometry the system also involves a renormalization backed on a derived k-algebra [VAA20], which, joined to the difference Intensity(EQ)/(QE)extensity [LAW12] and ultimately justifies the use of a monad. The key element of the model is the distinction between the k-scale of geometry (EQ : $\eta=u/v \in \mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{Q}_p$ or $\mathbb{R}$ as hyperbolic distance) and n^k with k and $n \in \mathbb{N}$ (QE) where n is considered as an extensity, or as Lawvere says a quantity of space (QE). As we figure in the schematic representation (Figure 2 and 7) the monadic implication imposed by the non-independent renormalization of the two classes of entities considered, ultimately constrains the field of definition of the intensity (Space of Quantities) which can then be reduced to the field of p-adic numbers: ($\eta=u/v \in \mathbb{Q}_p$). We will come back to this conclusion because this last justifies the use of of Grothendieck schemes (namely rings enriching vectorial spaces).

Indeed, in addition to the Cantor-Mandelbrot (CM) relation which expresses the non-linear link Intensity (EQ~η)/Extensity ((QE~n) such *as* $\eta^d.n=1$, with $d=1/\alpha$ the fractal dimension, by changing the Mandelbrot point of view into Cantorian one, TEISI model introduces a complex extension of Mandelbrot initial formula $\eta.(in)^\alpha=1^{\alpha}$ involving a dynamics constrained on the one hand by « i^α » with $i^2=-1$, namely the introduction of a phase angle via the factor " $(i)^\alpha$ ", phase observed empirically, on

[2] The concept of topos finds its source in the Aristotle rhetoric. It is a common locus merging together dual (dialectical) notions and their relative magnitude (for convincing). In mathematic a topos is a category characterized by (i) the existence of finite limits (ii) alongside the category of definition the existence of a category of objects called evaluators such that $B^A \times A \rightarrow B$ où B^A is named exponential and (iii) the presence of a "classifier" determined by a morphism $F : C^{op} \rightarrow Ens$, meeting certain bijection criteria. This type of category gives mathematical status to a bridge between various fields of mathematics, particularly geometry and algebra. A same universe can give birth to different toposes.

the other hand a discretization through. an introduction of the root of unit via the factor 1^{α}. The root of unit must involve divisibility and resonances between different rings or cycles. Let us add the bi-partition imposed to empirical data by the temporal normalization of frequencies via "τ" required when spectral measures are given by $i\omega\tau = in \in \mathbb{N}$, with: $i\omega=2i\pi\nu$ and $\tau \in \mathbb{N}$ the temporal norm is the degree of freedom of the experimentalist. It determines the size of the dial if "ω" is the velocity of the rotation of the tip of hand on this dial. However, this size necessarily responds to the decomposition of n and $\nu \in \mathbb{N}$ over the set of prime numbers (Fundamental Theorem of Arithmetic FTA extended in the frame of Ring theory FTA2 [KSY04], [LAB05]).

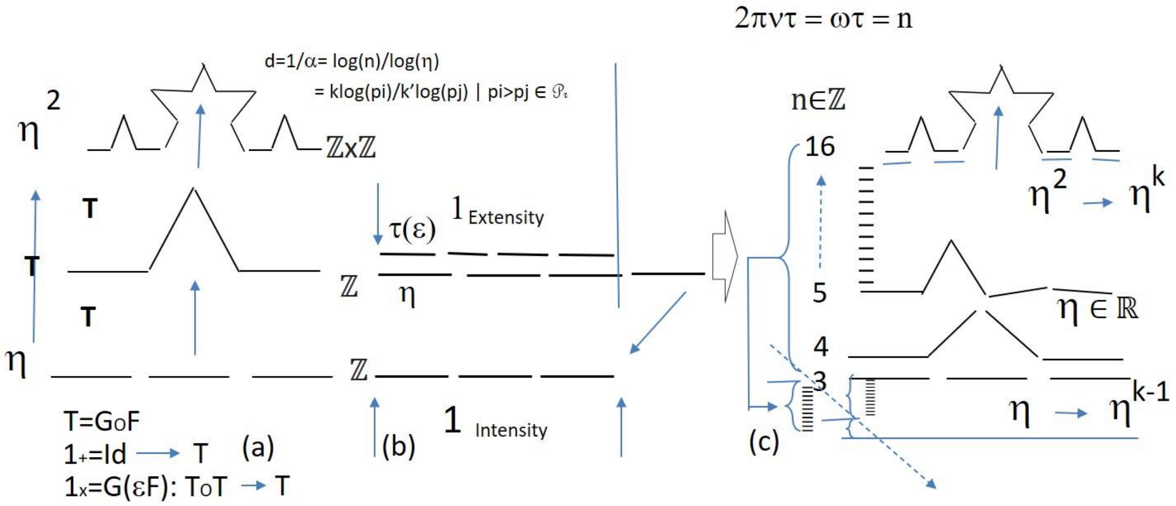

Figure 2: Schematic approach to the construction of the monadic functor (a) associated with the propagation of a discretized dynamic in a fractal geometry (Extended TEISI Model). The dual algebraic category at stake is given by the relation $\eta.n^{\alpha}=1^{\alpha}$. With $d=1/\alpha$. We will highlight the importance of the renormalization (b) leading to distinguishing the units of intensive entities (QE|1_x) and extensive entities (EQ|1_+) and the subsequent importance of the $\tau(\varepsilon)$ time constants and their distributions. This construction allows us to understand how the dual infinite foliation (c) (associated with $\mathbb{N} = \mathbb{N} \times \mathbb{N}$) in an octave can in practice be constructed in such geometries

The aim here is to justify mathematically the construction of a discrete model leading to formalize a convolution between the algebra associated with the quantified linear dynamic process (Flow/Force relationship) and the fractal geometry of dimension "d" as a discrete support for the transfer of physical extensive entities (QE). The approach aims to define not only an isomorphism between this algebra and its natural topology, but moreover to make the link between this topology and the physical support of transfer (electrode, heterogeneous material, etc) seen as determining the set of internal clocks required to identify the class of geodesics capable of being defined step after step on a real geometry: that of fractal structures of materials involved practically. The final aim is to identify the entropic properties, and therefore the long-distance correlations induced by the dynamic/geometry coupling, namely to relate entropy and geometry. In this epistemological framework, we will be particularly interested in the origin of the cohomological properties (i.e. surjection) associated with universality of the entanglements induced by fractality coming to superimpose and discretize the identity of the chosen local process. Furthermore, the link between the zeta function [BEM99] [BEN03] [BYT95] [COA96] [COJ03] [ELW75] [ELI94] [GUT90,13] [KEJOO] [PLM06] [SIG19] [STJ13] and the fibration of such dynamics via « $i\theta$ » for $s=\alpha+i\theta$, now recognized, in particular from the work of P. Riot and current authors [RIP19] [LMA10,22], will be revisited in the context of Craig's logic theorem (see also dual problems Burke-Chevalet), the zeta function formalizing (up to an isomorphism) through the functional relation $\zeta_{\alpha}(s)=F[\zeta_{1-\alpha}(1-s)]$ the existence of a bridge between the intensity (QE) distribution $(\eta)_{\alpha} \rightarrow Z_{\alpha}$ and the distributions of constants $(\tau)_{1-\alpha} \rightarrow Z_{1-\alpha}$. let us remind that this bridge ensures the completion of the empirical approach without guaranteeing, through an arithmetic approach of the continuum, the closure of the problem. This problem remains open-closed (clopen). In this vein, in spite of a new approach (a seen as the inverse of fractal dimension and α=1/2 related to compact diffusion and chance) not any demonstration of the Riemann Hypothesis (RH [BEM99] [COA96,03,14] [COJ03],[STJ13]) has emerged during this work. On the contrary, studies carried out over the past decade lead to conclude that the RH should have to remain a mere conjecture in set theory. Indeed, according to the latest studies [RIP23], RH would be linked to David Martin's axiom and thus to the forcing theory. It appears that RH cannot be approached analytically without circular reasoning, hence in certain respects, the relevance of criticisms based on an analytical approach of fractional operators [SAJ22].

We formulate the hypothesis that the obstruction to the closure of the site $Z_\alpha \oplus Z_{1-\alpha}$ onto its internal singularity defined by « α » is due to the irreducibility of the hypothesis of the continuous to the countable via a monad. We also hypothesize that this irreducibility is at the origin of the arrow of time and the feeling of time passing. If these hypotheses are correct, the question that arises is how can the continuous be taken into account by physics and consciousness while all empirical approaches relate to the field of the countable? This note try to lies this issue to the universal status of the Riemann function as an operational bridge between the countable and its possible boundaries and is also due to the fact that the zeros of this function are not accessible analytically (Riemann Hypothesis HR) but mainly spectrally at limit in relation with quantum mechanics [BYT95],[COA96], [ELI94,03],[GUT90], [KEJ00], [SIG19]. Our approach being based on dynamics, and more precisely on α–dynamics, it will be up to the specialist in pure mathematics to specify the conditions of validity of the analyzes which follow. The empirical data will serve as guide line to put forward assertions which, to our knowledge, have not been addressed by mathematicians at least in a form accessible to the intuition of Boeotians (adeles, ideles, sheaves, topos, sites, etc) but only by engineers keen on mathematics and in search for meaning and of new tools for thinking complexity [LMA10], [RIP, 17a,b,19].

Far from constituting a scientific failure, the observation of our impossibility of proving HR must be considered as an advance full of promise, not only for the future of science but also for all forms of creativity. The Riemann hypothesis reduced to an axiom leads to the fact that the invention cannot in any case become the result of a Turing program, therefore be reduced to a programmable digital calculation. We will show that the incompleteness of the dynamic representation related to the topos, leads to giving a monadic content to the arrow of time (hence the consciousness of the creation of the past by the future [WOF23]) and a physical meaning to the notion of anti-entropy [BAI09],[CNM22] .

The universal and dual role of the zeta function

Engineering approach of a ringed shell for zeta Riemann function

The zeta function being defined dually (by a sum as co product and a product) with $s=\alpha+i\theta$, $i^2=-1$ and $p \in \wp$ (set of prime numbers) :

$$\sum_{n\in\mathbb{N}} n^{-s} = \zeta(s) = \prod_{p\in\wp}(1-p^{-s})^{-1}$$

we can assert that the role of the zeta function as a “bridge” [CAO16] is at first due to the fact that zeta generalizes the identity operator between the additive series relating to integers (discrete universe) and the series multiplicative (may be leading to a continuous universe) here reduced to the prime numbers basis $\wp$. α points out self-similar extrinsic properties. We will underline that the latter can itself be expressed by means of an implicit additive sequence because : $1/(1-p^{-s})=1+p^s+p^{2s}+\ldots$if the first is associated to $\mathbb{N}$ the second is naturally associated to $\mathbb{N} = \mathbb{N} \times \mathbb{N}$, related to intrinsic self-similarity with

$\alpha_{\mathbb{N}} = 1/2$. The similitude of this entanglement with all octaves characteristics through changes of scale via the monoid ($\mathbb{N}$,x) leads to spread any given set into a multiplicity of subsets able to fill up a topological structure, seemingly in a compact manner[3]. Nevertheless, except in special cases fitting quadratic forms (α=1/2 and 1) , this spreading is unsuitable, for a ringed projection of the set of continuous geodesics [4] into $\mathbb{R}$ (the intuitive reason is due the non-equivalence between a real number coming from cantor diagonal approach and the set of prime numbers), henceforth a need to construct a representations of integral problematic in the complex plane $\mathbb{C}$ not reduced to $\mathbb{R}^2$ but more usefully to $\mathcal{P}_{\tau}^2$. You still need to know how to make mathematically sure this representation. If the examination of the dynamics Z_α can give rise to a model based on the convolution of the local process by the fractal geometry of the transfer support (TEISI model [LMA82]), not only is this model incomplete and gives rise to relevant criticisms [JSa20], but its geometrical completion from the set derivation [RIP19] is still unsatisfactory because it answers neither the question of derived k-algebra [VAA19] based on scaling order nor determines as assumed the completion-foliation ${}^{n}Z_{1-\alpha}$ as derivation of the same algebra into fields entities. The assumption that there may exist a link between the functional relationship equipping the zeta function $\zeta_\alpha(s) = F[\zeta_{1-\alpha}(1-s)]$ and the arrow of time, arises from this openness. The possible relationship might be not only due to the link between "n" and its decomposition on the basis of prime numbers $\mathcal{P}_{\tau}$ but much more with the lack of commutativity between the following adjunct morphisms : $Z_\alpha \longrightarrow \zeta_\alpha(s) \dashv \zeta_{1-\alpha}(1-s) \longleftarrow Z_{1-\alpha}$ that is to say, the categorical distinction $\pm\,\theta \neq \mp\,\theta$ (for $\alpha \neq 1/2$) when " $\pm i\theta$ " is seen as the fiber parameter of a new topos denoted $\zeta_\alpha(s) \bigoplus \zeta_{1-\alpha}(1-s)$ built above the site [COA13,19], [CAO22] pointed out previously, namely $Z_\alpha \bigoplus Z_{1-\alpha}$. Therefore it is at the junction $\alpha \rightleftharpoons (1-\alpha)$ reduced to RH when $\alpha=1/2$, that this adjunction hide a shift from commutativity to absence of commutativity (here based upon a change of the orientation of the complex plane) when the phase angle di-symmetrizes the capability of linear space-frequency representation. We easily understand that this abstract switching may puzzle as much the engineer than physicist looking for invariance.

Before taking the main step concerning the role of cohomology attached to the ringed spaces examined from an approach based on the notion of α-exponential, let us note that, the universality of the zeta function $\zeta_\alpha(s)$ expressed through Voronin's theorem [$\zeta_\alpha(s+it)_{t\to\infty} \to f(\{s^n\})$] [VOS75], [RIP17] -whatever the analytical function « f » and notwithstanding Bagchi's lemma with regard to self-similarity [BAG82],[MAJ13], in the absence of zero in the definition field-, implicitly impose the taking into account asymptotically two universes : the discrete (on the left) and the continuous (on the right). This theorem being an asymptotic theorem relating both the applications [$\zeta_\alpha\{n^{-s}\} \to f\{s^n\}$] and the duality product/coproduct which characterizes the definition of $\zeta_\alpha(s)$, the most subtle questions concern the links between inductive limits (+) and projective limits (x) limits which here cannot be handled analytically easily because the series based upon "α" are divergent and do not directly meet Cauchy's criteria. It is for this reason that filters (duals of ring ideals) must, as Riot does [LET13], be considered in order to deal with the characteristics of this type of limits. The internal duality of Voronin's theorem $n^{-s} \rightleftharpoons s^n$ having to be managed precisely at the limits whatever the divisibility of {n} on the basis of the prime numbers $\{\mathcal{P}_{\tau}\}$ is and whatever the bipartitions is chosen to express the duality (measure (ω)/normalization(τ)), this duality requires the arithmetic backing to $\mathbb{N} \cup \{0\}$ namely to work in the ring $\mathbb{Z}$ therefore in particular in all of its sub-rings $\mathbb{Z}p$ giving birth through divisibility and spreading to the expansion of $\mathbb{Q}$ into the set $\mathbb{Q}_p = P^{-1}(\mathbb{Z}p)$ [BHP15]. The distinction must also operate between the ordinal

[3] Part of this spread is due to the quadratic self-similarity of the set of integers $\mathbb{N} = \mathbb{N} \times \mathbb{N}$ but due to the presence of the factor α not only : there is a convolution between both structure of scaling

[4] This is in fact granted to the properties of the monoid (N,+)

character of “n” (QE) implicitly pointing to scales of hyperbolic distance and its cardinal with a field related to a space of quantities (EQ) [LAW12], here requiring a normalization, i.e. an additional class of duality (bipartition $n=\omega\tau$) over "{n}". To visualize the problem, we will refer to the definition of “n” as a cut between finite and infinite as defined by Riot looking for model of $\mathbb{N}$ (Fig.3 : we added to his diagram the additional indication of the link with the main components of the α-exponential). In cases where, as we assert, the ringed approach of $Z_\alpha \sim 1/1+(in)^\alpha$ (Fig2 and 3) is likely to give rise to a projective completion ${}^nZ_{1-\alpha}$ for taking into account the divisibility of “n”, there exists a topological category, without obvious physical meaning, but also denoted $Z_{1-\alpha}$ characterized by an opposite spectral orientation to that of Z_α in the complex plane such that its structure must be associated with the divisibility of Z_α spectral data but also in accordance with CM extended relationships. This operation can be investigated via Kan morphisms using the bi-partition of “n” and the fibers based all over spectral data categories using the factor « $\pm i\theta$ » as it appears to give birth to Riemann zeta function [RIP19]. This test must fit on the double writing of zeta: (i) as a summation ($\rightarrow\leftarrow$) on the basis of natural numbers with a countable inductive co-limit and (ii) as a Cartesian product ($\leftarrow\ \rightarrow$) of its subsets indexed by the set of prime numbers $\mathcal{P}_\tau$, a decomposition likely to lead now to a projective limit to which the continuum $\mathfrak{c}$ might be attached. This analysis is possible because $Z_{1-\alpha}$, as an arc of circle, can also be related to an hyperbolic space characterized by a distance $\eta=(1/\tau)^{1-\alpha}$.

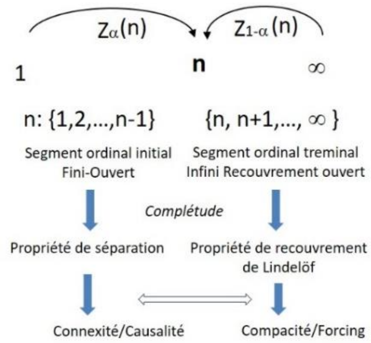

Figure 3 : There are different models for the set of natural numbers. Above, arithmetic approach of integers according to Philipe Riot for explaining compacity and connexity related to $\mathbb{N}$.

Then starting from the site $Z_\alpha \bigoplus Z_{1-\alpha}$ we are entitled to question the functional equality which characterizes the derived zeta functional relationship $\zeta_\alpha(s) \sim F\ \zeta_{1-\alpha}(1\text{-}s)$. Is it a relation of equivalence as suggested by the analysis and in this case we must consider $\zeta_\alpha(s)$ like a pure arithmetic characterization of a continuum or a representative of categorical adjunction bridging, up to an isomorphism, the countable and the continuous, by keeping an ambiguous mathematical status involved by the coupled renormalization ? In all cases the arithmetic, algebra, α–geometry and topology bridged together are strongly entangled rising mathematical questions far from than ended in this note, as shown for instance by the Langlands’ Program or the work of the school of thought of Pierre Colmez [DOG09], [COP08-1]. While there are many reasons for questioning the analytical functional zeta duality,

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s) = \pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)$$

this duality is overtaken by another one, born from the fibration of the site using « $\pm i\theta$ ». This stalk creates a links between both adjunctions: $\zeta_{1-\alpha}(1\text{-}s) \dashv \zeta_\alpha(s)$ and ${}^nZ_{1-\alpha}(\tau,p_i) \dashv Z_\alpha(n)$. Even if stalks are tangled like in a spaghetti dish, complexifying our representation of zeta, as basis of fibers, the empirical topos allows us to unravel the complexity by using dynamical hyperbolic distance

$1/\eta(n)=n^{\alpha}=(u/v)$ (Fig.1) which characterizes each state of the site duality $Z_{\alpha}(n) \oplus Z_{1-\alpha}(\tau)$ (if $p_i \in \mathcal{P}_r$ removed through projection). If this distance $(u/v) \in \mathbb{Q}$, is measured all over renormalized transfer function and its completion, namely when the geometry of transfer is α–fractal, the stacking of information within only one octave using $\mathbb{N}=\mathbb{N} \times \mathbb{N}$ (Fig.2) and the projection over $\mathcal{P}_r$ basis, relates this distance to the α-self similarity tree an leads the ideal representation of the distance now in accordance with $(u/v)|_p \in \mathbb{Q}_p$. The fractal category being dual $[\eta.(in)^{\alpha}=1^{\alpha}]$, the fields required by analysis of ring structure, involves the propagation of the link $\mathbb{Z}_p \dashv \mathbb{Q}_p$ all over the set of prime numbers $\mathcal{P}_r$. Through stalking it appears extended as the key entities for defining zeta $[(n^s,+)$ and $(p_i^s,x)]$ with $s=\alpha\pm i\theta$).

While the content of the fundamental theorem of arithmetic allows the unique projection of any natural number onto the set of prime numbers $\mathcal{P}_r$ above analysis may be featured in a logarithmic representation of the set {n} and then of zeta function, via $\alpha\log(n)$ [RIP17], pointing in a unique but non-bijective way, on the axes of the vector space of infinite dimension, referred as BB_{∞} space and characterized by the basis vectors : s or $\alpha\log(p_i)$ with $p_i \in \mathcal{P}_r$. BB_{∞} space looks like Banach or even Baire topological space, here extended to infinite dimension. BB_{∞} space is normed and scaled through the fractal cantor factor « α ». It is also scaled by the quadratic relation $\mathbb{N}=\mathbb{N}\times\mathbb{N}$ namely by a certain reference to $\alpha_{\mathbb{N}}$ = ½ related to bi-partition. $\alpha_{\mathbb{N}} = \alpha$ only in the framework of Riemann hypothesis. Due to the fact that stalking is backed on each "n" in BB_{∞} space, the quadratic relation holds if the scaling factor is "α" and not "s". "s" scaling is related to the ringed BB_{∞} space.

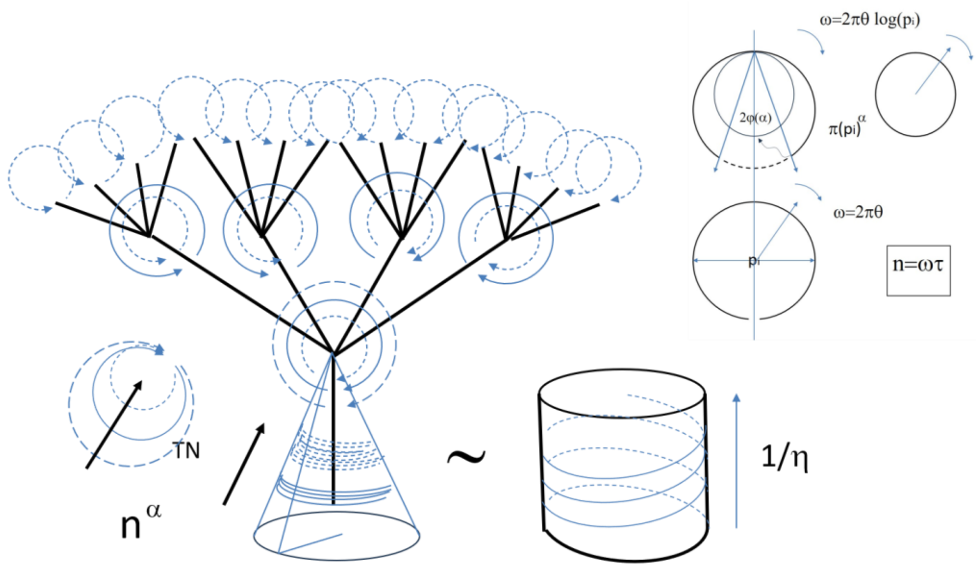

Figure 4: Intuitive image of the ringed fractal structure (view from a tree structure) when the algebraic ring structure is featured by means of cycles. This is only a first level of approach because the concept of scheme is made up of an infinity of such trees { $\mathbb{Q}_p$ }. The extended Cantor Mandelbrot metric (α) results from the relation $\eta.(in)^{\alpha}=1^{\alpha}$. In other words, the unity of the diagram, in its overall content, results from the coupling of the trees via the relative cycle sizes. The expression of this uniqueness is partially contained on the Riemann zeta function. Its purpose is to test, in particular through the "$\pm i\theta$" factor which plays the role of absolute clock, the link between rotation (QE: extensive entities) and pulsation (EQ: Intensive entities). p-adic numbers plays (via $\mathbb{Q}_p$ and cofinal algebra [BHP15]) a prominent role for rising long range correlations and staying the systems open, while $\mathbb{N} = \mathbb{N} \times \mathbb{N}$ push to close it. The arrow of time must be though within this framework.

More precisely, the empirical dynamic gives birth via fibration to a zeta function then featured by the set of cycles $n^{\alpha} e^{\pm i\theta\pi\log n} := \mathcal{O}(n) \in \mathbb{C}$ (Fig.4) with $\mathcal{O}(n)=\Pi_{pi}\, \mathcal{O}(p_i)$. Note that the sign of rotation in the complex plane being a priori indifferent, « $\pm\theta$ » its magnitude is defined modulo π. Represented as previously in BB_∞ space of infinite dimension, the projection of $\alpha\log(n)$ amounts to equipping the axes corresponding to each $n=\Pi(p_i)^{r_i}$ with cycle whose, what ever "r_i" is, external tempo is given by « $\pm\theta(\mathrm{mod}\pi)$ » while the diameter of the dial to which tempo applies locally is given by "log(n)". BB_∞ space is herein "ringed". We therefore associate to each "n" a structural rings sheaf (regular integral function) : $\mathcal{O}(\log n) = \bigotimes \mathcal{O}(\log p_i)$. The infinite vector space thus constructed is herein built upon non trivial set of cycles [COA13], [read if possible [COA22] and refer to [VAA19]) while the zeta function is sourced in a stalk representation calibrated via « $\pm i\theta$ » upon an external clock which can be considered like a Newtonian clock. Given $\pm\theta(\mathrm{mod}\pi)$, the entire process is in addition characterized by a set of multiple internal spatial pulsations (tempii of fields) due to the multiplicity of clock sizes which then equip the axes of BB_∞ space. Notwithstanding the powers "r_i" attached to "$p_i|_n$" the dial sizes depend only on "p_i^1" (p_i characteristics reduced to one). This result fits the proposal already done when ${}^nZ_{1-\alpha}(\tau, p_i)$ were obtained from the set derivation of "$n\rightarrow\tau x p_i$ "[RIP19], namely the bi-partition for normalization. Above analysis shows how ringed space related to zeta Riemann function can be obtained from primary engineering issues.

Paradoxically, this engineering work reveals that the non-linear (fractal) frequency versus scale relationship is another type of a space-time relationship which would have lost the notion of velocity here replaced by a multiplicity of clocks dispersed in an abstract irreducible fractal BB_∞ space. Then everything happens as if we could define zeta in an infinite dimensional space of base vectors calibrated by prime numbers and on which would be stacked a multitude of clocks tuned to a single external oscillator but with spatial phases detuned due to the variable dimension of the dials [LIG62]. Backed upon α–self-similarity, a global spatial pulsation tuning is nevertheless rebuilt in accordance to the roots of the unit. A relevant representation would be given by an infinite set of oscillators (metronomes) characterized by magnitudes (sound volumes) connected non linearly to the frequencies into a self-similar structure (distinction space-time) in the frame of $\mathbb{Q}_p$ p-adic fields. Each $\mathbb{Q}_p$ here associated to a self-similar tree whose lengths of branches are nonlinearly coupled to $p\in\mathcal{P}_\tau$ via the TEISI EQ/QE equation [$\eta.(in)^\alpha=1^\alpha$, $i^2=-1$]. Due to the fact that the whole system must fits the constraints here applied through the size of the ring $P^1(\mathbb{Q}_p)$ representation, fractal properties applied over the "α" or "s" normalization of all discrete data are imposed via the basis of BB_∞ space.

Sheaf of rings approach as a shell for Riemann zeta function

At this stage we must observe (i) the cycle defined by $\pm\theta\pi\log(n) \bmod \log(\pi)$ can be broken down into sub-rings whose tempo is given by the prime numbers which divide : $n=\Pi_{pi}(p_i)^{ri}$. (ii) due to the modular nature of the expression of « $\pm\theta$ » , the role of the valuation "r_i" of each prime number disappears from the tempo, otherwise indifferent to the absolute orientation of the complex plane [RIP19]. What happens now for the BB_∞ space ? Given the presence of cycles upon each axis, the representation space "α" or "s" $\log(p_i)$ axis [RIP17] with pi $\in\mathcal{P}_\tau$ can be rolled up over itself. Let observe that axis being rolled many times as necessary without phase effect, the evaluation is reduced to the single value "1". This approach of the role of class 1 modules sheds on the way, some lights about the mathematical question that has apparently been asked since a long time about the field of functions [SAD17][5]. At this stage the questions to be addressed are the following: find if exist *epimorphism relationship*s between $\zeta_\alpha(s)$ and $Z_\alpha(n)$ and *isomorphism relationships* between $\zeta_\alpha(s)$ and ${}^nZ_{1-\alpha}(\tau, p_i)$? Does the

[5] ""m" being a power of a prime [ELW75] number for a given "n" (valuation), it has long been part of the folklore that was necessary, for example to treat fields of functions (finite algebraic extensions), make m tend towards 1 and work in characteristic "1"". These two expressions have no meaning if we stick to an arithmetic which does not take into account the double normalization of morphisms (algebraic and geometric) and their possible absence of congruence. Above model gives an explanation coming from a self similarity matching the relations how $\mathbb{Z} \times \mathbb{Z}= \mathbb{Z}$ (octave constraint)

function $\zeta_{1-\alpha}(1\text{-}s)$ representable in a ringed space based on $Z_{1-\alpha}(\tau)$ via a stalk parametrized with $\mp i\theta\pi\log(p_i)$ (note the change in algebraic sign and introduction of the prime number) as projection excluding "p_i" ? These questions involve, as we have indicated in other writings [RIP19], the use of the Kan extension [LET14]. Fundamentally this question addresses the examination of the commutativity of a couple of operators: (i) "$\pm$ iθ " involving $Z_\alpha(n)$ to reach $\zeta_\alpha(s)$ and (ii) the operators of set derivation , namely $n \rightarrow \tau$ to change $Z_\alpha(n)$ into $Z_{1-\alpha}(\tau)$ [PRi19]. The approach of "n" using a cut (Fig 3) or filters [LET13] leads, as for algebra over $Z_\alpha(n)$ to thinking about an algebraic structure aimed at the non-local treatment of "n", by distinguishing (i) the role of ($\mathbb{N}$, +) as an expression of the succession up to a countable infinity (colim final category) then using duality and completion (ii) the role of ($\mathbb{N}$,x) defining "n" as an index of a finite set of subsets, using sheaves (lim: initial category), giving birth to a topological point of view. Then succession (+) and topology (x) enter into Topos as reciprocal completion. As shown incompleteness of α-exponential dynamics finds its origin and its resolution within a duality of a site : $Z_\alpha(n) \oplus Z_{1-\alpha}(\tau)$. The point of view relating to multiplicity [6] leads naturally to use an algebra over ideals, namely over the classes of multiples controlled in the frame of octaves. In a first step, the commutative ring structure (A,+,x) emerges as $\mathbb{Z}_p$ with $p \in \mathcal{P}_\tau$ to study how $\mathbb{Z}$ is managing the spectroscopic data "n" within $Z_\alpha(n)$, but also how $\mathbb{Z} \times \mathbb{Z}^*$ must also be expanded to $\mathbb{Q}$ for fitting the hyperbolic distance η=u/v of the current point of Z_α and $Z_{1-\alpha}$, from divisibility of cut-automorphism in the frame of non-linear Scale-Frequency TEISI equation : $\eta.(in)^\alpha=1^\alpha$ (fractal category). This equation is attached to the empirical transfer function $Z_\alpha \sim 1/1+(in)^\alpha$ [$\alpha \in \mathbb{R}$ with here $½ \leq \alpha < 1$; 1 being the exponential form here degenerate].

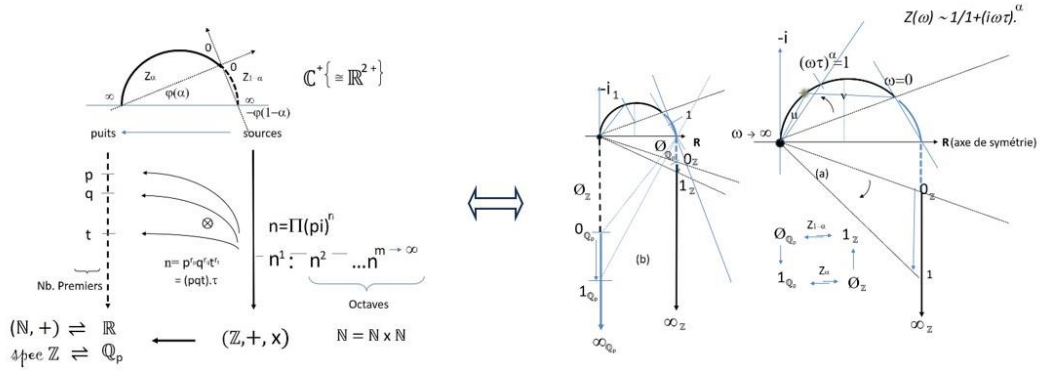

Figure 5 : this figure schematizes the obtaining of the Cole and Cole data $Z_\alpha(i\omega\tau)$ and the context by simple inversion of the line $D_\alpha := \mathbb{N}(+, x) \cup \{0\}_\alpha$ with respect to the point (0,0) of the complex plane, line to which a gap (segment $Ø_\alpha$) is imperatively assigned at the origin. This hole segment is associated with the foliated variety ${}^nZ_{1-\alpha}$,a pseudo-impedance constructed, as we have seen elsewhere, on the set derivative of Z_α. Given the circular geoedesic $Z_{1-\alpha}$ this structure can be reduced to an inversion of a dual line of D_α, namely $D_{1-\alpha} := \mathbb{N}(+) \cup \{0\}_{1-\alpha}$ this structure can be reduced to an inversion of a dual line of (0,1), - by in relation to a dual critical point of the complex plane this time (0,1), -the two lines being equipped with a dual gap at zero (hole segment associated with $\{0\}_{1-\alpha}$ and $\{0\}_\alpha$)-, namely $D_{1-\alpha} \cup Ø_{1-\alpha}$. The theory of Grothendieck Schemes leads to topologically assimilating the second line to the spectrum ($\mathcal{S}pec\mathbb{Z}$ équipé du segment vide $Ø_\alpha$) of the ring $\mathbb{Z}$ by making it, as an extension of $\mathbb{Q}$, either the analogue of the set $\mathbb{R}$ or more precisely the field of p-adic numbers $\mathbb{Q}$p. seen as the body of fractions of the integrated valuation ring $\mathbb{Z}$p (completed with $\mathbb{Z}$) We see that the central point of the problem certainly concerns the order of inclusion in $\mathbb{N}$ extended to $\mathbb{Z}$, but also the role of the order of divisibility therefore of the congruences an integrated ring, operating in the scales in play, and more irreducibly between the quotient structures taking here more specifically into account the quadratic self-similarity of $\mathbb{N}$ extended to $\mathbb{Z}$ but also the self-similarity of

[6] This discrete multiplicity responds to a Boolean (or Heyting) algebra. By following Stone's approach it is possible to construct a Boolean ring which opens the problem to a more general ring algebra. Stone's genius is to have designed this algebra by relying on the removal of the non-separable part of the coverings of sets [SEM13].

order "α " imposed both by empirical questions and by the dual formulations of the Riemann zeta function.

In the framework of the above site and topos, the idea is to use the concept of ring spectra, namely the set of all subset of $\mathbb{Z}_p$ to build *in fine*, through ring sheaves, (i) the Grothendieck scheme managing the topology of ${}^nZ_{1-\alpha}$ and thereafter via stalk (ii) zeta Riemann functions. The difficulty of the question raised by completion is due to dynamical endomorphisms which constrain the ring parameterization of the topos $Z_\alpha \oplus {}^nZ_{1-\alpha}$. These endomorphisms obviously come from the duality of the two semi lines of the complex plane which define the above topos by crossed inversion. The first, defining Z_α is parameterized in $\mathbb{Z}$ (fig. 1 and 5**).** But what about the parametrization of the semi line to be inverted defining ${}^nZ_{1-\alpha}$?

As highlighted above, the link of $Z_{1-\alpha}(\tau)$ with a set derivation and $\zeta_{1-\alpha}(1\text{-s})$ on the one hand [RIP19], and on the other hand, the work of Stone [SEM13] about ring algebra extension and Grothendieck concerning the concept of scheme [VAA19] and sheaves of local rings $\mathcal{O}_{\mathrm{spec}\mathbb{Z}}$, lead us to think that the topology of the completion initially based on $\mathbb{N}\cup\{0\}$ (foliated variety) is also originated from the spectrum of the ring $\mathbb{Z}$ (i.e. $\mathcal{S}pec\ \mathbb{Z}$), algebra based on the rings $\{\mathbb{Z}_p\}$, the set prime ideals of $\mathbb{Z}$. This idea is all the more interesting since the fundamental theorem of arithmetic that ensures the uniqueness and irreducibility of the splitting of empirical frequencies [« $n=\Pi(p_i)^{r_i}$ » avec $r_i \in \mathbb{N}$ et $p_i \in \mathcal{P}_r$] can be here extended to the set of prime ideals (with some reserves of the presence of maximal ideal overcame in the frame of categorical limits). In the line of Voronin's asymptotic theorem (additional complex parameter) the use of prime ideals (classes of multiples of $p_i \in \mathcal{P}_r$) can be seen as a subtle stacking and folding of indexed sheets, here based upon "p_i" within an unique octave « n^m » thus $\{p_i^m\}$ with $m \in \mathbb{N}$ in agreement via the "m" factor, with the theory of the valuation. This framework can be extended to $\mathbb{Q}$ via $\mathbb{Z} \times \mathbb{Z}^*$. Then, due to discretization and self-similarity, the spectrum does not open naturally onto the $\mathbb{R}$ extension but onto $\mathbb{Q}_p$ the p-adic extension of $\mathbb{Z}$. Therefore the extension of $\mathbb{Q}$ (n^α=v/u) into $\mathbb{Q}_p$ via $\mathbb{Z}$ spectra based on prime ideals becomes herein natural and the ringed arithmetic and geometric properties associated with the local rings $\mathcal{O}_{\mathcal{S}pec}$ warrants a relationship between $Z_{1-\alpha}(\tau,p_i)$, obtained through the bipartition of $n \rightarrow \{\tau,p_i^m\}|_n$ and set derivatives (as variety) and a topology of structures ${}^nZ_{1-\alpha}$ *scheme*, based upon an open ring space, furthermore characterized, as usual for p-adic fractal structure [BHP15], especially to α–self similar skeleton or trees as $P^1(\mathbb{Q}p)$. The p-adic distances that equipe these trees give rise to the non-archimedean metric already pointed out in our dynamical issues [LMA05]. In this framework long-range dynamic correlations can be highlighted by using cohomological approach. already implicitly contained in the building of fractal structure.

For example, the understanding of the topology hidden behind the $Z_{1-\alpha}|_n$ *scheme* requires to adjust a simplicial representation based on an associative product related the three-partition of data [for example : $\tau_i \times (p_j x p_k)=(\tau_i x p_j) \times p_k$ i.e. tetrahedral endomorphisms in $\mathbb{Z}_4 \rightarrow P^1(\mathbb{Q}_3)$] provided by spectroscopy $Z_\alpha[n=\tau p_k=(\tau_i\, x p_j) \times p_k]$ to the structure of the topos already pointed out. The removing of the valuation of the complete locus of the problem, gives a central role to ideals, therefore to open-close category (clopen). This property relates to cutoff definition of « n » between finite (closed/object) and infinite (open/arrow) categories (Fig.3), that can be featured into the topos $Z_\alpha(n) \oplus {}^nZ_{1-\alpha}(\tau, p_i)$. If so, it might be possible to build the diagonal adjunction $Z_{1-\alpha}(\tau, p_i) \rightleftharpoons \zeta_\alpha(s)$ related to ($\mathbb{N}$,x) and oppositely $\zeta_{1-\alpha}(1\text{-s}) \rightleftharpoons Z_\alpha(n)$ now related to ($\mathbb{N}$,+). This observation ensures the relevance of the dynamic topos $\zeta_{1-\alpha}(1\text{-s}) \oplus \zeta_\alpha(s)$ as the fibering of the site $Z_\alpha(n) \oplus Z_{1-\alpha}(\tau)$. representation but gives birth to a non-commutative square diagram based on monoidal α–*distinguo* ($\mathbb{N}$,+) $\neq$ ($\mathbb{N}$,x). This structure being based upon a distinct $\pm\theta$-parametrization of a couple of fibers, points out what could be the categorical impact of the θ-sign inversion $\pm\theta \neq \mp\theta$ (Fig.6). Obviously if α=1/2 this *distinguo* disappears $(\mathbb{N},+)_{Alg} = (\text{"}\mathbb{N}x\mathbb{N}\text{"}, x)_{Top}$.

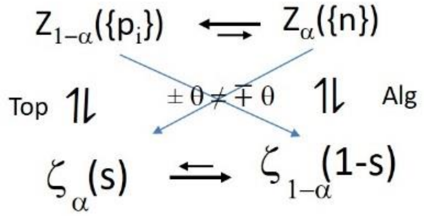

Figure 6: Relationship between ζ - dynamical topos and Z- site. This diagram points out the risk of absence of commutativity

Even if abstract mathematical tools play a central role here, they cannot easily be used because the required pure concepts are associated with the p-adic local Langlands program [DOG09], program always in progress [CAO19]. Nevertheless engineering requires such ideas and, due to the introduction of α-exponential linked with zeta function, it seems that engineering may provide the intuitive access to physical meaning of some advanced concepts [atomic patterns (atoms of the geometry at stake) / Riemann function as a bridge between universal logic operator (more generally the L functions)/ Classifying topos / self-similar structure / renormalizable continuous non derivable functions (if not automorphic)]. This approach via the engineering justifies the importance of the link between cohomology and an entropy extended to the geometry [BAP15] [VJP10], link which must lead understanding of the irreversibility of time (arrow of time) seen as the effect of a phase angle over categorical limits. What is at stake in local ring/ring spectrum dual approach, is none other than the ability of mastering complex geometries by using a divisible set of basic motives or morphisms involved into spectroscopic measurements when structure is complex and non separable. In the frame of ordered sets [ROG64] like fractal structures can be, we can design chains (and co-chains) matching order factors (inclusions / partitions: $n \longrightarrow \tau\{p_i\}|_n$), but also adapt the related algebras [(and/$\vee$/+/$\vdash$...) and (or/$\wedge$/x/$\perp$...)] by associating them to the physics of fields (forces) here addressed in the framework of ring A-theory and of spectrum approach ($\mathcal{S}pec$A, $\mathcal{O}_{\mathcal{S}pec\mathrm{A}}$) locally ringed. $\mathcal{O}_x$ is herein the structuring ring sheaf (quantity of space of morphisms: QE), locally isomorphic to the ring spectrum (space of the quantities of morphisms: EQ) pointing out over crossing of open sets. It is exactly at this level of abstraction that linear or non-linear relationship between extensities (QE) and intensities (EQ) must be taken into account. Indeed, as Lawvere highlights [LAW12], if category are involved in a process identity, then we have to point out a cutoff between extensities (QE: quantity of space) and Intensities (EQ: space of quantities) namely here, the compact and countable spectroscopic data (QE), and topologic correlations (automorphisms=endo-isomorphisms) involved by a singularity, always open, of the process at stake. If QE is constrained by the rules of arithmetic, EQ is then open upon functional space, defined here on integral functions and cycles. Like the dial of clocks, these cycles lead different kinds of information: at least (i) a tempo namely the velocity of the angle of clock pointer (here q) ; (ii) the spatial pulsation namely the spatial velocity of the pointer related to the size of the dial (here ω) and (iii) due to renormalizable characteristics of the dynamical categories at stake, size and tempo are here non-linearly related together leading the ringed functional space open to infinity. This space is a Grothendieck scheme.

Variety or Scheme : the engineering of multiplicity, foliation and entanglements.

The aim of this paragraph is to show that $M_{1-\alpha}$ thus the foliation of ${}^{n}Z_{1-\alpha}(\tau,p_i)$ [RIP19] sourced in the bipartition $n=\tau p_i : p_i \in \mathcal{P}_\tau$ and related to the topological completion of the Z_α (whose variable is a set of time constants « $\tau \in \mathbb{N}$ and $\eta \in \mathbb{Q}$ » such as $\eta.\tau^{1-\alpha} \sim 1^{1-\alpha}$) is practically, a Grothendieck scheme [VAA19] namely an open ringed space[7]. It is characterized by same kind of basis (normed by (s or α)log(pi)[RIP17]). While giving to the topology an overall identity role [COG92], this space is mainly dual: α–closed to a set of points (energy) with a zero measure and at the same time, 1-α–open set of unitary measure onto a vicinity (field) namely an exteriority (to understand via another way, let think about the colimit of cantorian construction or indexation of the quantum leaves of a $\mathbb{Q}_p$ tree). As we visualized by considering the topos $Z_\alpha \oplus {}^nZ_{1-\alpha}$ in figure 5 via a couple of projective semi straight lines inversions, the building of scheme requires to consider the epimorphism between a couple of parametrization of two straight lines ${}^\alpha D_{\mathbb{Z}}$ and ${}^{1-\alpha}D_{\mathbb{Q}p}$ respectively completed by two extended hole sets ${}^{1-\alpha}\emptyset_{\mathbb{Z}}$ and ${}^\alpha\emptyset_{\mathbb{Q}p}$ adhering to the preceding sets in zeros (Fig.5). Paradoxically, the couple of emptiness's accommodates the dual part to topos. For instance, the topologic components hidden within the algebra, thus are spread out, indexed, and correlated in the k-order of scales contained in the spectroscopic data via the hole set ${}^{1-\alpha}\emptyset_{\mathbb{Z}}$. Since the graded k-algebra being herein related to a shift of scale , the spreading of the hole set written ${}^\alpha\emptyset_{\mathbb{Q}p}$ must be related to the diagonalization of the overlaps of open sets (derived k-algebra) when passing from one sheaf of rings to another (gluing). The topology is then the result of the presence of the k-derived fields. Here we underline the fundamental role of the hole sets. This role is prominent for any open complex structure. Let take the example of fractal structure. Every point of the object is close to the void (algebraically the zero) which here defines a temporary exteriority (otherness as a component of the identity). Arithmetic correlations at long distances in scale operate generically through the set of prime numbers as infinite basis. This space opens the way to a diagonalization of our representations. The parameterization of the semi straight lines brings into play the duality between the ring $\mathbb{Z}$ (${}^\alpha D_{\mathbb{Z}}$ with nx=-nx) and the spectra of the ring. This last involves for the parametrization of ${}^{1-\alpha}D_{\mathbb{Q}p}$ the use of the field $\mathbb{Q}_p$ the set of p-adic numbers, set isomorphous to the spectrum of the ring $\mathbb{Z}_p$ for p $\in \mathcal{P}_\tau$ at a given scale k(η). This field extension, associated with self-similar trees related to fractal building, must then take into account the non-closed nature of the point in zero (practically the hole set required to represent whole dynamic). The valuation of the zeros is then based (via inversion) on the infinities of the two fields considered (internal/extensity(+) versus external/intensity (x) for fractal projective and inductive limits). The self-similar ring trees being related to $\mathbb{Q}_p$ this approach confirms the universal status of "fractal geometries" which emerge as a consequence of the closure of any fractional dynamics by means of the algebra/topology duality and by the natural extension of $\mathbb{Q}$ scale into $\mathbb{Q}_p$ through the spectrum of ring approach.

The Link to be questioned between foliated ${}^nZ_{1-\alpha}$ and $\zeta_\alpha(s)$ leads the present communication to open the way to more in-depth mathematical analyzes addressing in particular a generalization of the reasoning to L functions instead of the zeta function and to automorphic functions instead of self-similar spectral representations. We recall that the reasoning followed by the authors is in fact only based on the concept of α-exponential, in other words on the renormalizable empirical transfer functions known as Cole and Cole, i.e. $Z_\alpha \sim 1/1+(\iota\omega\tau)^\alpha$ equation which can be generalized locally using osculator circles . Meanwhile, the current point of the geodesic is characterized by

- On one hand, by a bi-partition extensive data (QE) $n =: \omega\tau \in \mathbb{N} \cup \{0\}$. (with ω an experimental spectroscopic data, normalized via τ through the characteristic of the physical process) defined by the algebraic sharing induced by the projection onto the set of prime numbers.

[7] A scheme is a locally ringed space locally isomorphic to a ring spectrum $\mathcal{O}_X := \mathcal{S}pec$ X. The scheme is said flat if the topology of X is a category whose objects are flat and none branched morphisms,, whose morphisms are natural transforms and morhisms of X-schemes. The flatness must be written in the framework of graded k-algebra here in scales $\mathcal{A}lg_k$. we deduce from this the existence of $\mathcal{S}tk$ fields (algebra functor on the category of simplicial sets S) as a sheaf in the flat topology.

- On the other hand, by an intensive data (EQ) η=u/v ou v/u $\in \mathbb{Q}$ or $\mathbb{R}$. This data is none other than a hyperbolic distance scale. Among the natural extensions is the set of p-adic numbers $\mathbb{Q}_p$ if scale $\eta \in \mathbb{Q}_p$ can be written $\eta = \pm p_i^m(u'/v')$ with $p_i \in \mathcal{P}_\tau$ and u', v' respectively prime in $\mathbb{Z}_{pi}$.
- The change from $\mathbb{Z}$ to $\mathbb{Q}$ and then $\mathbb{Q}_p$ must also respond to the symmetry of the transfer function with respect to the point 1=u/v=v/u, symmetry leading the inversion]0=:1/∞[whatever the status of infinity (Countable, Continuous or other) and to be normalized in a possibly different way ${}^{\alpha}D_{\mathbb{Z}}$ and ${}^{1-\alpha}D_{\mathbb{Q}p}$ depending of « α ».

Thus $\mathbb{Q}_p$ appear naturally including in the algebraic part of the approach undertaken. $\mathbb{Q}_p$ is then directly linked to the non-Archimedean distance on the tree associated with fractality. Fractality is an empirical representation of the hyperbolicity. There are therefore two distinct modalities for introducing p-adic numbers in our analysis: the topology of the ring spectrum and the algebra of hyperbolic distances. Both approach must, in fine, be matched together.

The representation of the transfer function Z_α in the complex plane being associated with the line ${}^{\alpha}D_{\mathbb{Z}}$ equipped with its hole set ${}^{1-\alpha}\text{Ø}_{\mathbb{Z}}$ is therefore parameterized by means of the ring $\mathbb{Z}$. The geometry (scale)/algebra (frequency) duality is then expressed through the Cantor Mandelbrot relation extended to the complex plane (CME) in accordance with the TEISI model, i.e. for α=1/d using the following intensity-extensity relation ηx$(i\omega\tau)^{\alpha}=1^{\alpha}$ with d=1/a fractal dimension of the geometry of the dynamic exchange interface. This relationship constrains the phasing of the divisibility of the two entities concerned (Figures 3 and 5) [8].

It is at this stage that the bipartition of "n" introduces the question of ring ideals and ring spectra, while the extension of the hyperbolic scale « η » leads a parametrization with $\mathbb{Q}_p$ p-adic set due to the possibly non-integer character of the embedding of this data. This extension is expressed through the completion of the process given by Z_α namely $Z_{1-\alpha}$. This observation leads to the duality of the line ${}^{\alpha}D_{\mathbb{Z}}$ and the line ${}^{1-\alpha}D_{\mathbb{Q}p}$. It is easy to understand that the limits of both classes of sets are almost certainly be different. At this step the notion of Grothendieck scheme must be introduced, that is to say of a locally ringed space locally isomorphic to the spectrum of the generic ring $\mathbb{Z}$. The diagram imposes not only that divisibility constraints require a phasing of the cycles in stake (with the presence of nilpotent operators Figure 5) but more generally that this phasing is imposed including at categorical limits, i.e. say about the non-closed points inseparable from the notion of schema. In the particular case of the α-exponential, the theoretical difficulty is due to the fact that the CME relation combines coupled renormalization constraints : (i) one external via the factor « α » related to $\mathbb{Z}_p$, where $\mathbb{Z}_p$ | ${}^{\alpha}D_{\mathbb{Z}}$ enter in duality with $\mathbb{Q}_p$ | ${}^{1-\alpha}D_{\mathbb{Q}p}$ through the ring spectrum of $\mathbb{Z}$, (ii) the second quadratic and internal whose function is to guarantee the quadratic arithmetic consistency of the respective divisibility rules. In addition, this divisibility requires defining the product unit valid for the whole duality. If this element is determined and only if this is the case, Grothendieck scheme approach can extend this divisibility to categorical limits and colimits. In particular the valuation, p-adic « m =: ord(q_p)» of a data « $q_p \in \mathbb{Q}_p$ »

[8] This relation involves the existence of a phase angle $(i)^{\alpha}$ given by $\varphi(\alpha) = \pm(1-\alpha)\pi/2$ in the complex plane with respect to the real axis. We will immediately note that it is more useful to take as the reference axis $\pi/4$ namely another angle $\psi(\alpha)$. Indeed the dual angle with respect to $\varphi(\alpha)$ is $\varphi_c(\alpha)|_{\pi/2} = \mp\alpha\pi/2$, is from $\varphi(\alpha)$ while the reference axis $\pi/4$, gives $\psi(\alpha) = \pm(1-2\alpha)\ \pi/4 = \pm\beta\pi/4$, which has as complementary as defined previously the phase angle $\psi_c(\alpha)|_{\pi/2} = \mp(1-2\alpha)\ \pi/4 = \mp\beta\pi/4$ with analog formulation. This change of reference thus leads to a simple change of orientation of the complex plane for any addition $Z_\alpha \rightleftharpoons Z_{1-\alpha}$. The reference angle $(i)^{1/2}=\pi/4$ is obviously related to the quadratic self similarity $\mathbb{N} = \mathbb{N} \times \mathbb{N}$ since then d=1/α=2 (the exchange interface then has a Peano geometry while the dynamics becomes a simple, strictly diffusive exchange process).

is given by the relation « $q_p = p^m.(u'/v')$ » if u' et v' are prime together, $m \in \mathbb{Z}$ but then ord(0)=∞. In other words, we confirm that in terms of the scale/quotient approach, the ideal zero is associated with the inverse of infinity. This nevertheless requires mastering the notion of unity and we will show that this new query is a key question far from obvious. The concept of scheme supplements the notion ringed varieties $^nZ_{1-\alpha}(\tau,p_i)$) - by non-closed points likely to justify the incompleteness of the projective completion previously represented by the topos $Z_\alpha \bigoplus {}^nZ_{1-\alpha}$. Therefore a scheme involves a meta incompleteness linked to open sets. Through the adjunction of hole sets Ø adhering to the origin of the generic lines $^\alpha D_{\mathbb{Z}}$ and $^{1-\alpha}D_{\mathbb{Q}p}$, this meta incompleteness is related to the relationship between the voids namely to the structure of the zero ideal within the topos. Indeed, since the a scheme is a locally ringed space $(U,\mathcal{O}_u)$ locally isomorphic to the spectrum of a ring (A,+,x), $\mathcal{Spec}$ A here seen as a set of prime positive ideals $[\{\mathbb{Z}p\}|\ p \in \mathcal{P}_\tau]$ but now it must include the zero ideal: $\{0_{\mathbb{Z}}\}$ as common term between all ideals. This additional term must be considered in the treatment of the sheaves of rings[9] i.e. in the structural sheaf $\mathcal{O}_{\mathcal{Spec}A}$ of functions above $\mathcal{Spec}$ A. Let observe the ring above $\mathcal{Spec}$ A is none other than A itself.

By identifying $\mathcal{Spec}A$ at the union of {0} and positive prime numbers[10], an open U of $\mathcal{Spec}A$ (the countable infinite set seen from "n" as a cut (figure 5 on the right of the diagram guaranteeing compactness)), is the complement of the finite set of prime numbers {p1, …, pm}. This set is given in the left part of the diagram in the Figure 5 (excludes $^{1-\alpha}Ø_{\mathbb{Z}}$ in the representation on the extended right). The topological open set U is then given by U $\mathcal{Spec}A \setminus \{p_1, \ldots, p_m\}$. The closure of the problem requires that the ring above U be $\mathbb{Z}[1/p_1, \ldots, 1/p_n] \subset \mathbb{Q}$, whose denominator only has p_i as prime factors. Note that "1" is not considered a prime number and constitutes the normalization factor; we will see very soon the importance of this remark. Thus for all x $\in$ X there exists an open neighborhood U of x in X such that que $(U,\mathcal{O}_u) \sim (\mathcal{Spec}\ A, \mathcal{O}_{\mathcal{Spec}\ A})$. in the empirical framework which is ours, namely measurements in the ring $\mathbb{Z}$, the field extension is backed by the local ring sheaf. The ring sheaf $\mathcal{O}_{\mathcal{Spec}A}$ upon the open set $\mathcal{Spec}$ A is defined by using the cutoff giving « n ». this extension is given along $^{1-\alpha}D_{\mathbb{Q}p}$. The isomorphous parametrization related to $\mathcal{Spec}$ A is given with $\mathbb{Q}_p$ as mathematical extension of $\mathbb{Q}$ but also physically, as previously shown, as a non-integer extension of the intensive field of hyperbolic scales {η=u/v} when the extensive field of spectroscopic data, parameterization of $^\alpha D_{\mathbb{Z}}$, is given in $\mathbb{Z}$. The approach consists initially of densifying the content of the left part of $^{1-\alpha}D_{\mathbb{Q}p}$ up to the limite $\mathbb{Q}_p$. We can compare this densification to the foliation of a set $\mathbb{Q}$ whose local fiber would be the set of prime numbers $\{\mathcal{P}_\tau\}$.

In physical terms the fundamental point to remember is the following: infinity for {ℤp} has a dual counterpart in {ℚp}}. However, the status of the two infinities is fundamentally different. The first corresponds to an co-limit based upon (ℤ, +), the second to a fractal type limit ($\mathbb{Z}_p$, x). In addition, trees associated to p-adic structures, namely to a virtual fractal sub-geometries, can have the length of their branch weighted by the amplitude of "η" to in accordance to the CME relation. We measure herein the complexity induced particularly at infinity both by duality based on the ring spectrum, and by the additional role played by the renormalization factor "α". This complexity still increases with the presence of hole sets whose virtual contents points onto the gap which separates each term of the duality of an absolute exponential reference (α=1) required to warrant the existence of topos. In this context not only does the ideal "zero" as the inverse of infinity on the ring ℤ have in practice an epimorphic counterpart in {ℚp}} but also the virtual content of each of the two hole sets exists themselves within a duality referred to exponential dynamics. This set of interrelations only allows standard arithmetic reduction in the case α=1/2 (RH); This case simply leads to verifying that the epimorphism pointed out above fits only the relation $\mathbb{N} = \mathbb{N} \times \mathbb{N}$. This expression is itself an image of

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[9] $\mathcal{O}_{\mathcal{Spec}\ A}$ the structural sheaf of A, tests the set of functions defined upon the opens isomorphous to $\mathcal{Spec}$ A. let remember that the presheaf category $P_{\mathcal{Sh}D}$ (C) ~ namely $[C^{op},D]$, which ignores, unlike sheaves, the questions of gluing open objects, is the category whose objects are functors. F : $C^{op} \longrightarrow D$ and morphisms the natural transformations between functors. This category can also be seen as the exponential category $D^{C^{op}}$ namely Hom(C^{op} , D).

[10] Note that zero is then in the neighborhood of all prime ideals and thus ensures the connectivity of their set.

of the CME relation η^2.in=1 (geometrically Peano interface and absence of exteriority) which ensures equipotency between a curve of dimension 1 and its matching with a plane. In the general matter, just as the fractal limit separates two distinct parts of the plane by a continuous infinity boundary, the infinities at stake in the issues as introduced, lead to an obstruction to the gluing of spectroscopic dynamic-limits and virtual co-limits of internal correlations. This obstruction justifies the use of the concept of scheme in place of infinite ringed vectorial space, concept characterized by the presence of irreducible singularities here unique punctuation over the topology due to the renormalization characteristics and the duality of zeros.

Now reduced to a fiber product {t} x {pi}, the dual foliation for ${}^{n}Z_{1-\alpha}$ can easily be imaged using fractal interfaces and the couple of required normalizations (Figure 2, 5 and). In all cases the set of bipartitions of n ={τ,p_i)} [RIP19] as well as the overlapping of open local rings [VAA19] structure the long-distance dynamic correlations by backing onto cohomological quotient groups, which feature the global unity of the α–process. This unity gives rise to a topology of schemes as ringed space open, except for $\alpha=1/2$ (RH), onto a non-symmetric punctuation, namely an exterior set different from internal. This situation gives rise to dynamic "$Z_{1-\alpha}$-site" which can be constructed from Z_α itself giving rise to $\zeta_{1-\alpha}$(1-s) seen, in complex space, as another dynamic site associated with the only spectroscopic data "n^α", bi partition now excluded. Kan extension allows us to compare both different topos in the framework of classifying classes [CAO16,17]. Before this step, the idea which naturally emerges is to design an isomorphism between the structural sheaves ($\mathcal{O}_{\mathcal{S}pec A}$) characterizing ${}^{1-\alpha}D_{\mathbb{Q}p}$ and the foliation of ${}^{n}Z_{1-\alpha}$ by assigning to this foliation an cohomology of scheme based on the double complex of chains in competition (decreasing for the scale η (EQ) and increasing for the number of points at stake n (QE) in the group associated with the construction of the fractal interface. Therefore while elementary the bi-partition of« n({t} x {pi}) » in a form which, for ring modular motifs, cancels « $(p_i)^1$ » the valuation of primes above $\mathbb{Q}$: log(n)= α[1 x Log(p_i)+ [(r_i-1) Log(p_i)+$\sum_{\mathbb{N},\mathcal{P}_*}$ $r_{j\neq i}$ Log($p_{j\neq i}$)]] with n=$p_i.\tau$ with $\tau\in\mathbb{Z}$ while nevertheless, "n" contains an infinity of factors "p_i" .

At this stage of the reasoning the α–exponential dynamical concept allows us to have disposal of zeta function ζ_α(s) seen as ringed site based on of the transfer empirical function Z_α(in=i$\omega\tau$) allowing the analysis topology of the renormalizable part (n^α) of the overall dynamic problem. This construction leads to a foliated manifold $M_\alpha(\pm\,\theta\,)$ parameterized by the variable "$\pm\,\theta \neq \mp\,\theta$". Above analysis suggests in addition to consider the existence of the categorical object ${}^{n}Z_{1-\alpha}$ (τ,p_i) seen as a topologic representation of overall α–dynamics as derived algebra on internal long range correlations (i) either as a laminated manifold $M_{1-\alpha}$ (p_i) via a set derivation with fractal background (intrinsic ringed by means of the adjustment in the complex plane between η (EQ) and n (QE) Figure 2 , 5 and 8) or, (ii) through a projective inversion of the spectrum of parametrization of the ring related to Z_α(in=i$\omega\tau$) giving an open Grothendieck scheme $M_{1-\alpha}$ (p_i) $|_{Z1-\alpha(\tau)}$, in other words a prime open ringed space, topology now associated to a set of p-adic numbers $\mathbb{Q}_p$. By using a relevant normalization this scheme might be able to equip ${}^{n}Z_{1-\alpha}$ (τ,p_i) of a useful canonical basis (motives). Furthermore, although without direct spectroscopic expression $Z_{1-\alpha}$ (τ) must formally be able to be expressed with the same formalism as Z_α(in=i$\omega\tau$) namely $\eta.[\tau/\tau_0)^{1-\alpha}]=1^{1-\alpha}$ and $Z_{1-\alpha}(\tau)$ ~ -1/1+$(\tau/\tau_0)^{1-\alpha}$] with disappearance of primes parameters. This opportunity obviously questions the status of $\zeta_{1-\alpha}$(1-s) and them the relation between $M_{1-\alpha}(\mp\,\theta)|_{Z1-\alpha(\tau)}$ and $M_{1-\alpha}$ (p_i) $|_{Z1-\alpha(\tau)}$.

The p-adic set $\mathbb{Q}_p$ backed by $\mathbb{Z}_p$ characterized by a fractal tree structure and by cyclic properties gives an analytic representation for self-similar long range correlations upon α–dynamics in the same way that two very distant points on a 2D sheet of paper can find themselves very close if the sheet is wrinkled in 3D and that 1D void plays the role of an extended zeros. Much better than in $\mathbb{R}$, the set of $\mathbb{Q}_p$ can expressed any proximity. This can be done by means of p-adic absolute distance value. In addition we know that then there exists at least one fixed point likely to be close to all the other points of the $(1/\alpha)$D set. This notion of proximity can easily be defined in a p-adic and non-Archimedean framework and directly linked to the hyperbolic α–dynamic properties (tree) here associated with the self-similarity and dual renormalization characteristics. This is what were already suggest in previous notes [PRI19]: namely to give a foliation parameterized by $p_i \in \mathcal{P}_\tau$ to ${}^nZ_{1-\alpha}(\tau,p_i)$ to the site. The further step here consists to introduce the concept of Grothendieck scheme and comparing it to the derived set-approach. This operation can then make it possible to distinguish and shed light on the importance of questions, mainly hidden within the symmetries and related to the content of the zeros, (or the hole sets related to exponential dynamics already pointed out). We can argue using the sets ${}^{\alpha}D_{\mathbb{Z}} \cup \emptyset_{1-\alpha,\mathbb{Z}}$ and ${}^{1-\alpha}D_{\mathbb{Q}p} \cup \emptyset_{\alpha,\mathbb{Q}p}$ of the complex plane leading by inversion to the empirical data related to Z_α and $Z_{1-\alpha}$. Figure 7 gives an idea of an arguing which is exposed above and which can be mathematically expanded from a categorical approach.

$$
\begin{tikzcd}
Z_{1-\alpha}(\{p_i\}) \arrow[r, phantom, "\rightleftharpoons"] \arrow[dr] & Z_\alpha(\{n\}) \arrow[dl] \\
\mathbb{Q}_p \cup {}^{\alpha}\emptyset_{\mathbb{Q}p} \arrow[u] \arrow[r, phantom, "\rightleftharpoons"] & {}^{1-\alpha}\emptyset_{\mathbb{Z}} \cup \mathbb{Z} \arrow[u]
\end{tikzcd}
$$

Figure 7: schematic representation of the coupling of the adjoint categories leading to leafing through the geometry associated with the completion of the empirical data produced by the fractional operators.

Weaving of enriched holes and the status of norms

As it was shown in the TEISI model extended (CME) to ensure the completion of the physical model, the internal (algebra) and external (topology) renormalization constraints generally distinct, characterizing both $Z_\alpha(n)$ and ${}^nZ_{1-\alpha}(p_i,\tau)$ leads the crumpled universe on which the dynamics unfolds to be featured by a fractal geometry. This geometry is open at its limit in scale (except for $d \neq 2.1$). This limit is dual: extensive with a zero measure and intensive with a unitary measure. This kind of geometry is associated with tree structure adapted on the one hand with the quadratic constraint of the representation support (octave shift) $\mathbb{N} \times \mathbb{N} = \mathbb{N} \subset \mathbb{Z}$ for $Z_\alpha(n)$ and on the other hand with the p-adic structure $\mathbb{Q}_p$ attached to ${}^nZ_{1-\alpha}(p_i,\tau)$ so that the symmetries involved by ideal "p^m" lead to the projection of spectral characteristic such that : $P^1(\mathbb{Q}p) := \mathbb{Z}p$. In other words, notwithstanding the role of hole sets, the role of the α-Cantorian metric manifests itself first through completion, that is to say the renormalization of derived algebra via the factor « $1-\alpha$ », i.e. topology.

It is only at this stage that the dual structure of the empirical data (intensive versus extensive) attached to the fractional derivation operator $Z_\alpha(n=p_i \times \tau)$ and to its topology given through ${}^nZ_{1-\alpha}(p_i,\tau)$ lead to assign a practical role to the non-closed points of the Grothendieck scheme theory. As shown above, openings at the edge appear in the form (Figure 5) of two holes ${}^{\alpha}(\emptyset)$ and ${}^{1-\alpha}(\emptyset)$ associated with the structure of infinities, themselves associated with the parameterizations of $\mathbb{Z}$ and $\mathbb{Q}$p. These holes are seen as a "rift" between two ridges: i.e. ${}^{\alpha}\emptyset_{\mathbb{Q}p}$ for $\mathbb{Z}$ et ${}^{1-\alpha}\emptyset_{\mathbb{Z}}$ for $\mathbb{Q}_p$. As shown in the diagram above (Figure 7), each of them is in adjunction with a transfer function ${}^{\alpha}\emptyset_{\mathbb{Q}p} \rightleftharpoons Z_\alpha(\{n\})$ and ${}^{1-\alpha}\emptyset_{\mathbb{Z}} \rightleftharpoons {}^nZ_{1-\alpha}(\{p_i\})$ while their geometrical supports permute with the categories of complementary transfer functions. Completion permutation and inversion then obviously call into question the commutativity of overall diagram of morphisms, based on lines and holes, (spectroscopic (n) and virtual and topologic (correlations)) and transfer functions and zeta functions. Taking into account the restrictive nature of the indexations, we can demonstrate that commutativity is only acquired in particular cases (α=1/2 : HR and in the degenerate limit $\alpha=1$). But then a question arises that we did not expect: the two categories in the construction being distinct in terms of scaling laws, the respective normalizations which play a fundamental role in the expression of the fractions concerned: $(1/n)\in\mathbb{Q}$ and $(1/p^m)\in\mathbb{Q}_p$. , have, except in special cases, no reason for being identical. The status of the units « 1_{α}, and $1_{1-\alpha}$ » arises here in two different ways, both related to symmetries upon the transfer functions, ways which must take into account the sign of rotation when considering the morphisms related to the determination of the unit associated to both fields $\mathbb{Z}$ and $\mathbb{Q}$. The construction of the unit from the complementary diagram of all the selected sets is given (Figure 1, and 5) using projective approach.

Analysis of the results leads us to focus on a first characteristic: the parametrization zero of $Z_{\alpha}(n)$ and $Z_{1-\alpha}(\tau)$ are located on lines of respective base points ($(0,0)_{\alpha}$ and $(1,0)_{1-\alpha}$, symmetrical with respect to the median axis of the complex plane. These lines help to define the initial point of the lines and arcs of circles which correspond to them by inversion and permutation. However, the definition of these arcs only requires the data of a triplet of points. The ends being fixed, at this stage only one data point remains to be specified. Ockham principle leads to the point of the triplet necessary for the definition of « α » being the unit point such that the intensity ${}^{\alpha}\eta=u/v=v/u={}^{\alpha}1$ for $Z_{\alpha}(n)$; the unit point ${}^{1-\alpha}\eta=u/v=v/u={}^{1-\alpha}1$ associated to $Z_{1-\alpha}(\tau)$ is deduced geometrically. The spectroscopic data being acquired by inversion of the lines open onto the holes ${}^{\alpha}D_{\mathbb{Z}}$ U $\emptyset_{1-\alpha,\mathbb{Z}}$ and ${}^{1-\alpha}D_{\mathbb{Q}p}$ U $\emptyset_{\alpha,\mathbb{Q}p}$ inverting morphisms is enough to define the units associated with the sets $\mathbb{Z}$ and $\mathbb{Q}_p$. The reader will refer to Figure 1 and 5 to understand our approach. The geometrical projection, shows that we can change the reference and use the one given by α=1/2. In this hypothesis, the arc of a circle can be obtained by basing the reasoning on the symmetry of the angle deviations which intersects the median line of the complex plane therefore the units " ${}^{\alpha}1$ " and " ${}^{1-\alpha}1$ " shifted by using β = (2α-1) into symmetric expression "${}^{(1/2\pm\beta(\alpha))}$ 1". Referred to $\mathbb{Z}$, the unity " ${}^{\alpha}1$ " or "${}^{(1/2+\beta(\alpha))}1$" is a function of « α » which has all the more magnitude as « $1/2\leq\alpha<1$ » is small and therefore approaches ½ (determined by the median axis). Conversely, "${}^{1-\alpha}1$" or "${}^{(1/2-\beta(\alpha))}1$" required for the normalization of $\mathbb{Q}_p$ varies in the opposite direction . It follows that the amplitude of the unit "${}^{1-\alpha}$ 1", is all the more important than « α » is big (if $\alpha\to1$, $\emptyset_{\alpha,\mathbb{Q}p}$ tends to infinity and "${}^{1-\alpha}$ 1" is undefined). In other words, due to the discrepancy between the required units, the arithmetic attached to $Z_{\alpha}(n)$ and its completion $Z_{1-\alpha}(\tau)$ are generally distinct (except in one case β=0 to refer to Markov chains and pure stochastic process) because it requires a measure of dissymmetry: $\sigma = |\,{}^{1-\alpha}1/\,{}^{\alpha}1|$. Therefore, now we denote $\mathbb{Q}_p{}^{alg}(\alpha)$, the p-adic category which could eventually be normed through the $\mathbb{Z}_p$ standard, reserving $\mathbb{Q}_p{}^{top}(1-\alpha)$ for matching with the dynamic current issue. In other words, Figure 5 is misleading. If the extensities and intensities must both be standardized, not only is the status of the unit not the same, but also the magnitude to be assigned to the units represented are only similar protectively; there must therefore exist an external unit allowing this projection to be fixed. This unit is not only arithmetic but also physical. Examination of the diagrams shows that the reference factor necessary for fixing these quantities is none other than unity, appearing in the denominator of the transfer function $Z_{\alpha}\sim1/[\mathbf{1}+(i\omega\tau)^{\alpha}]$ in other words to the reference exponential associated with the topos $Z_{\alpha}\oplus{}^{n}Z_{1-\alpha}$ (the limits are finite by compactification and the classifier exists since « $\tau\in\mathbb{Z}_{p^m}$ » can address all conceivable discrete dynamic homomorphisms in a bijective manner while addresses the algebraic component of the foliation of ${}^{n}Z_{1-\alpha}(p_i,\tau)$. As shown, in practice the unit is fixed by the "entropy" linked to the α–exponential function serving as formal support for the formal exponential associated to the topos and vice versa. The approach used therefore confirms that except for the case $\alpha=1/2$, the norm attached to $\mathbb{Z}$ and then $\mathbb{Q}$ is different from the norm attached to $\mathbb{Z}_p{}^m$. Fortunately the renormalization characteristics adjunct this difference via a tensor structure equipped with a constant angle $2\psi(\alpha)=\pm(2\alpha-1)\pi/2$, namely, a change of vectorial basis of the empirical spectra $2\psi(\beta)=\pm\beta.\pi/2$. Both approaches are related to the hyperbolic structure of negative curvature in such a way that combination leads a flat space. The $\psi(\beta)$ angle is here standardized to the situation where the process is only ruled by a pure chance ($\beta=0$, $\psi=0$ involving pure diffusion, HR). This angle cause an asymmetry in orientation of the complex plane since the reversal of the direction of rotation implies taking into account a phase shift equal to $4\psi(\alpha)$ (obviously the shift is zero modulo (π) for $\alpha=1/2$ leading the equivalent $\pm i\theta \sim \mp i\theta$ for the complex factor of the zeta function if HR is valid) . If $\pm i\theta$ represents a temporal flow measured on a dial of clock, this flow is associated with the entropy pointed above in a subtle and complex matter taking into account all the morphisms pointed above, unveiling a distinction between entropy and neg-entropy (neg-entropy as entropy of remaining opportunities after acting [BRL56] : normalization status such that entropy $\sim \log(\alpha/1-\alpha)$). Both dual sets have a similar thermodynamical status only for $\alpha=1/2$.

In practice, fractality leads to an arithmetic norm attached to the ring of spectroscopic data (tempo), not commensurable with the external norm of the lengths of internal long range correlations attached to the algebra of the local rings. Thank to renormalization, this difference leads to a tensorial behavior involving usual covariant factors in physics (entropy) but also contravariant factors which will give rise to anti-entropic factors, the memory in stake being associated here with the difference between entropy and negentropy. In the context of our analysis, if $\alpha=1/2$ this memory is reduced to nothing because the processes only depend on the "now" and therefore on pure chance (Markov chains : $\alpha=1/2$). In other words, once reality is crumpled, the "analytic unit" is not only crumpled but intrinsically modified to take into account, in the frame of entropic approach, the non-linear *distinguo* between extensity/algebra (QE) and Intensity/topology (EQ), and the disappearance of the velocity. This distinguo quantifies the meta incompleteness of the closure involved by the duality considered. Identity of reality is not the picked up equally according to analytic or topologic approaches. As suggested via toposes approach It differs mainly by an irreducible distinction between inductive limit (+) and projective limits (x). This difference creates at edges an absence of isomorphism between algebra and topology, an discrepancy only reducible if $\alpha=1/2$. This is implicitly expressed through the functional relationship which characterizes the zeta function by bringing into stake (i) the gamma function, the first characteristic is to combine addition and multiplication while the second characteristic is to bring into stake Euler's constant which is assumed to be transcendent and finally to link the characteristic to a cut in the complex plane, obstructions to any perfect analytical access. We are thus led to bring into the model even at the limit, a categorical dual-monad (increasing and decreasing) whose morphisms must be mediated by the first non-countable cardinal set ω_1, (Martin's Axiom [RIP23]) which appears here (Fig. 10) as the inductive limit (colimit) of the spectroscopic data attached to $Z_\alpha|_\mathbb{N}$ but also as the projective limit of the p-adic set attached to the topology of de ${}^nZ_{1-\alpha}|_{\mathbb{Q}p}$ giving rise to the continuous $\mathfrak{c}$ and discrete $\mathbb{R}^2\sim\mathbb{C}$. If $\alpha\neq1/2$ right and left in $\mathbb{C}$ are only identical up to an isomorphism involving a phase shift when changing sign of rotation, namely sign of "θ". Let's clarify this point !

Crumpled reality, sortilege of the virtual and fractal score of the arrow of time

Dual approach to the physical meaning of the arrow of time

Since Galileo and Newton then Einstein the concept of time has been preempted by physicists [SML13] [COP92] [DAZ00] [KLE07] [ROC12-17] [SID20] [FRM70]; as not only succession but as an adherence to space through the notion of continuity and derivative (velocity). Time flows eternally and, modulo the masses which deform space, the notion of velocity as derivation is valid everywhere in the same way as, always backed by the notion of energy, the physical laws. Standard time is ordinal. This is a set reference index. Anterior and posterior exclude the "present" (now) which is categorical but not set-oriented and is related to a transition arrow over an order of inclusion [ROG64]. Surprisingly, this time variable is also that of the standard model and quantum mechanics. It formalizes transitions between states ruled by von Neumann algebra. We can show that the standard time variable emerges from these algebras. They make time a unique and reversible parameter born from a group of modular automorphisms when an energy type constant is at stake [TAM70] [ARH78] [COA 13-17] [RIP17] Acting on this appropriation, phenomenology has attempted to conceive the time of consciousness [GRG68] which would not be not the time-variable of physicists We will leave it to neuroscience experts and philosophers to confirm whether or not this approach has achieved its goals . In a different way we suggest to develop a conception of time for physicists which we will describe as contemporary to contrast it with modern physics based on Noetherian principles of invariance. We will assign to this new approach the helping of physicists confronted with multi-scale complexity, entanglement, non-separability, information systems and developments such as creativity and forcing, irreducible to the use of a standard time variable. We will consider problems which expressed in Fourier space, that is to say in the complex plane, do not have an inverse transform therefore time representation.

In this context, consider a reference clock and its dial of unit radius (Figure 8). Let's call $\theta_{ref.}$ (modπ) the angle of the complex plane and λ_{ref} the distance traveled by the tip of the needle on the arc of the dial. This clock determines a vibration « $\exp(\pm i\pi\theta_{ref})$ » The variable $\theta_{ref.}$ (modπ) that is to say the angle given by the hand provides the standard time given by this so-called reference clock.

We can then build a new temporal formalization suggested by the Riemann zeta function. To do this, consider a second clock whose dial is larger than the first so that the radius is given by log(n) (we will deal with log (1/n) later). We then have a second clock which we will describe as internal. Let us call θ_{α}(modπ) the angle of the complex plane and λ_{int} the distance traveled by the tip of the needle on the arc of the second dial (Fig.8).

The information given by the clocks being based on a common congruence mod (π) we can consider the arithmetic of this duality in accordance to two distinct points of view: either (i) computing the number of turns over the dial and the crossing of the module from the reference clock via parameter $\theta_{ref.}$ (modπ) and deduce an angle θ_{α}, or (ii) taking into account the same parameters via an angle noted θ_{α} mod(π) notwithstanding the reference data. It is quite clear that if the angular variables are coupled but they can only be coupled by means of the lengths traveled respectively by the tips, and therefore associated with distinct arithmetic variables which we will denote n, m $\in \mathbb{Z}$. We can then give a physical and more precisely temporal meaning to the duality formalized above.

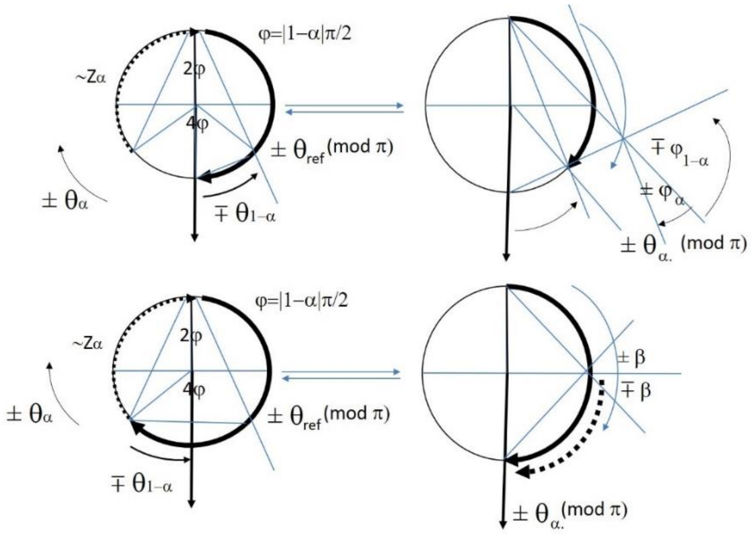

Figure 8 : Schematic representation of the coupling between external tempo (Newton time) and internal tempo ruled by the scaling given via « η ». Zeta Riemann function scans this coupling through the bi partition $\theta_{ref}=\theta_{\alpha}.\log(1/n)$. The factor log(1/n) backs the self-similarity. Correlations are built at resonances, that is to say for all the values of integer partitions. These couple space and the multiples of the phase angles by bringing together the opposite sign rotations. The positive time and the negative time are involved together in the structure. The arrow of time births if both items are different, almost surely (in fact except for $\alpha=1/2$)

As suggested by the zeta formalism it is possible to describe $\forall$ n,m : $\lambda_{ref} \sim \theta_{ref} \sim \theta_{\alpha}$. Log(n), explicit relationship when the external reference clock sets the module (Fig. 8 above). On the other hand, when the internal clock sets the same module we have $\lambda_{int} \sim \alpha\theta_{ref} \sim \theta_{\alpha}$. Log(m)= θ_{α}. αLog(n)= θ_{α}. Log(n^{α}). Let us observe that in the ringed duality considered m and n are linked by the standard Cantor Mandelbrot scaling relation (CM) with $n^{\alpha} \sim 1/\eta$=m. We are then led to observe that que λ_{ref}- λ_{int} $\Longrightarrow \varphi=\theta/2=(1-\alpha)\pi/2$ (Fig.8). This approach which consists in particular of computing in a modular way for discrete values of θ_{ref} et θ_{α} by taking the congruence to the value « π » on the external clock or on the internal clock makes it possible to approach discretization and temporality in a renormalizable environment by understanding, by means of topos engineering, the dual content of the algebra of morphisms which is hidden behind the Space-Time duality or more precisely here Scale-Pulsation. .

We can then show that the notion of the arrow of time is in practice hidden behind the categorical limit behavior of such dual structures. They make time no longer a unique and reversible parameter born from a group of automorphisms associated with the notion of speed (energy) and diffusion coefficient (action), but the result of a weaving of dynamic relationships and correlations in a context imposing multi-scale couplings that, due to the presence of a memory, no longer respond to Noetherian principles. More precisely, this arrow of time is the result of our relationship to the size of the objects in our empirical world. In order to be easily quantified, our analysis first relies on the self-similarity of the structures considered and the k-algebras [VAA19] here indexed on scaling. The arrow of time and the irreversibility of human actions, and therefore of life [BAI09], [CHO23] would arise from the absence of an equivalence relation between monadic limit and and monadic colimit depending on whether the scales are taken in the increasing direction (analysis) or decreasing (synthesis), or even if the physical topos is considered from the concrete process and its algebraic component (analysis) or from the topological completion of this (synthesis). In each case the cohomological treatment of morphisms on complexes of chains involves entropic terms [VIJ10] [BAP15 The key point here is that the Cantor Mandelbrot relation makes this chain a bi-category and that the coupling of the index k either with η or with n results, except in the case α=1/2 (Riemann hypothesis and Martin's axiom) the appearance of a distinction between the two chains always computed in an increasing way while in inverse direction (time plus and time minus). The breaking of symmetries of correlated chains gives rise to tensor discretization jointly involving η or n, therefore in particular contravariant terms which are usefully expressed by means of the universal self-similar zeta function. This leads to simply explaining how time and its arrow live in separate but dual universes and in a way very similar to what distinguishes vector spaces and linear forms in the linear world.

The reason why zeta plays a central role is due to its role aiming the test for the differencing in cardinality between Z_α et $^nZ_{1-\alpha}$, difference similar to that which, distinguishing in the limit $\mathbb{N}$ and $\mathbb{N}$ x $\mathbb{N}$ allowed Cantor to pose, through diagonalization, the hypothesis of the continuum. $Z_{1-\alpha}$ cannot be compared to $^nZ_{1-\alpha}$ but with $\zeta_\alpha(s)$ which corresponds to fibering by means of $\pm\ \theta\ |_\alpha$ (Fig.6 and 7). Notwithstanding the intrinsic foliation/partition of Z_α giving birth to $^nZ_{1-\alpha}$, and $\zeta_\alpha(s)$, we can consider the foliation of $Z_{1-\alpha}$, by means of $\mp\ \theta\ |_{1-\alpha}$ giving birth to $\zeta_{1-\alpha}(1\text{-s})$, which amounts to consider not $\pm$ $\theta\ |_\alpha$ (log(n) therefore the external modulo over extensity (QE) but $\mp\ \theta\ |_{1-\alpha}$ log (m) over intensity (QE) as defined above (Fig.8). In other words we do not count the same quantity on Z_α (quantity of space) and on $Z_{1-\alpha}$ (space of quantities) from which it results that in the general case $\pm\ \theta\ |_\alpha \neq \mp\ \theta\ |_{1-\alpha}$. More precisely the quadratic expected arithmetic consistency $\mathbb{N}$ = $\mathbb{N}$ x $\mathbb{N}$ is ultimately finds for the spectroscopic data since $\eta n^\alpha.\eta n^{1-\alpha} = \eta^2 n$, namely the basic energy definition. Therefore the inside splitting concerning both the transfer function and the zeta function, distinguishing quantity of space and space of quantities points more precisely in fine on the difference within physical behavior between universal complex systems and the system ruled by a pure chance that identifies both variables [THF97].

Mathematical approach of the arrow of time

At this stage we have to considered fundamental mathematical tools to relate local and global. Co-homologycal groups can easily be considered in this aim, and easily imaged of fractal geometry and category theory. These groups require definition of complexes of chains of k-morphisms over k±1morphisms (between points, between point-to-point links, between k-simplexes triangulating topological structure, etc), namely the morphisms matching the links between two levels of lattice and/or levels of graduated partitions of a set E →{P(E)}. The idea is to consider a complex of chains M_* : $\partial_n = M_n \rightarrow M_{n-1}$ based on categories with $\partial_n\ \partial_{n+1} = 0$. We then can consider the quotient groups $H_n(M_*, \partial_*) = \ker(\partial_n)/\mathrm{Im}(\partial_{n+1})$ and for the complex of co-chains M^* : $\partial^n = M^n \rightarrow M^{n+1}$ with $\partial^n\ \partial^{n-1} = 0$ the quotient groups $H^n(M^*,\ \partial^*) = \ker(\partial^n)/\mathrm{Im}(\partial^{n-1})$. Practically these groups can be used to handle the local global relationship, more precisely local → global approach for the co-chains (analytic) and global → local approach for the chain (synthesis). We owe Daniel Bennequin school of thought [BAP15], [VJP10] [COP08-1], for a preliminary analysis of the link between the properties of cohomology and entropy. Due to abelian properties, this analysis leads to the expression of the entropy function in the standard probabilistic form S~ -ln[Pr/(1-Pr)] where Pr is a probability measure. Entropy then appears to any engineer as the logarithm of an occupation of a number of acquired action sites "x_0Pr" (real) compared to the number of action sites still possible (virtual) "x_0(1-Pr)". Considered in the frame of CME and TEISI model, while the renormalization of both variable EQ and QE requires non-linear relationship, abelian hypothesis fails. As we have seen the non-commutativity is almost sure and we can expect that the two probabilities (if probability concept keep its meaning) does not necessarily concern the same entities. We have in particular to consider the TEISI transition from the Mandelbrot geometrical approach dynamic approach involving Cantor dimension $d \rightarrow 1/\alpha$ This transformation implies a generic normalizations for scale η, (QE x) but a non-generic one for the tempo n: (EQ +), because nongenericity related to the root of unity ($\tau(\varepsilon)$: Fig.5). At the limit of the fractal deployment, therefore at infinity, the scale « η (EQ) » measurement support is unitary and the tempo « n (QE) » measurement support has a zero measure. Following Bennequin Vigneaux idea we can consider (η^d,n) as new object in a category and the morphism ∂^+: $n \rightarrow \eta^d$ introduced between ker(∂^n) and Im(∂^{n-1}). The aim is to keep the issue in the framework of tempo along the fractal folding. Therefore considering ∂_n and $\partial_{n\pm1}$ within fractal framework we can build an adjunction of natural transformations between these new "objects", transformations either based on F (for growth) or G (for decay) (Fig. 2). Then the monad is ruled by a functor T=GoF, ToT=T with 1x=G(εF) using the "ε" extensity renormalization factor already pointed out practically : namely $\tau(\varepsilon)$. We can then reduce the homology/co-homology problem to a monadic issue characterized by an intermediate structural level (characterized by the relation η^d.n=1 in Mandelbrot fractal geometry and η.(in)$^\alpha$=1 $^\alpha$ within TEISI dynamic extended framework) aiming to refer only to a natural number relevant stacking. This construction must include behaviors at the monadic categorical limits and colimits. Therein the monad remains the set of morphisms between three different levels taking into account the morphisms between, the countable cardina ω_0, the cardinal of the continuum $\mathfrak{c}=2^{\omega_0}$ achieved via a couple of sub-morphisms T taking into account an intermediate limit or colimit "object" (ω_1), which would be the smallest analytically conceivable set, which is no countable [RIP23].

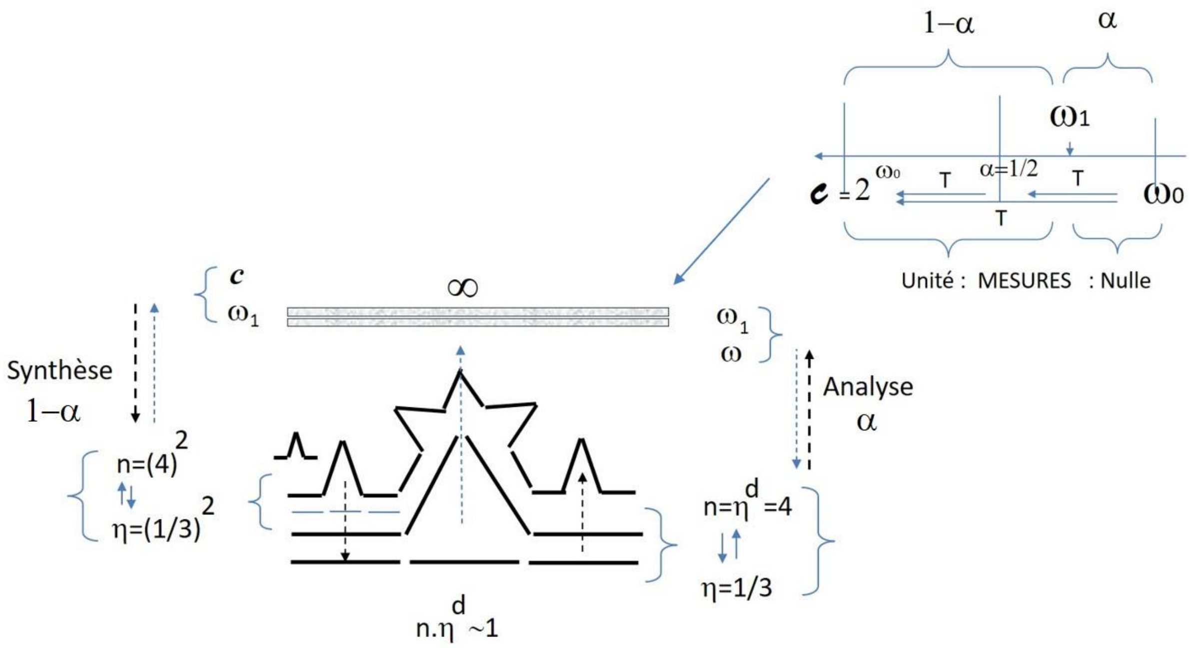

Figure 9: Schematic representation of behavior at the edge of monadic deployment. Given the dual category at stake at the heart of this deployment, the monad acquires at the infinity a status requiring a duality of the factor $\omega_1 \rightarrow \Delta\omega_1$ which depends on the direction of propagation of the monadic deployment. The duality of the local foundations of this entity is shown by the emergence of a global arrow of time, therefore of the principle of causality. $\Delta\omega_1$ is associated to a shaped punctuation boundary.

Due to the relation T^2=T, and given the application $\mathfrak{c}=2^{\omega_0} \hookrightarrow \omega_0$ the position of ω_1 in the final category ensures the "top|down/bottom|up" or folding/unfolding symmetry when ω_1 can be located in the "middle" of the category (a rough reading of D. Martin axiom using monadic spectacles whatever the meaning of "middle"). In accordance with Riot and al. hypotheses [RIP19] and above analysis, the monadic expression of the morphisms attached to α–exponentials only guarantee this "location" when $\alpha=1/2$ (HR~AM [RIP23]). In all other cases, there is a conflict between Riemann or Martin's hypotheses and the categorical limit of the monadic structure for the categories in stake (Continuous, Countable and Boundary). Due to distinct status and relationship between η(measure over α)$|_{1-\alpha}$ (with external co-support) and n (counting, $1-\alpha$)$|\alpha$ (for internal measure support), the monadic approach fails for closing the system at infinity. This obstruction can be expressed through the properties conveyed by the Riemann zeta function seen as a characteristic of a dynamic site backed on the site $Z_\alpha \bigoplus Z_{1-\alpha}$. Since we consider not the site as universal support, but different classified toposes based on ${}^nZ_{1-\alpha}$, ζ_α(s) and $\zeta_{1-\alpha}$(1-s), the bi-orientation of the complex plane is distinguished for classifying if $\beta \neq 0$ and $\alpha \neq (1-\alpha)$ namely, if the angles of phases are such that $+\varphi(\alpha|_\beta) \neq -\varphi\ (1-\alpha|_{-\beta})$ mod $\pm\pi/2$. The requirement for taking into account the non-linearity between space of quantities (η, $1-\alpha$, $\mathbb{Q}_p{}^{top}$) and quantity of space (n, α, $\mathbb{Z}_p$) and vice versa using (η, α, $\mathbb{Q}_p{}^{alg}$) and (n, $1-\alpha$, $\mathbb{Z}_p$) both backed to the site $Z_\alpha \bigoplus Z_{1-\alpha}$ and zeta functional relationship, involves that orientation of the plane is not only due to the fundamental impossibility of establishing a strictly arithmetic epimorphism between $\mathbb{Z}_p$ and $\mathbb{Q}_p{}^{top}$ but due in general, to the lack of common normalization (see definition above) has for consequence: $\mathbb{Q}_p{}^{alg}(\beta) \neq \mathbb{Q}_p{}^{top}(-\beta)$. Epimorphism can only be achieved via an additional operation which, as we have shown above, adjusts of the units to the adjunct fields in stake, so that by referring the analysis to strictly stochastic processes (α=1/2) we can write $\mathbb{Q}_p{}^{alg}(\beta)= \mathbb{Q}_p{}^{top}(-\beta)$. This equality is only ensured β=0, specific values involving the disappearance of the role of the orientation of the complex plane (case of quantum mechanics).

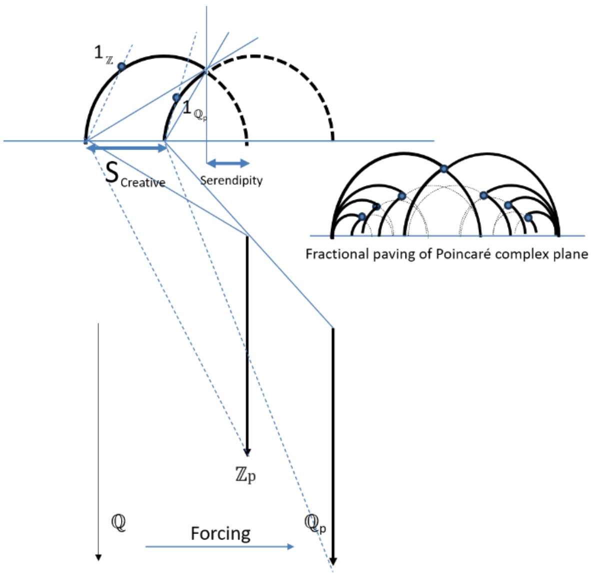

Figure 10: Stone like representation of the difference between Creativity and Serendipity. With respect to Fig.5, we use $Z_\alpha(\iota\omega\tau)$ entropy production symmetry to inverse the location of $Z_{1-\alpha}(\tau)$ and by the way the sign of information direction. Entropy of complex systems stays an extensive entity (QE). Therefore in a close system (real plus virtual) entropy is conservative but due to highlighted duality, negentropy, namely the entropy of information, must be shared between a couple of two entities: (i) Serendipity relate to chance, fitting i-information (complex correlations) over exponential and prime numbers chains (α=1/2). (ii) Creativity obtains using (-i)-information, by forcing the system taking into account the paving and the scaling of the complex plane. The logic of creative is then based partly on non-separable correlations and on projective geometry and partly on the additive structure defined by Stone [SEM13] to think Boolean rings.

By designing a link between the site $Z_\alpha(n) \bigoplus Z_{1-\alpha}(\tau)$ and a ringed site (distribution of cyclic integrals) and starting from the hypothesis of a priori neutrality of this orientation, the zeta function implicitly plays the role of test function with respect to above epimorphisms. Assuming $\pm i\theta = \mp i\theta$, fibering of the site introduces jointly (i) the tempo of a deterministic Newtonian clock given by par $\pm\, i\theta$ mod $\pm\pi$. but also (ii) a modular forms associated with $\Omega \sim \pm\, ik\theta.\log(n)$. This construction gives rise to a foliated variety $M_\alpha(s)$ [RIP19] of which at least one sub-manifold should be isomorphic to $\mathbb{Q}_p^{alg}$ (β). By backing upon the factor $\pm\, i\theta$ as Newtonian tempo, "Ω" responds to a multiplicity of sizes of clock-dials $1/\eta\sim\pi\log(n)$, involving spatial pulsations and resonances between them. These sizes, -linked together by the presence of a phase angle, flagging a constant curvature of the underlying hyperbolic structure - lead the filling of the physical process with a multitude of internal clocks ($\{\tau\}$ and modular forms of which only the resonances (of spatial origin) give rise). Now, for a given constant curvature of the geometry, all of variable sizes cycles are constrained by the presence of an angle of parallelism which, here discretized and infinitesimal in the complex plane, leads to distinguish left from right, upstream from downstream, the cause and the effect etc. This orientation is taken into account in the expression of zeta through the distinction $s\neq 1-s$ and therefore the distinction $M_\alpha(s) \neq M_{1-\alpha}(1\text{-}s)$, but this treatment does not correspond to the physical problem of completion as introduced by α–exponential dynamics. Indeed it presupposes the indifference of the direction of rotation and the arbitrary nature of the notation "s" since $M_\alpha(1\text{-}s') \neq M_{1-\alpha}(s')$ if $s|_{1\text{-}s}=1\text{-}s'|_{s'}$. Moreover zeta scanning gives the same status to Z_α and $Z_{1-\alpha}$ This assumption is physically false and mathematically highly limiting. We can schematize this problem by noting how the ringed varieties are characterized according to the use we make of zeta when we seek to link the sum or coproduct ($\sum$) and product ($\prod$) operators:

Analytic zeta approach : $\pm\,\theta = \mp\,\theta \pmod{\pi} \iff \sum | \alpha \rightarrow \zeta_\alpha(s) \rightarrow \alpha | \prod$ and Idem for (1-s)

Scheme zeta approach: $\pm\,\theta \neq \mp\,\theta \pmod{\pi} \iff \sum | \alpha \rightarrow \zeta_\alpha(s) \| 1\text{-}\alpha | \prod \rightarrow \zeta_{1-\alpha}(1\text{-}s)$

In other words, the analytical approach indifferently associates product and co-product with s or 1-s. The scheme approach, fitting our physical problem, makes a distinction associating s with the co-product and 1-s with the product. The confusion only makes physical sense in the case looking like quantum mechanics, namely when α=1/2 (RH et AM).

We see here why, in all the previous publications of the authors, while pointing out the link between the α-exponential dynamics with the zeta functions, underlined the problems raised by the breaking of symmetry of the orientation of the complex plane for entangled physics problems. Accurate questions are raised well beyond the simple questions of quantum mechanics, namely beyond only the use of von Neumann' algebras [VAA19] . For the first time in our knowledge, engineering requires an analysis based on Grothendieck schemes, ask for the role of the virtual in terms of power and sheds light, in a completely explicit manner, on the topological origin (dual punctuation torus) of the arrow of time. This arrow results in practice from the hyperbolic character of the co-limit punctuation of quantity of space denoted coproduct ($\sum$), open to a dual punctuation attached to the limit space of quantities, denoted product ($\prod$).Figure 10, below gives an schematic indication about the structure of colimit (Initial) and limit (Final) sub-varieties, about the link with the fine triangles irreducible to the tree associated with the hyperbolic spaces [MGr13] and on the fundamental meaning of the arrow of time as an asymmetry of the positioning of ω_1 [AM]with regard to topological duality punctuation

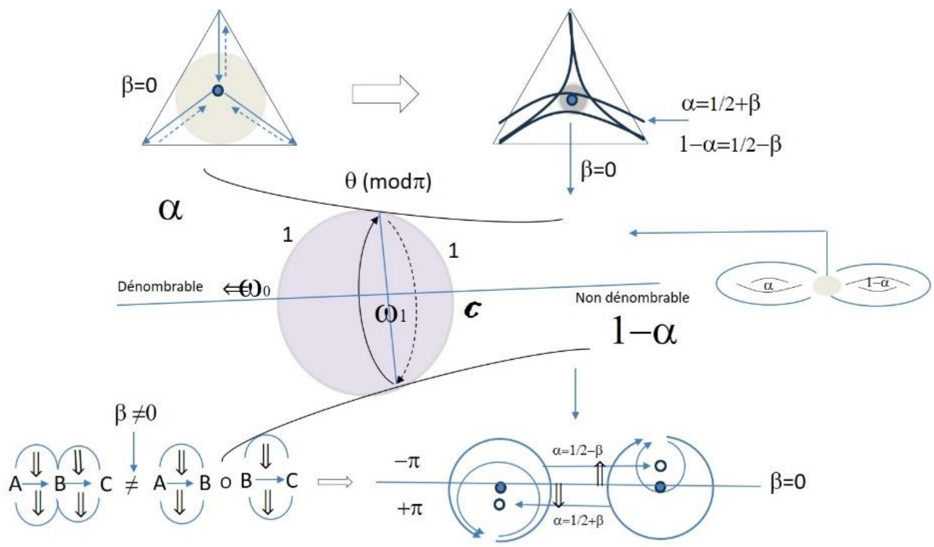

Figure 11: schematic representation of the structure at the edge of a punctuated torus when the punctuation originates from a fractal geometry while the torus corresponds to a set of geodesics of fractional origin. The punctuation is thick $\Delta\omega_1$ and the common boundary obtained via extension of the 2-hole donut structure to infinity, appears as a sphere to match zeta properties. The duality: Algebra (A) versus Geometry (G) manifests itself either by a symmetry of both topologies ($\alpha=1/2$, $\beta=0$) or by an asymmetry ($1>\alpha>1/2$, $|\beta|>0$). The analytical approach corresponds to the A→G approach, while the synthetic approach corresponds to the creative G→ A inverse approachThe completion being imposed for reasons of coherence of action and functioning of the brain, the existence of consciousness corresponds to the brain's grasp of this asymmetry whose origin is monadic thus fundamentally local via a thickening of the "now".

Conclusions

This note is much more a program outline than a fulfilled task. Although it fits, so far back, in the footsteps of Chatelet, [CHG10], Colmez [COP08], Connes [COA17] or Planat [PLM03,06,11] the work to be carried out to ensure the mathematical quality of this preliminary is considerable. It is not up to engineers or even physicists to engage in this way. Therefore, why can we nevertheless assert that here, engineering brings an additional stone to an already very elaborate mathematical edifice [CAO16,17]? While often required by urgency (climate crisis, geopolitical and economic chaos, sociologic stress and migrations, social networks, needs of pedagogical renew, compression of the unit of time, blow up of information amounts and inequalities, confusion of real and virtual, IA cognitive dangers, quantum engineering, etc) and necessity, it seems that management of entangled systems highlights issues whose complexity is far beyond the "solutions" currently mathematically accessible. In addition to the introduction of concepts unfamiliar to the community of physicists (except partly if concerned by quantic physics) and even more so to that of engineers, it seems to us that access to the concepts of fibers, sites, topos, scheme, motives, etc, here seen through spectra of fractional operators, presents several advantages. The first is that it physically features issues which until then remained abstract. The geometries hidden behind the scheme concept and ringed space have real experimental content especially if related to the concept of entropy, chaos control and the improvement of engineering devices [TOD87]. These last are always far from equilibrium or even, in mechanics Noether principles [KSY04], ergodic context [GAG94] and in quantum statistics, KMS conditions [COA17], [RIP17b]. Mathematicians could use even the defects of this note, because they are the ones whose for decades, justified and guided our laborious applied research up to new zeta fractional approach [ALM10]. The universe of engineers, sociologist or even politics is always crumpled, rumple, strained, curved, entangled and its data more often far from understanding. It is irrelevant to smooth them as most standard physical approaches do [LIU85], [SAB97], including within self-similar frameworks which seems very universal. Engineering issues might guide us as shown above. The hyper complex materials are mainly made up of objective or virtual holes, voids, but full of fields. Engineers need them to balance Action (L2t-1), Energy (L2t-2) and Power (L2t-3), namely the control of velocity of operational exchanges of extensive entities (QE) transfer efficiency when energy stored is transfered. The materials required to promote power are convex, related, dense but full of defects. To summarize,the geometry of action is mainly fractal. Affirming a self-similarity seems to be a simplification which is not only due to our cognitive limits but to the way in which energy/power balancing operates in the absence of any reference to the size of the object considered. This is at least what is confirmed by the universality of the empirical data attached to it and even more so by the emergence of self-similar objects under the effect of the irreversibility of physical processes [SAB97], and furthermore via the arrow of time. The relationship between certain form of forcing, innovation [HAA13], creativity [CHG10] and open self-similar structures seems unavoidable [RIP19,23]. Obviously, these relationships must be related to irreversibility of time and to entropy.

Whether this notion is approached through irreversible physics or from the Algebra-Topology duality, none of the cognitive issues can be detached from the velocity of entropy output. This last notion expresses the impossibility to make an analysis of the real fully self-coherent without any introduction of "virtual"[CHG10] namely the openness. At least at infinity, circular reasoning necessarily unveils a flaw, a groove, a rib, a cut. Any Baron of Münchhausen would be powerless to extricate himself alone from it. He needs at least otherness! It is this " initial/final fault" which gives the conceptual value to the notion of α-exponential; this distribution provides a local representation of this fault. It opens the way to a new class of tiling of the complex plane. In this regard, the empirical transfer functions of Cole and Cole multiply the heuristic value of our models hugely by illuminating and compacting our relationship to infinity and by pointing out one more time, that several classes of infinities co-exist. The concept of schema obviously does not ignore their existences any more than does number theory after Cantor, but the notion α-exponential and its applications highlight their self-similar multiplicity, in particular by giving a role to all of the p-adic numbers and quotient rings on which a non periodic tiling can be based.

It is they who project the constraints associated with the Context (geometry) / Process (Algebra) coupling onto an irreducible basis of cycles (integrals) whose combination represents not only the universe but its scaling handling (or approach via inclusion order [ROG64],[ROC12,17]) that we can have of it. Playing with the size, the different types of scanner devices are implicitly at least ,the expression of this approach . Because the sizes must be connected to each other, the use of the concept of ring mathematical concept is required. However, the approach remains open and the topologies involved "clopen" set. The schemes over sites contains non-closed points and this is precisely the origin of the need to consider projective spaces and not just affine ones. It remains to be understood why, within very entangled systems, annelation, although linked to all prime numbers and zeta fnction [PLM03,06,11], is not generally a matter of pure chance as it is the case in quantum mechanics (a simple special case). This specificity obviously originates from the difference brought by the quadratic self-similarity due to arithmetic $\mathbb{N} = \mathbb{N} \times \mathbb{N}$ ($\alpha = 1/2$) and the geometrical open self-similarity imposed when the factor $\alpha \neq 1/2$. It is with this gap that the Cole and Cole transfer functions and their completion are seen as a Topos, abstract place where something happens; as well the Riemann zeta function seen as an expression of a symmetric arithmetic site among the simplest. All of this points out the need for an extended program of understanding of the role of the “virtual” [CHG10] and forcing in physics. The zeros of the zeta function [BYT95],[COA14] formalize the particular case where geometry and its algebra become self-dual and creativity becomes serendipity. In this context of openness, the time of creativity is in no way the time of the Newtonian clock of lean management. The creative time would rather looks like a weaved multitude time-ornaments [WOF23] or symmetries [JEF19], enriched by the sense, the tonic, the harmony the timbres, the resonances [ADT62], the "je ne sais quoi" [JAV83], which, based on silences [LIG62] [POY23], makes music the discrete set of clocks of our consciousness, and the soul of the arrow of time ("what remain in music when we remove all the components of the musicality" [WOF23]) .

To conclude, we can affirm that the best image of the problem raised by fractional operators is undoubtedly the way in which music sculpts the temporal component of our relationship to the world [JEF19], [WOF32]. Certainly, our common metronome imposes an analytical tempo which is intended to be universal. He is so. However, this reference does not induce belief in the paradigm of modernity: the notion of velocity/derivative/efficiency/ergodicity [GAG94], [SEC90]. The postmodern relationships with the world require more advance concepts [CHG10],[COA90],[DEP77],[GOF37,45],[JEF19],[SEC02] Standard paradigms confer, without our being aware of them particularly a central role to the notion of energy. Ignorance of Noether's principles leads even the most knowledgeable minds to make memory and energy coexist without care while they require a Isbell duality [VAA19]. It must be said in their defense that this cohabitation is possible when chance is considered the sole source of creativity, let say serendipity. Hence the essence of modern physical thought which makes quantum mechanics the ultimate in the complexity of the world. However, this is only the tip of an iceberg of ignorance which, from biology to human intelligence, is confronted not with analytical complexity (local to global) but with intuitionist complexity (global to local) shaped by the competition between classifyong toposes [CAO17]. Our relationship with time is the clinical sign of cognitive capacities trapped by the indicated paradigm. Music is, on the contrary, the expression of an ordered resistance to unspeakable limitations hidden within implicit space topologies pointing out the meaning of a "je ne sais quoi" (JAV83).

It is through self-similarity and through the monadic expression that is imposed to the dynamics that the limits and co-limits attached to our approach to reality are summoned locally. they are expressed physically through the notion of entropy which we could dare to write in the form S=-log($\alpha/1-\alpha$) or even more formally S=-log(∑(A) / Π(G)). This concept which can be written in the categorical framework is related to cohomological approach for entangled set closure obstructions. However, things are more subtle if we want to distinguish true creativity from serendipity. In the framework of the theory of n-categories, long-distance physical correlations are expressed through morphisms and natural transformations whose limits and colimits take on a generic identicalness status. Now the link between these notions and the notions of entropy can be expressed very simply on the real axis of the complex representation of the α-exponential. As has been shown, the reference point is pure chance. In this case, the entropy of the process is strictly equal to the negentropy and the possible states are purely chance. There is no creative process and any action is a stochastic one. The only degree of freedom is the deviation of the 2D self similar metric. In the case of compact chance only this deviation is possibly creative. We call it serendipity. This remark leads us to understand the meaning of the factor "α". It unbalances entropy and negentropy and it leads to the affirmation that anti-entropy, that is to say the creative potential, does not relate to the function log(S/1-S) as we would have imagined by following the modern thought, but of the factor Log(S(1/2)/1-S(1/2)) that is to say precisely the deviation from chance. This gap can also be expressed through the zeta function, not by considering the zeros, but by considering the functional relation which expresses the gap in question, and further the irreversibility of time when it is expressed at average of the "iθ" factor. The arrow of time is not an illusion but the result of the monadic character of our relationship to the world which requires that we take into account the continuous (the fields) jointly with the countable (of singularities), a conjunction which imposes the emergence of the notion of temporality as a non-linear distinction between space and time, that is to say a disappearance of the notion of velocity and a thickening of the "now". The attempt of modern physicists to reduce our relationship to the world to the use of the resulting energy is then defeated and it will be up to contemporary thought to take the necessary cognitive step forward to approach physics. entangled complex systems which undoubtedly constitute the greatest challenge facing future sciences and policy [DEG15].

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Références :

[ADT62] Adorno Theodor, Philosophie de la nouvelle musique [« Philosophie der neuen musik »] Gallimard (1962) , Paris, coll. « Tel » (nº 42), 1962 (1re éd. 1948 (de))

[ARH78] Araki, H.: *On KMS states of a C∗-dynamical system. In C∗-algebras and Applications to Physics*, R.V. Kadison editor. Lecture Notes in Mathematics 650 . 1978, Springer, New York

[BAG82] Bagchi G. (1982) *Join universality theorem for Dirichlet L Functions*, Math. Z. 181, 319-334.

[BAI09] Bailly François, Longo Giuseppe, *Biological organization and anti-entropy* , Journal of Biological Systems, 17 (01) (2009) 63-96,.

[BAP15] Baudot P and Bennequin D, *The homological nature of entropy*, 17 (2015) 3253-3318

[BEM99] Berry Michael and Keating Jonathan, *The Riemann Zeros and Eigenvalue Asymptotics* , SIAM Review, 1999 (41), 236-266.

[BEN03] Berline Nicole and Sabbah Claude, *La function zêta* Edition Ellipses (2003) Paris

[BHP15] Bhattacharya Prasit, *The p-adic integers as final coalgebra*, arXiv :1504.01408v1 [MathNT]20 Mar 2015 (see also Bhattacharya P., Moss L., Ratnayak J. and Rose R., *Fractal set as final coalgebra by completing an initial algebra*)

[BLD05] Blanco-Laserna David, *Emmy Noether matemàtica ideal*, Nivola Libros y Editiones (2005) Madrid,

[BON68] Boccara Nino, *Les principes de la thermodynamique classique,* PUF (1968) Paris.

[BRC23] Bruter Claude, *The Principle of Stability, within the pantheon of mother ideas*, Cambridge scholars publishing (2023) Cambridge (ISBN13:978-1-5275-6857-0).

[BRL56] Brillouin Léon, *Science and Information Theory*, New York, Academic Press, 1956, 1962, 2013, (ISBN 978-0-486-49755-6)

[BYT95] Bytsenko A.A., Cognola G., Elizade Emilio, *Ten Physical Applications of Spectral Zeta Functions*1995 Springer

[CAO16a] Caramelo Olivia, *Les topos de Grothendieck comme ponts unifiant en mathématique*. Exposé IHES Bures sur Yvettes 14/12/2016

[CAO 16b] *Théorie topologique de Galois*, Advances in Mathematics 291 (2016) 646–695

[CAO 17] *Theories, Sites, Toposes : Relating and Studying Mathematical Theories Through Topos-Theoretic* bridges, Oxford University Press (2017) Oxford. (see for instance www.bing.com/videos/riverview/relatedvideo?&q=olivia+caramelo+&&mid=713FD860D4ECE6D264F4713FD860D4ECE6D264F4&&FORM=VRDGAR)

[CHG92] Choquet Gustave, *Cours de topologie*, Masson Paris 1992

[CHG10] Chatelet Gilles, *L'enchantement du virtuel*, Edts Charles Alunni et Catherine Paoletti Edition de la rue d'Ulm (2010) Paris

[CNM23]Chollat-Nami Mrie et Longo Guiseppe, Entropie, *Négentropie et anti-entropie, Le jeu des tensions pour penser le vivant*, Lîla-Entropy Project Entropie Vol1 (2023)

[COA90] Connes Alain, *Géométrie non commutative*, Interedition (1990) Paris.

[COA96] Connes alain, *Formule de trace en géométrie non commutative et hypothèse de Riemann*, C.R. acad. Sci. Paris 323-1 (1996) 1231-1236

[COA13] Connes Alain, Dixmier Jacques et Chéreau Danye, *Le théâtre quantique* : *l'horloge des anges d'ici bas* Odile Jacob (2013) Paris

[COA14] Connes Alain, *Arithmetic site*, C.R. acad. Sci. Paris 352-1 (2014) 971-975

[COA17] Connes Alain, *Mécanique Statistiques quantique des* $\mathbb{Q}$*-réseaux*, Cours du Collège de France, Analyse et Géométrie (2017) Paris.

[COA19] Connes Alain, *Un topo sur les topos* IHES Bures sur Yvettes 18/05/2019

[COA22] Connes Alain and Caterina Consani , *Cyclic theory and the pericyclic category* , arXiv,2208-08339,1 [Math AT]17 Aug.2022

[COJ03] Conrey J.B. *The Riemann Hypothesis*, Notices of the AMS 50 (2003) 341-353

[COP92] Coveney Peter and Highfield Roger, *The arrow of time* (Introduced by I. Prigogine) Fawcett Colombine (1992) New York.

[COP08] Colmez Pierre, *Représentations de* $GL_2(Q_p)$ *et (j, G)-modules*, 216 pages, à paraître à Astérisque, http ://people.math.jussieu.fr/ colmez/publications.html.

[DAZ00] David Z Albert *Time and Chance*, Harvard University Press (2000) Cambridge MA

[DEP77] Deligne, Pierre (1977). *Séminaire de Géométrie Algébrique du Bois Marie – Cohomologie étale – (SGA 4½). Lecture notes in mathematics (in French). Vol. 569. Berlin; New York: Springer-Verlag. pp. iv+312. doi:10.1007/BFb0091516. ISBN 978-3-540-08066-4.*

[DEG15] Duffuant Guillaume et neuf autres membres du Comité National des Systèmes Complexes, *Visions de la complexité. Le démon de Laplace dans tous ses états*, Natures Sciences Sociétés 23 (2015) 42-53

[DOA09] Döring Andreas, *Topos and 'Néo-Realist' quantum Theory*, Quantum Field theory, B. Fauser, J. Tolksdorf and E Zeidler Eds 25-47 (2009) Birkhäuser Verlag Bale.

[DOG09] Gabriel Dospinescu math.ens.psl.eu/shared-files/10543/?dospinescu.pdf (voir également Autour de la correspondance locale de Langlands p-adique pour $GL^2(Q_p)$ Les cours Peccot, Edition Spartakus Paris 2015 et thèse de doctorat mathématique X 2012).

[ELW75] Ellison William John et Mendès France Michel, *Les Nombres premiers* Ed. Hermann(1975) Paris

[ELI94] Elizade E., Odintsov S.D., Romeo A., Bytsenko A.A., Zerbini S., Zeta *Regularization Techniques with Applications* 1994 World Scientific

[ELI03] Elizade E., Moretti V., Zerbini S., *Analytic Aspects of Quantum Fields* 2003 World Scientific

[FRM70] Von Franck Marie-Louise *Nombre et temps: psychologie des profondeurs de la physique moderne*. Ed. la fontaine de pierre (1970) Paris

[GAA19] Alina. Gavrilut, Ioan. Merches, M. Agop, *Atomicity through fractal measure theory*, Springer, Zurich, 2019

[GAG94] Gallavotti Giovanni, *Ergodicity, ensembles, irreversibility in Boltzmann and beyond*, arXiv,chao-dyn/9403004 10 Mars 1994

[GOF37] Gonseth Ferdinand, *Les mathématiques et la réalité*. *Essai sur la méthode axiomatique*, Edition Alcan (1936) Paris

[GOF45] Gonseth Ferdinand, *La géométrie et le problème de l'espace*, Edition du Griffon (45-55) Neuchatel, CH.

[GRG68] Granel Gérard, *Le sens du temps et la perception chez Husserl*, NRF Gallimard (1968) Paris

[GRM13] Gromov Michael, *In a search for a structure* (2013) Entropy (part 1)

[GUT90] Gutzwiller Martin *Chaos in classical and quantum mechanics*, 1990 Springer.

[HAA13] Hatchuel Armand, David Albert and Laufer Romain, *The new foundations of management science*, Presses des Mines (2013) Paris.

[HOC98] Houzel Christian, *Histoire de la théorie des faisceaux*, Publication de la Société Mathématique de France (1998) Paris

[JAV83] Jankelevitch Vladimir, *La musique et l'ineffable*, Essais Le point, Le Seuil (1983) Paris

[JEF19] Jedrzejewski Franc, *Hétérotopies Musicales : modèles mathématiques pour la musique, Ed.* Hermann (2019) Paris

[KEJ00] Keating J.P., Snaith N.C., *Random matrix theory*, Commun. Math. Phys., (2000), **214**, 57–89

[KLE07] Klein Etienne, *Le facteur temps ne sonne jamais deux fois*, Flammarion(2007) Paris

[KSY04] Kosmann-Schwarzbach Yvette, *Les théorèmes de Noether*, Ed. De l'Ecole Polytechnique, (2004) Saclay.

[KUT85] Kuhn Thomas *La structure des révolutions scientifiques* Flammarion (1983) Paris.

[LAW97] Lawvere William and Schanuel Stephen *Conceptual mathematics : a first introduction to categories* Cambridge University Press (1997) Cambridge.

[LAW12] Lawvere William, *Categories of space and quantity , the space of mathematics,* Eds Javier Echeveria, Andoni Ibassa, Thomas Marmam, de Gruyter (2012) 14-30

[LEH04] Lebesgue Henri, *Les lendemains de l'intégrale* (lettres à emile Borel) Vuibert (2004) Paris.

[LET13] Leinster Tom, *Codensity and ultrafilter Monad*, arXiv : 1209.3606v3 [Math.CT] 10 Jul 2013

[LET14] Leinster Tom, *Basic category theory* Cambridge University Press (2014) Cambridge.

[LET22] Leinster Tom, *Entropie and diversity axiomatic approach arXiv* 2012.02113v3[q-bioPE] 22 oct 2022

[MLS71] Mac Lean Saunders, *Categories for the working mathematician* Springer (1971) London

[lIG62] György Ligeti, *Poème symphonique pour 100 métronomes* :
https://www.youtube.com/watch?v=ZcBxNJKZa-A

[LIU85] Liu S.H. *Fractal model for ac response of a rough interface* Phys Rev. Lett. 55 (1985) 529-532

[LMA82] Le Méhauté Alain and al. *Transfert d'énergie sur interfaces autosimilaires*, C.R. Acad Sciences Paris (1982) Paris

[LMA83] Le Méhauté Alain and Crepy Gilles *Introduction to transfer and motion in fractal media : the geometry of the kinetics* , Solid states Ionics, 9-10(1983) 17-23

[LMA84] Le Méhauté Alain *Transfer process in fractal*, J. Stat. Phys. 36 (1984) 665-670

[LMA91] Le Méhauté Alain, *Fractal geometry and applications*, Penton Press (1991) Londre

[LMA95] Le Méhauté Alain, Nivanen Laurent et Nigmatullin Raoul, *Les flèches du temps* Ed. Hermes (1995) Paris

[LMA10] A.R.P. Le Méhauté, A. El Kaabouchi, L. Nivanen, "*Riemann Conjecture and fractional derivatives*", J. Computers in Maths with Applications, 59 (5), 1610-1613, 2010.

[LMA20] Le Méhauté Alain, Héliodore Frédéric, *The history of the invention of Lithium Batteries (1975-1985) : an example of Zeta mamagement*. Hyperion International Journal of econo-physics and modern economy , (2020) Bucarest.

[LMA22] Le Méhauté Alain, Riot Philippe, Gavrilut Alina, Tayurkii Dmitrii, beyond Entropy : *the zeta function International Conference on mathematical analysis and applications in Science end Engineering* Porto Portugal 2022

[LOG11] Longo Guiseppe, *Incomplétude*, Traduction de l'italien (dans 'la Mathematica') en français pour les mathématiques editions CNRS (2011) Paris.

[MAB77] Mandelbrot Benoit, *Fractal Form Chance and dimension*, Freeman (1977) San Francisco

[MAB83] Mandelbrot Benoit, *The fractal geometry of nature*, Freeman (1983) San Francisco

[MOA71] Moles Abraham, *L'art est L'ordinateur,* Casterman (1971) Paris

[OlS74] Oldham K.B. and Spanier J., *The fractional Calculus*, Acad. Press New York (1974) N.Y.

[OUA83] Oustaloup Alain , *Systèmes asservis d'ordre fractionnaire Théorie et pratique* (1983) Masson PARIS

[OTS05] Le Méhauté Alain, Oustaloup Alain, Trigeassou Jean Claude and Sabatier Jocelyn, *Fractional differentiation and its applications* U Book on deman (2005) ISBN 3-86608-026-3

[PES06] Perrine Serge, *Approche catégorique des correspondances de Galois*, Cours de 3ème cycle Institut Supérieur des Matériaux et Mécaniques 21-22 Mars 2006 Le Mans

[PLM03] Planat Michel ArXiv:quan-ph/0304101v217 jun2003

[PLM06] Planat Michel Huygens, Borh, *Riemann and Galois phase locking*, International Journal of modern physics B 20/11,12,13 (2006)1833-1850

[PLM11] Planat Michel, Solé Patrick and Omar Sami, ArXiv: 1012-4665v3 [math-ph] 16 fev 2011

[POY23] Pogorolich Yvo, Ivo Pogorelić — Wikipédia (wikipedia.org)

[RIP17a] Riot Philippe et Le Méhauté Alain, *Autosimilarité Fonction zêta et conjecture de Riemann* Revue REE Vol 1, Paris 2017

[RIP17b] Philippe Riot and Alain Le Méhauté, *From the arrow of time in Badiali's quantum approach to the dynamic meaning of Riemann's hypothesis*, Condensed Matter Physics (2017) Vol. **20**, No 3, 33001: 1–13.

[RIP19] P. Riot,, A. Le Méhauté, D. Tayurskii, *Fractional Dynamics, Riemann Zeta Function and Self Similarity through Category Theory*, in Advances in Special Functions and Analysis of Differential Equations (Chapter 14) Agarwal Praveen, Agarwal Ravi P., Ruzhansky M. Editors. CRC Press Deleware, 2019

[RIP23] Riot Philippe *La fonction zêta ou l'ombilic des mathématiques pour traiter de la complexité*, Lîla-Entropy Project, Vol 2, Entropie (à paraitre) Paris

[ROC12] Rovelli Carlo, *Et si le temps n'existait pas ? Un peu de science subversive* , Ed. Dunod (2012) Paris

[ROC17] Rovelli Carlo, *L'ordre du temps*, Flammarion (2017) Paris

[ROG64] G. C. Rota *Foundations of combinatorial theory* I Z. Wahrscheinlichkeit theo. Verwandte Geb. Z. (1964) page 340 - 368

[SAB97] Sapoval Bernard, *Universalité des fractales* Flammarion (1997) Paris

[SAD17] Sagnier Aurelien, *An arithmétic site of Connes-Consani type for imaginary quadratifs fields with class number one*. ArXiv:1703.10521v2 [Math.NT] 9 may 2017

[SAJ20] Sabatier J., Farges C., and Tartaglione V. *Models for modeling long memory behaviour* Mathematics 8 (2020) 196-210

[SCL50] Laurent Schwartz *Théorie des distributions*, Hermann, 2 volumes, 1950/1951[26], new edition 1966

[SCO86] Scofield J.H. *Comment on fractal model for ac response of a rough interface* Phys Rev. Lett. 56 (1986) 268-272

[SEC91] Series Caroline *Geometric Methods of Symbolic Coding* In *Ergodic Theory, Symbolic Dynamics and Hyperbolic Spaces* Eds C. Series, T. Bedford et M. Keane , Oxford University Press, 1991.

[SEC02] David Mumford, Caroline Series and David Wright *Indra's Pearls* . Cambridge University Press (2002) Cambridge UK.

[SEM13] Serfati Marcel., *Sur l'abstrait et le concret en mathématiques et l'axiomatique dans l'œuvre de Marshall stone*. Notae Philosiphicae Formalis, 2(2) (2013) 175-191

[SID20] Sibony Daniel, A la recherche de l'autre temps, Ed. Odile Jacob (2020) Paris

[SIG19] Sierra German, *The Riemann Zeros as spectrum and the Riemann hypothesis*, Symmetry 11-494 (2019) 1-27

[SML13] Smolin Lee, Time Reborn, Hougthon Mifflin Harcourt (2013) Boston

[STJ13] Steuding Jörn, *Ergodic Universality Theorems for Riemann Zeta Function and other L-Functions*, Journal de théorie des nombres de Bordeaux 25(20013)471-476

[TAM70] Takesaki M., in *Tomita's theory of modular Hilbert algebra and its applications* , 1970 Berlin Springer

[THF97] Thierry François, Claude Simon : *une expérience du temps*, Ed. Sedes (1997) Paris

[TOD87] Tondeur Daniel and Kvaalen Eric, *Equirepartition of entropie Production An optimaly criterion for transfer and separation processes*, Industrial Engineering and Research (1987) 26 (1) 50-56 DOI: 10.1021/ie00061a010; see also : *Optimisation thermodynamique: avancées, défis et verrous*, Communication colloque SFGP 2003.

[TRC03] Tricot, Claude, *Courbes et dimensions fractales*, Springer (2003) New York

[TRC08] Tricot, Claude, *Géométrie et Mesure fractales*, Ellipses (2008) Paris

[TRB11] Trautwein Samuel, Röder Esther, Barozzi Giorgio, *Topological properties of Zp and Qp and Euclidean models*; https://www2.math.ethz.ch/education/bachelor/seminars/hs2011/padic/report 5, pdf

[VAA19] Valence Arnaud, *Montrer et démontrer : la dualité d'Isbell ou les voies à doubles sens des raisonnements scientifiques*, Edition de la Sorbonne (2019) Paris

[VJP10] Vigneaux Juan Pablo, *Coefficients multinomiaux Cohomologie . Entropie Faisceaux, Théorie des Information, Théorie de l' Statistique non-extensive Théorie des types topos* PhD Paris Sorbonne (2019)

[VOS75] Voronin S. *Theorem of universality of the Riemann Zeta function*, Izv. Akad. Nauk. SSSR, 39, 475-486; Reprint Maths, URSS Izv, 9, (1975) 443-455 .

[WOF23] Wolff Francis, *Le temps du monde*, Ed. Fayard (2023) Paris

ANNEX : **Musique . Ring Spaces, Temporality and Arrow of Time**

The easiest practical approach of above analysis is probably to consider the musical science [IIG62] [ADT62] [MOA71] [JAV83] [KUT85] [CHG10] [JEF19] as a Grothendieck topos matching above issues. The French philosopher Francis Wolff states in his book " le temps du monde" (World Time) [WOF23], "pure time is what remains when we have removed from music what makes musicality" and he add, " …what remains of the world when its intelligibility has been removed". Time then appears as the result of a screening [CAO15-17]. In accordance to above analysis and as shown by musical engineering, this screening is first of all spectral. Time is in fact not a dynamic imperative. As shown by α-exponentials dynamics, there exist concrete dynamics without time parameters; they are not, however, without temporalities as shown by the stacking structure of ${}^{n}Z_{1-\alpha}$ when seen as an expression of a parametrization ring spectrum (see above). The screening (filtration) gives rise to a field of temporalities different from $\mathbb{R}$, more rich more complex. Like music, timeless α-dynamics can be represented by their spectroscopic components (frequencies, correlations, superpositions, intersections, resonances, etc.). The approach proposed above, in particular by the bi-partition $n=\omega\tau$, then overlaps the 3 main components of "Wolff's Philosophical Time [WOF23]"; Succession (1), permanence (2) and simultaneity (3) can easily be formalized spectrally in the following way: succession ($\rightarrow\partial_n E\rightarrow$: $n\hookrightarrow n+1$), permanence ($n=w_1\tau_1= w_2\tau_2$) simultaneity ($n_i=w_i\tau$, et $n_j= w_j\tau$). Based on fractal geometries, the dynamical topos (event-driven by definition) $Z_\alpha \bigoplus^n Z_{1-\alpha}$ is, in this context, a co-limit since the set $\mathbb{N}$U{0}, that is to say the parameterization ring $\mathbb{Z}$ has been completely analyzed. The field of temporalities is based on a dual structure (Fig.8) which admits an accumulation, an extension and not just a simple translation (interoperability).

Let us eliminate, as F. Wolff suggests, all the arithmetic correlations, there remain 4 "edges": the anterior/posterior in the sense of succession; a "thick-now" the now here thickened first in the frame of the extended CM relation $[\eta.(in)^\alpha=1^\alpha]$ and second in the sense of a dual multiplicity on octave ($\mathbb{N}=\mathbb{N}$ x $\mathbb{N}$), namely the concentration of all the information foliated virtual within a single octave since $\forall$ m $\in \mathbb{N}$: $m\alpha\log(n)=m\log(1/\eta)$ finally tree structure of correlations related to the role of $\mathbb{Q}p$ associated with α–self-similarity in place of $\mathbb{R}$. At no time was time considered here, however the practical analysis was based (i) on a spectroscopically multi-thickened now, and (ii) on the generally non-linear link between an approximation size of the details of the world (η) and a number which is not a time but an externally standardized pulsation. In this spectral context, the posterior (trivial) and the elsewhere (virtual) appear according to distinct but dual statuses: the anterior as Algebra a+b$\Longrightarrow$c (History: in the course of the river of time the past comes to us according to Wolff) and elsewhere as Topology on the set of possible partitions of our measurements (zero or infinity close to any open singular event; choices, bifurcations, catastrophes at any point) dual to Algebra. Let us observe that there is no other choice than chance if Topology and Algebra fits faithfully and completely the standard arithmetic ($\mathbb{N}=\mathbb{N}$ x $\mathbb{N}$, $\alpha=1/2$). ; in fact the effect of geometric self-similarity disappears and only the distribution of prime numbers can justify foliation in $\mathbb{N}$. In the latter case, the ½-exponential has an inverse Fourier transform. The quadratic self-similarity above combines the Pythagorean theorem with temporal linearity. Noether's principles ensure, among other things, the invariance of a quantity called energy (ML^2t^{-1}) which itself gives meaning to the local velocity whatever the geodesic considered, hence the emergence of a unique variable like automorphism parameter. Time is then born, as A. Connes suggests, from spectral analysis if energy is considered as an invariant (KMS theory [RIP17]).

Conversely, in the general case there is no adhesion because $\alpha \neq 1-\alpha$. It is at this stage that the role of the Riemann zeta test function takes on its full physical meaning. It scans in $\mathbb{R}$ the topos as bridge between dynamic Algebra and Topology in discrete complex plane $\mathbb{C}$. The current point at stake takes $\eta \sim n^{-\alpha}e^{-i\theta_\alpha \log(1/n)}$. soit encore $\eta \sim n^{-\alpha}e^{-i\theta_{ref}}$ avec $\theta_{ref}=\theta_\alpha.\log(1/n)$. We are then led to admit that there exists a reference "angle" $\pm\,\theta_{ref} \in \mathbb{R}$ associated with a clock matching the complex plan. Notwithstanding the value of "n", this clock external to the process considered has a dial of unit diameter which determines a reference pulsation including $\forall\,\theta_{ref} \in \mathbb{N}$, the partition leads to a pulsation $\theta_\alpha \in \mathbb{N}$ and to a singular tempo log (1/n). The latter can be seen as the radius of the dial of a clock internal to the process (Fig. 8 a and b). We will observe that at this stage the direction of rotation is not only indifferent but more widely included in the overall behavior. In fact, the parameterization of the arc Z_α is symmetrical with respect to the pivot denoted "1" and log(n) and log (1/n) play the same role. In the category of hyperbolically parameterized dynamic arcs limit and co-limit play the same role except for inversion: lim (∞)=1/co-lim(0). In a self-similar α–dynamic system ($1>\alpha>1/2$), all succession is coupled to a deterministic and organized dispersion, i.e. leads to anti-entropy. We have seen that by taking pure chance as a reference $\alpha_0=1/2$ we reveal a symmetrical parallelism angle « $\pm\beta\pi/2$ » depending on the direction of rotation. We will however observe from the mathematical form of the zeta function that, given that $\alpha\ (\pm) \rightleftharpoons [1-\alpha](\mp)$ the sign of the power term which changes depending on whether we consider log(n) or log (1/n) does not have the same physical meaning as the change in direction of rotation of the complex plane.

Thus as described by philosophers [GRG68],[WOF23] the musician does not sculpt time as is very commonly claimed, but he sculpts white noise. It is in relation to the disorder of white noise that music positions itself, by distinguishing itself from the chaos of sound [lIG62]. It turns out that incidentally this noise contains a time. What music creates precisely is a universe capable to classify toposes, as artistic works that does not contain time. The syntax (a language) that the music sets up then contains its own semantics because this syntax, built on a spectral field of scaling of temporalities is dualy oriented. The syntax gives birth to an arrow of time (a meaning), therefore an outside, a virtual, therefore a meaning from which its semantics emerges. Language not only precedes meaning here but, as consciousness perfectly perceives it, gives it its human substance.